

Macroeconomic Priorities and Crash States*

Abstract

This paper reproduces Lucas's analysis of the costs of business cycles in an economy with a low probability, crash state in consumption growth. For reasonable parameter values, it is shown that the presence of a crash state dramatically increases the costs of consumption volatility. Specifically, for relative risk aversion around 5, households in the US economy would, in aggregate, pay over \$60 billion (approximately 3% of consumption in 2001) to eliminate consumption uncertainty. The conclusion is that stabilization policy is important not for its effects on second moments but in reducing kurtosis by lowering both the probability and severity of a crash state.

- *JEL Classification: E1, E32, E6*
- *Keywords: business cycles, crash states*

Kevin D. Salyer
Department of Economics
U. C. Davis
One Shields Ave.
Davis, CA 95616-8578

Contact Information:

Phone: (530) 752 8359; e-mail: kdsalyer@ucdavis.edu

*I thank Tim Cogley, Oscar Jorda, and Katheryn Russ for helpful comments and suggestions.

1 Introduction

In an elegant and dramatic demonstration, Robert Lucas, Jr. (1987) argued that the welfare costs of consumption volatility, as exhibited in post-war U.S. data, are extremely small - for reasonable values of risk aversion, the typical household would pay much less than one percent of their annual consumption to eliminate **all** volatility. This result clearly posed a challenge to economists involved in stabilization policy analysis since the cost of the paper and ink involved in such work, not to mention the conferences, likely exceeded the potential benefits to society. Not surprisingly, there were many responses to this challenge and these were in large part motivated by the simple environment that Lucas used for his demonstration. Notably, he assumed a representative agent with time-separable preferences. Papers have been introduced that permit non-expected utility (Tallarini (2000)) while others attempt to model the asymmetric distributional effects on household consumption caused by business cycle activity (Krussell and Smith (1999, 2002)). In his 2003 American Economic Association Presidential Address (given sixteen years after his original demonstration), Lucas (2003) reviews this literature and concludes, “I argue in the end that, based on what we know now, it is unrealistic to hope for gains larger than a tenth of a percent from better countercyclical policies.” (page 1).

In reaching this conclusion, Lucas points out that estimates of the costs of business cycles are inextricably linked to analysis of the equity premium puzzle since, in both types of analyses, the volatility of households’ (or investors’) marginal utility of consumption

plays a critical role.¹ It is not surprising, therefore, that modifications that helped to resolve the equity premium puzzle (e.g. non-expected utility) were also used to study the costs of consumption volatility. But in the list of modifications that Lucas surveys, one proposed resolution to the equity premium puzzle is noticeably absent: the inclusion of a low probability crash state. As shown in Rietz (1988), the presence of such a state can indeed explain the equity premium puzzle (and the associated risk-free rate puzzle). Recently, Barro (2005) extends the Rietz analysis and provides some empirical estimates of crash state scenarios. As he states, “...I extend Rietz’s analysis and argue that it provides a plausible resolution of the equity premium and related puzzles. Included in these other puzzles are the low-risk free rate, the volatility of stock returns, and the low values of typical macroeconomic estimates of the intertemporal elasticity of substitution of consumption.” (page 2) I demonstrate below that it can also dramatically increase the costs of business cycles.

I first quickly review Lucas’s example and then cast the analysis in a discrete-state setting which includes a rare, catastrophic state. This discrete setting is then calibrated to match post-war consumption data. For reasonable parameter values for risk aversion, I show that the costs of business cycles are roughly ten times as large as that implied by Lucas’s analysis. Using consumption data for 2001, these estimates imply that households would pay close to \$70 billion to eliminate consumption volatility, i.e. roughly 3% of aggregate consumption. The conclusion is that the gains from stabilization policy are not seen in reducing the second

¹ Of course, the covariance of investors’ marginal utility and returns is also critical for the equity premium.

moments of consumption but lowering the probability and severity of crash states.

2 Lucas's Analysis

Lucas asks the simple question: Suppose household's current consumption path is growing at the constant rate of μ . If uncertainty was introduced into this path, how much would household's have to be compensated to be indifferent between the random and non-random consumption streams. Assuming CRRA utility, this formalizes to finding the value of λ that solves:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \frac{[(1 + \lambda) c_t]^{1-\gamma}}{1 - \gamma} \right\} = \sum_{t=0}^{\infty} \beta^t \frac{[Ae^{\mu t}]^{1-\gamma}}{1 - \gamma} \quad (1)$$

Lucas assumes that, in the stochastic case, consumption is described by the following process:

$$c_t = Ae^{\mu t} e^{-\frac{\sigma^2}{2}} \varepsilon_t$$

where the innovation is assumed to be lognormally distributed with mean 0 and variance, σ^2 . Lucas makes use of the result that, if $\ln z_t \sim N(\mu_z, \sigma_z^2)$, then $E(z_t) = \exp[\mu_z + \frac{1}{2}\sigma_z^2]$. In particular, $E\left[e^{-\frac{\sigma^2}{2}} \varepsilon_t\right] = 1$. Define the stochastic component of utility by:

$$x_t = \left(e^{-\frac{\sigma^2}{2}} \varepsilon_t\right)^{1-\gamma}$$

This definition implies that

$$\ln x \sim N\left(- (1 - \gamma) \frac{\sigma^2}{2}, (1 - \gamma)^2 \sigma^2\right)$$

So that

$$E(x_t) = \exp\left[\frac{-\gamma(1 - \gamma)\sigma^2}{2}\right]$$

Use this in eq. (1) so that λ is defined by the solution to:

$$(1 + \lambda)^{1-\gamma} \exp \left[\frac{-\gamma (1 - \gamma) \sigma^2}{2} \right] \sum_{t=0}^{\infty} \beta^t \frac{[Ae^{\mu t}]^{1-\gamma}}{1 - \gamma} = \sum_{t=0}^{\infty} \beta^t \frac{[Ae^{\mu t}]^{1-\gamma}}{1 - \gamma}$$

Canceling terms and taking logs yields:

$$\lambda \approx \frac{\gamma}{2} \sigma^2$$

Lucas estimates $\sigma^2 = (0.032)^2$ (defined by the standard deviation of the residual from regressing log of annual per-capita real consumption on a time trend over the period 1947-2001). Hence, we have the following estimates

Table 1: Costs of Consumption Volatility in the Lucas Economy

γ	λ
1	0.000768
3	0.00154
5	0.00246
8	0.00410

Clearly, the costs of consumption uncertainty are quite small.

3 An Alternative Characterization

An alternative estimate of the costs of random consumption can be obtained by assuming a discrete state process for consumption growth. Here, I employ a representation first used by Rietz (1988) in order to study the equity premium puzzle. Specifically, it assumed that (gross) consumption growth follows the following discrete-state Markov process

$$\mu_t = \begin{cases} \mu_1 = 1 + m + \delta \\ \mu_2 = 1 + m - \delta \\ \mu_3 = k(1 + m) \end{cases}$$

with transition probability matrix (where the entry in row i and column j represents the conditional probability of going to state j from state i)

$$\Pi = \begin{pmatrix} \pi & 1 - \pi - p & p \\ 1 - \pi - p & \pi & p \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

States 1 and 2 are normal growth rate states while, under the assumption that $k < 1$, state 3 represents a “crash” state or catastrophic state; note that the crash state is assumed to have no persistence. I examine below different values for the severity of the crash (determined by k) and the likelihood of a catastrophic state (denoted by p); for given values of these parameters, the remaining parameters are chosen so that the mean, standard deviation, and first-order autocorrelation of μ_t match the data.

Within this setting, the costs of business cycles are measured by the value of λ that solves

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{[(1 + \lambda) c_t]^{1-\gamma}}{1 - \gamma} \right] = \sum_{t=0}^{\infty} \beta^t \frac{[c_0 \mu^t]^{1-\gamma}}{1 - \gamma} \quad (2)$$

The right-hand side of eq.(2) can be expressed as:

$$\frac{c_0^{1-\gamma}}{1 - \gamma} \frac{1}{1 - \beta \mu^{1-\gamma}} \quad (3)$$

To compute the left-hand side expression, rewrite this first as:

$$\frac{(1 + \lambda)^{1-\gamma}}{1 - \gamma} E_0 \left[\sum_{t=0}^{\infty} \beta^t c_t^{1-\gamma} \right]$$

and define the value function, $v(c_t, \mu_t) = E_0 [\sum_{t=0}^{\infty} \beta^t c_t^{1-\gamma}]$. Given this definition, the value function must satisfy the functional equation:

$$v(c_t, \mu_t) = c_t^{1-\gamma} + \beta E_t [v(c_{t+1}, \mu_{t+1})] \quad (4)$$

As discussed in Ljungqvist and Sargent (2004), the assumption of CRRA preferences implies that the value function can be written as separable in current utility and a function of the current growth rate.

$$v(c_t, \mu_t) = c_t^{1-\gamma} w(\mu_t) \quad (5)$$

Let $w_i = w(\mu_i)$. Then the unknowns (w_1, w_2, w_3) are the solution to the three equations defined by eq.(4) and using eq.(5)

$$w_i = 1 + \beta E_i [\mu_j^{1-\gamma} w_j] \quad (i, j) = 1, 2, 3. \quad (6)$$

Once the values of w_i have been determined, the unconditional expectation of lifetime utility is computed using the unconditional probabilities associated with Π (determined by the eigenvector associated with the eigenvalue of 1). Denoting this unconditional expectation as $E(w)$, the costs of business cycles are determined by the value of λ that solves:

$$(1 + \lambda)^{1-\gamma} E(w) = \frac{1}{1 - \beta \mu^{1-\gamma}} \quad (7)$$

3.1 The costs of business cycles in the crash state economy

To obtain quantitative estimates of the cost of business cycles in the crash state economy, I calibrate the discrete state Markov process in the manner described in Mehra and Prescott (1985). Namely, the parameters are chosen so that the mean, standard deviation and first-order autocorrelation of μ_t are broadly in line with that of annual US per-capita consumption from 1948-2001.² These moments are given in Table 2.

Table 2: Sample Moments

$E(\mu_t)$	1.02
$Sd(\mu_t)$	0.032
$Corr(\mu_t, \mu_{t-1})$	0.09

Since these three moments are insufficient to determine the five parameters describing the Markov process (π, p, m, v, k) , I choose values for the severity of the crash state (k) and the probability of the crash state (p) that are roughly in line with those used by Rietz. I look at two cases: Economy 1 represents a 50% fall in normal consumption ($k = 0.5$) with probability (p) of .001. Hence, this represents a truly catastrophic scenario (the drop in consumption experienced in one year is equivalent to the fall in output during the first three years of the Depression) which occurs roughly once every 1000 years. In Economy 2,

² In the data, the standard deviation of μ_t is 0.02 while the first order autocorrelation is 0.3. I use the figures reported in Table 2 so that the 3 state Markov model yields sensible parameter values (specifically so that $\pi < 1$) for the two cases studied. A problem arises in that the assumed lack of persistence in the crash state implies negative serial correlation in the process. To overcome this requires a high value of π . The standard deviation in Mehra and Prescott's analysis was 0.32 (using a much longer time series for consumption) while they found consumption growth to be negatively autocorrelated. As mentioned earlier, Lucas used an estimate of the volatility of consumption based on a trend stationary specification of per-capita consumption.

consumption falls by 25% ($k = 0.75$) of its normal value and this state occurs just a bit less than fifteen times every 1000 years.³ The discount factor is held constant at $\beta = 0.96$ while relative risk aversion takes on the values reported in Table 1, i.e. $\gamma = (1.5, 3.0, 5.0, 8.0)$. The implied costs are presented in Table 3. For comparison, I also include the costs produced in the discrete state economy with no crash state (i.e. $p \approx 0$). That is, since the environment is slightly different than that studied by Lucas (1987), it is useful to establish that the costs of business cycles are indeed small in the absence of a crash state.

Table 3: Costs of Consumption Volatility in Crash State Economies

γ	<i>Economy 1</i>	<i>Economy 2</i>	<i>No Crash State</i>
1.5	0.0034	0.020	0.0030
3.0	0.017	0.031	0.0054
5.0	0.031	0.042	0.0059
8.0	0.116	0.065	0.0056

In 2001, aggregate consumption of nondurables and services in the U.S. was roughly \$2 trillion (using chain-weighted year 2000 prices). Hence, for relative risk aversion in the range of 3 to 5, the numbers in Table 3 imply that the presence of a crash state produces costs of approximately \$60 billion for the U.S. economy.

³ These parameter values are chosen primarily for illustrative purposes; clearly much more work needs to be done in establishing estimates of the parameters k and p . Barro's (2005) recent study of GDP for the US and several developed and developing economies would support the values used in Case 2. Recently, Thomas, et.al (2000) report that per-capita consumption fell by 34% during the recent financial crisis in Indonesia.

4 Discussion

By comparing the values in Tables 1 and 3, it is clear that the presence of a crash state in consumption significantly increases the costs associated with consumption uncertainty relative to an economy in which consumption has a symmetric distribution. One conclusion to draw from this is that the modeling of policymakers preferences as the sum of squared deviations from some target (whether inflation or full employment GDP) value, the common practice in almost all applied policy analysis, may be misguided: As Lucas (1987) demonstrated and reflected in the figures given in Table 1, these costs are insignificant. Instead, the results here argue that reducing the likelihood and severity of tail events is a policy objective with real welfare consequences.

Consider the welfare consequences in Economy 2 of reducing the probability of a crash state from 0.015 to 0.0075 - that is, reducing the probability by half. The welfare gain in terms of consumption (determined by the difference in the value of λ given in eq. (7)) is presented in Table 4:

Table 4: Welfare Gains in Economy 2

γ	$\lambda_{p=0.015} - \lambda_{p=0.0075}$
1.5	0.009
3.0	0.013
5.0	0.019
8.0	0.033

These marginal gains are fairly substantial relative to those presented in Table 1. Hence

stabilization policy that focuses on the rare but catastrophic event would represent an improvement over current cyclical concerns.

References

- Barro, R. J., 2005, "Rare Events and the Equity Premium," *National Bureau of Economic Research Working Paper* 11310.
- Krussell, P. and A. Smith, Jr., 1999, "On the Welfare Effects of Eliminating Business Cycles," *Review of Economic Dynamics* 2(2), pp. 245-72.
- Krussell, P. and A. Smith, Jr., 2002, "Revisiting the Welfare Effects of Eliminating Business Cycles," Working Paper.
- Ljungqvist, L., and T. Sargent, 2004, *Recursive Macroeconomic Theory*, 2nd ed. Cambridge, USA: MIT Press.
- Lucas, R. E., Jr., 1987, *Models of Business Cycles*. New York: Basil Blackwell.
- Lucas, E. E., Jr., 2003, "Macroeconomic Priorities," *American Economic Review* 93(1), pp. 1-14.
- Mehra, R. and E.C. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics* 15, pp. 145-61.
- Rietz, T. A., 1988, "The Equity Risk Premium: A Solution," *Journal of Monetary Economics* 22, pp. 117-31.
- Tallarini, T.D., Jr., 2000, "Risk-Sensitive Real Business Cycles," *Journal of Monetary Economics* 45(3), pp. 507-32.
- Thomas, D., E. Frankenberg, K. Beegle, and G. Teruel. 1999. "Household Budgets, Household Composition and the Crisis in Indonesia: Evidence from Longitudinal Household Survey Data." Paper prepared for the 1999 Population Association of America Meetings, New York, March 25-27.