The outside option channel of central bank asset purchase programs: A tale of two crises *

Changhyun Lee†

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Abstract

I suggest a new channel through which central bank asset purchase programs could have effects on asset prices: The outside option channel. After the global financial crisis, central banks have widened the variety of assets they can purchase. Secondary markets for the majority of the newly targeted assets are characterized by the OTC market structure with matching frictions and bargaining features. In bargaining, the central bank's asset purchase announcement could affect the outside option value of the asset seller by providing one more option of selling the asset to the central bank to the seller. The effect of the outside option channel materializes even without actual purchases by the central bank since once the asset seller is matched with the buyer, she would exploit the announcement to require a higher price in the bargaining but not actually sell the asset to the central bank. I show how the outside option channel could work through a two-period model and discuss its empirical relevance by comparing two episodes of the Fed's asset purchases during the global financial crisis and the COVID crisis.

JEL codes: E50, E58, G12

Keywords: Unconventional monetary policy, OTC markets, Asset pricing

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†University of California, Davis. Email: chzlee@ucdavis.edu

1 Introduction

This paper suggests a novel mechanism, the "outside option channel", through which central bank asset purchases influence asset prices. In a frictionless Walrasian asset market, the asset price is determined by the aggregate demand and supply curves of the asset. In a frictional over-the-counter (OTC) asset market, however, the asset price is determined through negotiations between buyers and sellers. The intuition of the outside option channel is as follows. If an asset price is determined through negotiation as it is in the OTC environment, the central bank's asset purchase announcement will affect the outside option values of the asset seller or buyer in the negotiation and consequently the asset price. The central bank's asset purchase program provides one additional option for the asset seller: selling the asset to the central bank. This would elevate the outside option value of the asset seller and thus the asset price.

Why should we care about negotiations in OTC markets regarding the central bank's monetary policy? It is because the world's central banks nowadays trade assets in sufficiently frictional OTC markets where negotiations do matter. The global financial crisis (GFC) forced them to develop new ways of supporting the collapsing economies. The systemic nature of the crisis, coupled with a low-interest rate environment, constrained the efficacy of conventional short-term interest rate adjustments. One of the unconventional monetary policy tools introduced to overcome such constraints is the direct asset purchase by the central banks. For example, the Fed purchased the agency MBS during the GFC and corporate bonds in the recent COVID crisis.

One important difference arises in the secondary market structures between the US Treasuries and the newly targeted assets. US Treasuries have particularly liquid secondary markets crowded with many buyers and sellers, and with efficient matching processes. In this sense, their secondary markets are well approximated to Walrasian markets, where negotiation does not play an important role in determining the asset price therefore leaving little room for the outside option channel to work. Conversely, secondary markets for the majority of the newly targeted assets are characterized by inherent matching and negotiation frictions. Corporate bonds, for example, are notorious for the illiquid nature of the secondary market. Secondary market trades of corporate bonds entail substantial search costs, and the prices are determined by negotiations between the matched sellers and buyers rather than by the aggregate supply and demand curves for the bond. (Duffie et al., 2005) Therefore, when the central bank operates in such a market, it can intervene in the negotiation process by manipulating the outside option value of the seller or buyer through its announcement about asset purchases.

I construct a two-period model illustrating how the outside option channel could work and how different types of financial shocks have different policy implications in terms of the outside option channel. In the first period, a representative firm issues bonds to investors in a frictionless primary market to raise funds for production. At the start of the second period, idiosyncratic asset management cost shocks are realized to investors. After observing the cost shocks, the investors could trade the bonds with each other in a secondary OTC market, where matching frictions exist and the price is determined by bargaining between the asset seller and buyer. In this setting, the central bank's asset purchase announcement in the secondary market could increase the outside option value of the asset seller, enhancing the seller's position in the negotiation. This leads to an increase in the secondary market prices. Then the primary market price also rises since investors anticipate they could sell the asset at a higher price in the secondary market. This in turn increases both the investment and output by loosening the firm's funding condition.

Importantly, this effect of the outside option channel could materialize itself without actual purchases by the central bank. Once an asset buyer and seller are matched, there is no need for the seller to actually break the bargaining with the buyer and sell her assets to the central bank. Instead, the seller can use the central bank's announcement as leverage to require better terms of trade in the bargaining. Therefore, the amount of resources that the central bank should spend to absorb the bonds in the secondary market to keep the promise of the asset purchase announcement depends on the relative sizes of asset sellers and buyers. When there is a relatively small number of sellers, the central bank need not undertake substantial asset purchases, as sellers easily find matches within the abundant buyer pool. Once they are matched, the outside option channel would work through the bargaining process between the pair of private agents, without actual purchases by the central bank. Conversely, if a relatively large number of sellers compete with each other to be matched with a small number of buyers, many of those sellers will be left without selling their bonds in the secondary market. Then they have no choice but to sell the assets to the central bank. In this case, the central bank should expand its balance sheet substantially to absorb those unmatched sellers' bonds. Although I focus on asset purchases throughout this paper, the central bank's announcement of selling off assets would have the opposite effect by symmetry.

This could explain why the two episodes of the Fed's asset purchase programs unfolded in quite different ways. The first was the agency-MBS purchases via the first round of quantitative easing (QE1) during the GFC and the second was the corporate bonds purchase programs, the Primary and Secondary Market Corporate Credit Facility (PMCCF/SMCCF), during the COVID crisis. Despite the consensus that both were successful in raising the targeted asset prices and hence stabilizing the markets, the striking difference between those two programs lies in the size of the actual purchases done by the Fed: while the Fed had to purchase more than \$1 trillion of MBS during the GFC, it only absorbed \$14 billion of corporate bonds during the COVID crisis. These two numbers respectively correspond to 89.6% and 1.9% of the maximum size of the purchase promised by the Fed in the two crises.

I argue that the COVID crisis could be characterized by a small number of sellers and the GFC is well illustrated with a large number of sellers in the secondary market. The GFC was a serious system-wide crisis. Almost all financial institutions suffered from a lack of liquidity and the poorly designed financial derivatives worsened informational frictions, leaving no buyer in the secondary MBS market. Since MBS sellers could not find buyers in the secondary market, the outside option channel could not be activated and hence the Fed had to absorb the huge amount of MBS in its balance sheet. On the contrary, the turbulence in the corporate bonds market during the COVID crisis was initiated by massive redemption from open-ended fixed-income mutual funds, which took up only a small part of the whole financial system. In order to meet the redemption requests, those mutual funds were forced to dump their corporate bonds in the secondary market at dislocated prices, pulling down the aggregate bond prices. However, other players in the financial system, especially banks, stayed healthy relative to what they had been during the GFC. Therefore, when the Fed's corporate bonds purchase program was announced, the troubled mutual funds could find many buyers in the secondary market, which is the condition needed for the outside option channel to work — the outside option channel works through the bargaining process between a buyer and seller. As a result, the Fed could increase corporate bond prices without purchasing the bonds on a large scale.

The model is built on the OTC asset market literature pioneered by Duffie et al. (2005). They assume exogenous idiosyncratic shocks on valuations for the same asset, motivating investors to trade the assets in the secondary OTC market, and capture the illiquid nature of trades in OTC markets by introducing matching frictions and bargaining structures between asset sellers, buyers, and market makers. Duffie et al. (2005) discuss the effects of the microstructure of the OTC market on bid-ask prices and welfare but do not focus on monetary policy. My model is a variant of their model modified to analyze the central bank's asset purchase policies in OTC markets.

Some New Monetarists incorporate money as a medium of exchange into the Duffie et al. (2005)'s framework and examine the role of monetary policies, especially focusing on the money stock growth and the resulting inflation rate. In Geromichalos and Herrenbrueck (2016); Geromichalos and Jung (2019); Geromichalos and Herrenbrueck (2022); Mattesini and Nosal (2016), idiosyncratic shocks on the consumption opportunities in the decentralized goods markets (DM) generate heterogeneous liquidity needs for agents since only money can serve as a medium of exchange in the DM. The agents who have the opportunity sell real assets and acquire money to use in the DM, and the others buy real assets in the OTC asset market. In this environment, the price of the real asset at issuance could involve a liquidity premium even though it cannot be directly used in the DM because consumers can dispose of the asset in the OTC asset market to secure liquidity. The effects of inflation on asset prices and trade volumes in the secondary OTC market vary across the model assumptions and parameter values.

Instead of the DM, Lagos and Zhang (2019, 2020) generate heterogeneous asset valuations through idiosyncratic preference shocks on dividend goods from equity relative to consumption goods produced by labor. Money plays its role as a means of payment in the OTC asset market where agents rebalance their portfolios according to the realization of the preference shocks. The equity price in the competitive centralized market then carries a speculative premium stemming from the expectation of selling it to a high-valuation investor in the following secondary OTC asset market. As the cost of holding money increases, more investors prefer equity to money, decreasing the marginal buyer's valuation of the equity in the OTC market. This decreases the equity price by reducing the speculative premium and the trade volume in the OTC market since it is equity buyers but not sellers who are liquidity-constrained and thus cannot buy enough equity. Geromichalos and Jung (2024) construct a model encompassing both the DM goods market and an OTC asset market with agents having heterogeneous asset valuations, where money plays its role as a medium of exchange in both markets. They show that, in contrast to the conventional view, inflation could be welfare-improving in such an environment.

This study diverges from the New Monetarist papers cited above in that I examine the effects of unconventional monetary policy of direct asset purchases, while those papers only consider the traditional dimension of monetary policy, the adjustment in the money growth rate. For this reason, I abstract from money and assume an economy where only a real good and private asset exist.

There are of course other New Monetarist works dealing with unconventional monetary policies. Williamson (2016) examines QE in the form of long-term bond purchases by the central bank under two different regimes, the channel system without interest on reserves and the floor system with interest on reserves. In his model, short-term government bonds are better collateral in the DM than long-term government bonds by assumption, and thereby the central bank's purchase of long-term bonds increases the total stock of collateral in the private sector. On top of that, he shows that the floor system could be more effective in running QE since it relaxes the central bank's balance sheet constraint. Other papers discussing interest on reserves from the New Monetarist perspective include Williamson (2019); Rocheteau et al. (2018).

Geromichalos et al. (2021) extend Lester et al. (2012) and construct a model where the direct and indirect liquidity of assets are endogenously determined. They assume that all DM sellers accept money but have to pay an information cost to accept real assets as a means of payment. Then the extent to which the real asset is accepted in the DM determines the direct liquidity of the asset. If a DM buyer is matched with a DM seller who only accepts money, the buyer should enter the OTC asset market and exchange real assets for money in order to use it in the DM. How easily one can sell the asset in the OTC asset market determines the indirect liquidity of the asset. In this setting, they capture the effect of eligibility policies, where the central bank widens the variety of assets that can serve as collateral, by a decrease in the information cost. This enhances the direct liquidity of the real asset and hence improves welfare under a reasonable parameterization.

Madison (2019) adds informational frictions à la Akerlof (1970) to Geromichalos and Herrenbrueck (2016), by introducing the distinction between good and bad assets. The agents who have the opportunity to access the DM and hence try to sell their non-money assets in the OTC asset market face additional informational friction that the asset quality is the private information of the seller. The good asset holders thus should signal their type to the buyers by suggesting terms of trade that are sub-optimal compared to the full information benchmark. To reduce the inefficiency, the central bank can exchange the information-sensitive assets for information-insensitive government securities. Since the quality of government securities is known to both parties of the seller and buyer, the seller no longer has to sacrifice the optimal terms of trade to signal its type.

Camargo and Lester (2014); Chiu and Koeppl (2016) use similar informational frictions in the OTC asset market but abstract from money. The frictions could bring a halt in trading in the OTC market since when the average asset quality is too bad, the potential buyers find it optimal not to make offers at all to avoid holding bad assets as a result of the trade. They then discuss different policies to rejuvenate trades in the OTC market. Chiu and Koeppl (2016) show that the central bank's purchases of bad assets could increase the share of good assets in the market and the value of bad assets. This enhances the incentive for buyers to take the risk of buying bad assets, leading to a resumption of trades in the OTC market. Camargo and Lester (2014) consider the central bank making loans to investors. This practically acts as a subsidy for purchasing bad assets because when a buyer acquires bad assets from bargaining, it can choose to default on the loans and transfer the loss to the central bank. They see the Public-Private Investment Program for Legacy Assets introduced during the global financial crisis as such a policy.

Lagos et al. (2011) have dealers and investors in their model and capture the features of financial crises by temporary adverse preference shocks for the asset on investors. Since dealers are not exposed to such shocks by assumption, they can pile up their inventories of assets during the crisis and resell them in the recovery phase when the asset price rises back to the steady state level. In this way, dealers can serve the market as liquidity providers. When the market friction is sufficiently large, however, it could be optimal for the dealers to accumulate no inventory, since it takes too much time for them to resell assets in their inventories. In this case, the central bank's direct asset purchase in the secondary market could improve welfare by providing liquidity to investors in place of dealers. Rocheteau and Wright (2013) also discuss how liquidity provision by the government could prevent the shutdown of the DM by eliminating the possibility of bad equilibria.

While all those papers deal with the effects of unconventional monetary policies in OTC asset markets, none of them discuss the ramifications of the central bank's announcements on outside option values of agents bargaining the terms of trade in OTC markets, as I do in this paper. Therefore, I contribute to the literature by shedding light on the unrecognized channel of monetary policy through the OTC structure.

Finally, a number of New Keynesian models examine the effects of large-scale asset purchases by the central bank. It is understood as public intermediation replacing private intermediation in turbulent times (Curdia and Woodford, 2011; Gertler and Karadi, 2011, 2013) or the exchange of liquid assets with liquid assets (Del Negro et al., 2017; Kiyotaki and Moore, 2019). This study is fundamentally different from their approach in that they posit Walrasian asset markets with financial frictions while I consider the OTC market structure with matching and bargaining frictions.

The paper proceeds as follows. Section 2 compares the two episodes of the Fed's asset purchase programs during the GFC and the COVID crisis to clarify the motivation of the paper. Section 3 constructs a model to illustrate how the outside option channel is in play. Section 4 discusses the different nature of the two crises and how they molded the differences in the actual execution of the Fed's asset purchase programs within the model framework. Section 5 summarizes the results and concludes.

2 Motivation: A tale of two crises

In this section, I discuss two asset purchase programs operated by the Fed during the two crises. The first one is the MBS purchases amid the GFC. MBS was one of the most troubled assets at the outbreak of the GFC since the crisis *per se* originated from the distress in the mortgage market. At the same time, it was one of the most systemically important assets because highly-rated MBS had been widely used as collateral in financial contracts (Gorton and Metrick, 2012). Although emergency measures were needed to resuscitate the MBS market, the Fed's hands were tied to the zero lower bound. Against this backdrop, the Fed took the unprecedented step of directly purchasing agency

MBS. The Fed announced \$500 billion of agency MBS purchase in November 2008 and expanded the size of the purchase up to \$1.25 trillion in March 2009. The MBS holdings on the Fed's balance sheet peaked at \$1.12 trillion in June 2010 before the start of the second round of quantitative easing. Now, 15 years after the announcement of QE1, the consensus acknowledges its effectiveness in bolstering MBS prices and decreasing mortgage rates (Stroebel and Taylor, 2012; Hancock and Passmore, 2011, 2015; Gagnon et al., 2011).

Amid the COVID crisis, massive redemptions from fixed-income mutual funds caused dislocations in the corporate bonds market in March 2020 (Falato et al., 2021; O'Hara and Zhou, 2021; Ma et al., 2022; Liang, 2020). According to Fred, the yield spreads between 10-year Aaa and Baa corporate bonds and the US Treasury were 1.17 and 1.97%p on Jan 21, the first business day after the first case of COVID in the US, and surged up to 3.20 and 4.23%p on March 20. In response to this turmoil, the Fed announced its first-ever corporate bond purchase programs, PMCCF and SMCCF, on March 23. The effects were quite dramatic. The spreads ceased rising further right after the announcement and receded to 1.96 and 3.41%p by May 11, one day before the actual purchase by the Fed started. By July 23, four months after the announcement, these spreads reduced further to 1.46 and 2.58%p, respectively. Consequently, the Fed succeeded in stabilizing the turbulence without executing substantial corporate bond purchases: it had purchased only \$14 billion through SMCCF and nothing through PMCCF by the end of the programs in Dec 2020 out of \$750 billion, the initially promised upper limit of the purchase.

Figure 1 summarizes the discussion in this section. It shows the relative sizes of the actual purchases (green circles) to the maximum purchases (blue circles) announced by the Fed. The left figure corresponds to the MBS purchase during the GFC and the right figure to the corporate bond purchase during the COVID crisis. The difference is quite notable at a glance: through QE1, the Fed had conducted nearly all (89.6%) of the maximum purchase, while it acquired virtually nothing (1.9% of the maximum purchase) through PMCCF and SMCCF. Despite the differential sizes of purchases, both programs are evaluated as successful in increasing targeted asset prices and consequently stabilizing the markets as discussed above. The main goal of this paper, therefore, is to elucidate how this was possible.

Figure 1: Relative sizes of the actual purchases (green circles) to the maximum purchases announced (blue circles) by QE1 (the left circles) and PMCCF/SMCCF (the right circles)

3 Model

In this section, I construct a two-period model to illustrate how the outside option channel could work. The model has two types of agents, a representative firm and a unit measure of investors. Only one type of good and asset exist: final goods and corporate bonds. The firm issues bonds to investors in the primary market and the investors trade bonds with each other in the secondary market. There is no aggregate uncertainty but the investors face idiosyncratic asset management cost shocks, which makes the secondary market nontrivial.

3.1 A representative firm

A representative firm maximizes profits at the end of the second period. It has the production technology but no endowment at the start of the first period. It thus issues one-period discount bonds that repay one final good per unit of bond at the end of the second period to investors in the first period to raise funds for production in the second period. In the second period, the firm produces final goods and pays back the bonds. Specifically, the firm solves the following problem:

$$\max_{k,b} Ak^{\alpha} - b, s.t.\ k = pb, \alpha \in (0,1). \tag{1}$$

In equation 1, A is the aggregate productivity, α is the capital share, b is the amount of bonds issued, p is the price of the bond denoted in units of the final good, and k is capital. That is, it is assumed that the final good can be used as capital by the firm and the capital fully depreciates after the production. From the first order condition, we have the following bond supply function.

$$b = (\alpha A p^{\alpha})^{\frac{1}{1-\alpha}}. \tag{2}$$

3.2 Investors

A unit measure of investors are ex-ante identical and maximize their expected profits at the end of the second period. They get endowment m as final goods at the start of the first period but do not have the production technology. They can store the endowments with zero rate of return or purchase corporate bonds in the first period. Since each investor is atomic, it takes the price of the bond in the primary market as given.

At the start of the second period, they have Duffie et al. (2005) type idiosyncratic asset management cost shocks: λ portion of the investors become H-type and have a high cost c_H, and the remaining $1-\lambda$ portion of investors become L-type having a low cost c_L, where $0 \le c_L < c_H < 1$. Each type $i \in \{L, H\}$ has to pay c_i per unit of bond it holds at the end of the second period. These cost shocks could be interpreted in many ways. They could, for example, reflect different views of investors about the profitability of the same asset. Alternatively, one could understand the shocks as reflecting different liquidity needs.[1]

After observing their types, investors enter the secondary OTC market. Naturally, H-type and L-type will become asset sellers and buyers, respectively. Different from the frictionless primary market, the secondary OTC market is characterized by matching frictions and the price is determined by bargaining between the matched seller and buyer. There is only one chance of matching. That is, if an investor is not matched with its counterpart in the first round of the matching, it does not have

[1]See Duffie et al. (2005) for more detailed discussion.

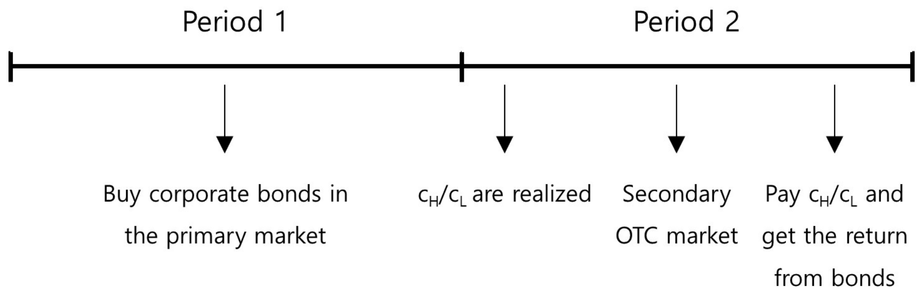


Figure 2: Timing assumptions

any additional chance of matching in the secondary market. After the OTC market closes, each type $i \in \{L, H\}$ pays c_i and gets one final good per unit of bond it holds at the end of the second period. The timing assumptions are summarized in Figure 2.

Once an asset seller and a buyer are matched, they exchange m' units of final goods with b' units of bonds through the Kalai bargaining where the asset buyer's bargaining power is $\theta \in (0, 1)$. More specifically, they solve the following problem.

$$\max_{m',b'}\{(1-c_L)b' - m'\} \text{ s.t. } (1-c_L)b' + m' = \frac{\theta}{1-\theta}(m' - (1-c_H)b'),\ b' \le b_H,\ and\ m' \le m - pb_L,$$

where b_i is the bond held by the i-type investor, $i \in \{L, H\}$.

(3)

Note that the buyer and seller surplus are $(1-c_L)b' - m'$ and $m' - (1-c_H)b'$, respectively. Therefore, from the first constraint in equation 3, it is clear that the Kalai bargaining allocates the total surplus to both parties proportional to their bargaining powers. For simplicity, I assume that the endowment m is large enough so that the asset buyer's liquidity constraint, $m' \le m - pb_L$ does not bind. Then, the bargaining solution is $b' = b_H$ and $\frac{m'}{b'} = 1-(\theta c_H + (1-\theta)c_L)$. That is, the asset seller hands over all its bonds to the buyer and the price of the bond, $\frac{m'}{b'}$, is the weighted average of the valuations for the asset by the seller $(1-c_H)$ and the buyer $(1-c_L)$, where the weight is given by their relative bargaining powers.

Based on this bargaining solution, we can set up the value functions of H/L-type investors having $\tilde{b}$ units of bonds and $\tilde{m}$ units of final goods before the OTC market opens as follows:

$$
\begin{aligned}
W^H(\tilde{b},\tilde{m}) &= \frac{M(\lambda,1-\lambda)}{\lambda}(\tilde{m}+(1-\theta c_H-(1-\theta)c_L)\tilde{b}) + \left(1-\frac{M(\lambda,1-\lambda)}{\lambda}\right)(\tilde{m}+(1-c_H)\tilde{b}),\\
W^L(\tilde{b},\tilde{m}) =& \frac{M(\lambda,1-\lambda)}{1-\lambda}(\tilde{m}-(1-\theta c_H-(1-\theta)c_L)E(b')+(1-c_L)(\tilde{b}+E(b')))\\
&+\left(1-\frac{M(\lambda,1-\lambda)}{1-\lambda}\right)(\tilde{m}+(1-c_L)\tilde{b}).
\end{aligned}
\tag{4}
$$

In equation 4, $W^i(i\in\{L,H\})$ is the value function of the type i investor, M is the matching function in the secondary market, and $E(b')$ is the expected unit of bonds that any H-type investor will bring to the bargaining. Then, an investor's profit maximization problem in the first period is summarized as follows:

$$
\begin{aligned}
&\max_{\tilde{m},\tilde{b}}\{\lambda W^H(\tilde{m},\tilde{b})+(1-\lambda)W^L(\tilde{m},\tilde{b})\},\ \text{s.t. } m=\tilde{m}+p\tilde{b}\\
&\iff m+\max_{\tilde{b}}\{M(\lambda,1-\lambda)(1-\theta c_H-(1-\theta)c_L)\tilde{b}+(\lambda-M(\lambda,1-\lambda))(1-c_H)\tilde{b}\\
&+M(\lambda,1-\lambda)((1-c_L)\tilde{b}+\theta(c_H-c_L)E(b'))+(1-\lambda-M(\lambda,1-\lambda))(1-c_L)\tilde{b}-p\tilde{b}\}.
\end{aligned}
\tag{5}
$$

From the first order condition about $\tilde{b}$, we have the bond demand equation 6. The interpretation is quite intuitive. If an investor purchases one unit of corporate bond in the primary market at the price p, it will get one unit of final good at the end of the second period. The investor, however, has to pay c_L if it becomes an L-type investor with the probability $1-\lambda$. This cost is reflected in the second term of the right hand side in equation 6. The third term shows the expected cost when it becomes an H-type investor with the probability λ. In that case, it can try to sell the bond in the secondary market. If the investor is matched with an asset buyer with the matching probability of $\frac{M(\lambda,1-\lambda)}{\lambda}$, it can sell the bond and then practically pay only $\theta c_H+(1-\theta)c_L$. If not, it has no choice but to pay c_H.

$$p = 1 - (1-\lambda)c_L - \lambda\left(\frac{M(\lambda, 1-\lambda)}{\lambda}(\theta c_H + (1-\theta)c_L) + \left(1 - \frac{M(\lambda, 1-\lambda)}{\lambda}\right)c_H\right) \tag{6}$$

3.3 An equilibrium

In principle, there exists an infinite number of equilibria in the primary market since investors are indifferent in purchasing any amount of bonds at the price in equation 6. Here, I focus on the symmetric equilibrium where all investors purchase the same amount of bonds in the primary market.

Definition 3.1 (The symmetric equilibrium). *The symmetric equilibrium is $\{b^*, p^*\}$ such that (i) the firm maximizes profit by issuing b^* units of bond for given p^* and (ii) each investor maximizes its expected profit by purchasing b^* units of bond for given p^* and under the expectation that all other investors would purchase b^*. More specifically, the symmetric equilibrium is $\{b, p\}$ satisfying equations 2 and 6, and $E(b') = b$ in equations 4 and 5.*

Any investor's expectation about other investors' behavior matters in terms of the definition of the equilibrium since it determines how many bonds it could purchase in the secondary market from its counterparty when the investor becomes L-type. In the symmetric equilibrium, $E(b') = b^*$ in equations 4 and 5. The sufficient endowment condition that prevents the asset buyer's liquidity constraint from binding is then given by $m \geq p^*b^*$.

Graphically, the symmetric equilibrium is represented in Figure 3 as the intersection point of the bond supply curve in equation 2 and the bond demand curve in equation 6. Then, the investment and output in the equilibrium are $k^* = p^*b^*$ and $y^* = A(p^*b^*)^\alpha$, respectively.

In this model, there are two sources of financial shocks: λ and $c_i (i \in \{L, H\})$. The increase in λ implies the proportion of investors who get the high-cost shock c_H rises, and the increase in c_i captures the increase in the asset management cost itself. Both would make the bond less attractive to the investors and thus shift the demand curve downward as in Figure 4, which leads to a decrease in the equilibrium price. Since the funding condition gets tightened, the firm would invest and produce less. These two sources of shocks that seemingly have similar effects on the economy, however, will have different implications concerning the central bank asset purchase policy.

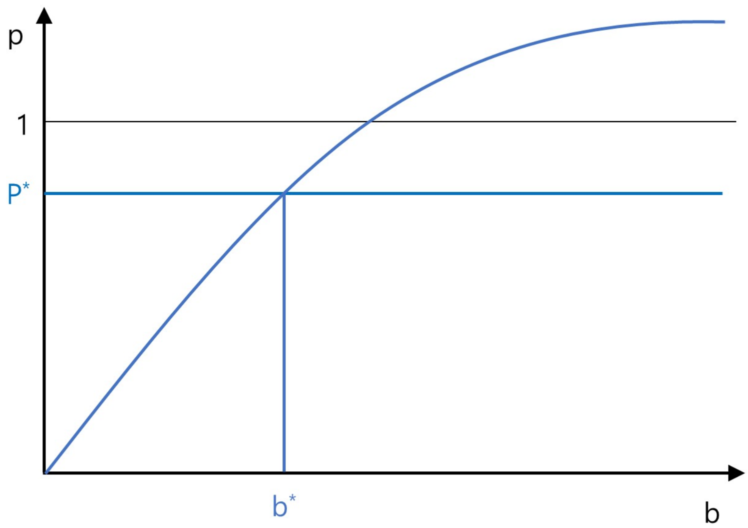


Figure 3: The symmetric equilibrium

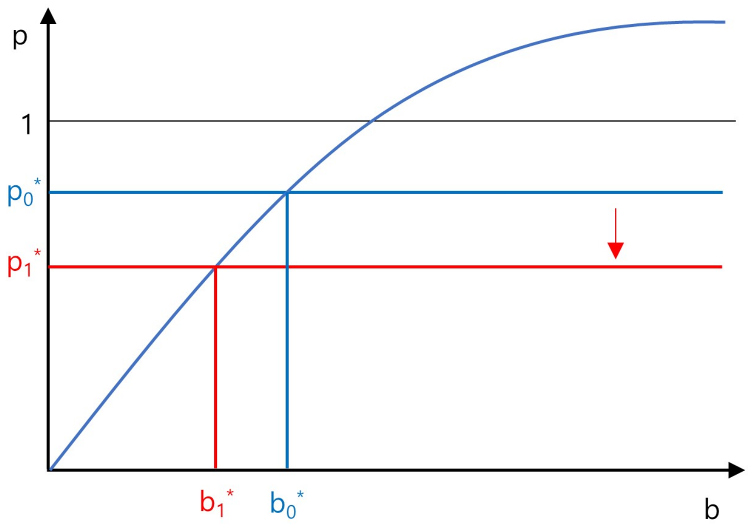


Figure 4: The effect of adverse financial shocks

3.4 Central bank asset purchases

Now, let's incorporate the central bank into the model. I assume that the central bank has endowments m_{CB} as final goods at the start of the first period and purchases bonds in the secondary market. More specifically, consider the central bank announcing infinite bond purchases at the price $p_{CB} \equiv 1 - r_{CB}$, where $r_{CB} \in (c_L, c_H)$. For simplicity, I assume that there is no friction for the investors to sell bonds to the central bank. That is, any investor can sell any amount of bonds to the central bank at any time in the second period at the price p_{CB}. m_{CB} is assumed to be large enough for the central bank to operate the asset purchase program.

What changes would it bring to the bargaining between the asset seller and buyer? Now the seller's outside option is not holding the bonds until maturity but selling them to the central bank. Note that since $c_L < r_{CB} < c_H$, the L-type asset buyer does not have the incentive to sell the bonds to the central bank but the H-type asset seller has. Therefore, it only increases the outside option value of the seller. The seller could threaten the buyer to pay at least higher than $1 - r_{CB}$ because otherwise it can leave the bargaining and sell the bonds to the central bank at $1 - r_{CB}$. Since the threat is credible, the buyer should offer better terms of trade to the seller. Consequently, the bond price will rise in the bargaining.

Importantly, note that once a seller is matched with a buyer, the seller would not actually break the bargaining and sell the bonds to the central bank. Instead, the seller would use the central bank's announcement in increasing the bond price within the bargaining because then she can obtain a higher price than $1-r_{CB}$ at which the central bank would purchase her bonds. In this way, the outside option channel could work only with the announcement even without actual purchases.

Technically speaking, the central bank's asset purchase announcement changes the seller surplus in the bargaining from $m'-(1-c_H)b'$ to $m'-(1-r_{CB})b'$, and thus the resultant price in the secondary market becomes $\frac{m'}{b'} = 1 - (\theta r_{CB} + (1-\theta)c_L)$ instead of $1 - (\theta c_H + (1-\theta)c_L)$. Accordingly, we can rewrite equations from 3 to 6 by substituting c_H with r_{CB}. Then, we have the new bond demand curve in equation 7.

$$p = 1 - (1-\lambda)c_L - \lambda\left(\frac{M(\lambda, 1-\lambda)}{\lambda}(\theta r_{CB} + (1-\theta)c_L) + \left(1 - \frac{M(\lambda, 1-\lambda)}{\lambda}\right) r_{CB}\right) \tag{7}$$

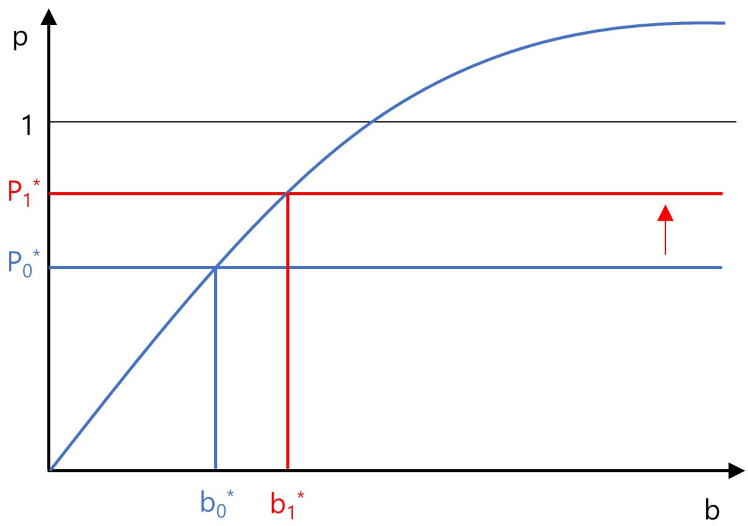


Figure 5: The effect of central bank asset purchases

Since $r_{CB} < c_H$, the price given in equation 7 is higher than the one in equation 6. Investors are now willing to pay more for the bond in the primary market because they understand that even if they become H-type in the second period, they can sell the bonds at higher prices to other L-type investors or to the central bank. This is represented as the upward movement of the demand curve in Figure 5. As a result, the equilibrium bond price and issuance rise, promoting investment and production. The central bank's asset purchase program is effective in raising the asset price and stimulating the economy.

3.5 The size of the actual purchase

More interesting is the changes in the central bank's balance sheet. As noted earlier, the outside option channel could work without actual purchases. Indeed, it is only the unmatched H-type investors who will sell their bonds to the central bank in the model because L-type investors are not attracted by the price suggested by the central bank and the matched H-type investors will sell their bonds to their counterpart L-type investors. Therefore, the amount of final goods that the central bank should spend to purchase bonds, $\tilde{m}_{CB}$, to operate the asset purchase program is

$$\begin{aligned}\tilde{m}_{CB} &\equiv (1-r_{CB})\lambda\left(1-\frac{M(\lambda,1-\lambda)}{\lambda}\right)b^* \\ &= (1-r_{CB})\lambda\left(1-\frac{M(\lambda,1-\lambda)}{\lambda}\right)\left(\alpha A\left(1-(1-\lambda)c_L-\lambda\left(\frac{M(\lambda,1-\lambda)}{\lambda}(\theta r_{CB}+(1-\theta)c_L)+\left(1-\frac{M(\lambda,1-\lambda)}{\lambda}\right)r_{CB}\right)\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}.\end{aligned} \tag{8}$$

In equation 8, λ is the measure of H-type investors, $\left(1-\frac{M(\lambda,1-\lambda)}{\lambda}\right)$ is the probability for any H-type investor being unmatched, and b^* is the amount of bonds held by each investor in the symmetric equilibrium.

As is clear in the equation, the magnitude of the central bank asset purchase depends on λ, the size of asset sellers. This differentiates the two types of adverse financial shocks, the increase in the share of investors getting a high cost c_H ($\lambda \uparrow$) and the increase in the cost itself ($c_H \uparrow$), in terms of their effects on the central bank's balance sheet when the central bank runs an asset purchase program.

For the sake of simplicity in the analysis, here I discuss only the case where the matching function is as efficient as possible, $M(x,y) = min\{x,y\}$, but more generally, proposition 1 holds.

Proposition 1. *Assume that $\lambda_1 \geq \lambda_2$ iff $\frac{M(\lambda_1,1-\lambda_1)}{\lambda_1} \leq \frac{M(\lambda_2,1-\lambda_2)}{\lambda_2}$. For any two different types of adverse shocks, an increase in λ and an increase in c_H, that cause the same amount of bond price decrease in the primary market, the central bank should spend more final goods in purchasing bonds in response to an increase in λ than an increase in c_H to achieve the same size of price increase.*

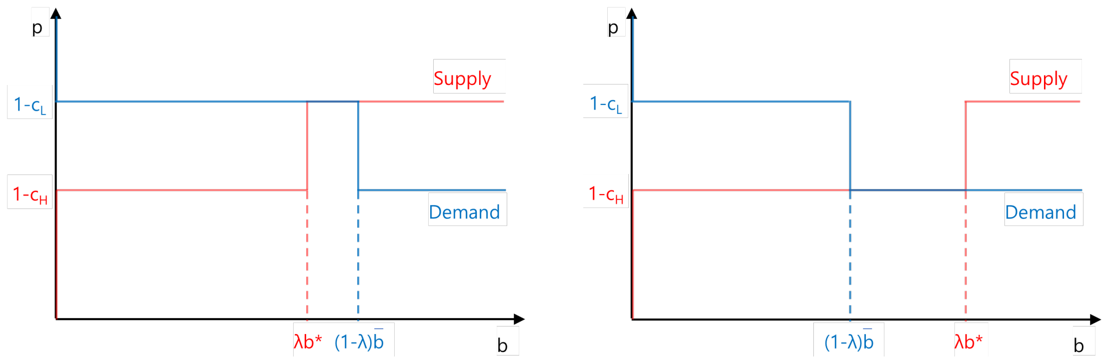
Proof. Refer to section A. □

The assumption on the matching function used in proposition 1 implies that any seller(buyer)'s matching probability decreases as the share of sellers (buyers) increases, which is quite weak and reasonable. Note that $M(x,y) = min\{x,y\}$ does not imply there is no friction in the OTC market. The main friction here is that each investor has only one chance of matching. That is, if any investor is not matched with another investor or cannot reach a settlement on the terms of trade in bargaining, she cannot have a second opportunity to search for another investor. This essentially differentiates the outside options of the matched seller and buyer, making room for the outside option channel to work.

First, consider the shock where λ increases above 0.5. Assuming $M(x,y) = min\{x,y\}$, $\tilde{m}_{CB} = \lambda\left(1 - \frac{1-\lambda}{\lambda}\right) b^* > 0$ from equation 8. That is, the central bank has to actually purchase bonds to increase the bond price. In contrast, if c_H increases while λ remains low below 0.5, $\tilde{m}_{CB} = \lambda\left(1 - \frac{\lambda}{\lambda}\right) b^* = 0$: the central bank can raise the bond price only with announcement but without actual purchase.

This difference comes from the negative relationship between a seller's matching probability and the number of sellers in the secondary market. Intuitively, when there is a small number of sellers while a large number of buyers are searching for their counterparties every here and there, the sellers could find the buyers more easily. Note again that the matched sellers will not sell their bonds to the central bank but only use its announcement to increase the bond price in the bargaining. This reduces the hassle for the central bank to purchase the bonds. Its words have effects even if not supported by actions. If there are a bunch of sellers with few buyers, however, there must be many unmatched sellers who have no choice but to sell bonds to the central bank. In this case, the central bank balance sheet will expand to absorb the unsold stock of bonds.

This has important implications for the optimal asset purchase policy. Although not explicitly modeled here, the central bank's asset purchase could have various kinds of costs. It may, for instance, have to set up a special purpose vehicle and hire asset management experts to practically operate the purchases. There could be a political burden in using resources from taxpayers to support wealthy bankers and entrepreneurs. Maybe the central bank is less efficient in trading assets in nature since it does not have as strong an incentive to maximize profits as private financial institutions have. This inefficiency of the central bank in financial intermediation is one of the most conceptually important assumptions in the literature as well. For example, Curdia and Woodford (2011) assume that the central bank's intermediation cost is higher than the private bank's cost in normal times. They justify the central bank's asset purchase only in turbulent times when the negative cost shock on the private sector's intermediation is so large that the central bank has the advantage in costs. Gertler and Karadi (2011) also assume that the central bank is less efficient in making loans but find its comparative advantage in that the central bank is not under the balance sheet constraint. Considering the OTC structure, this paper contributes to the literature by showing that the optimal policy depends on the nature of shocks since the amount of resources the central



(a) Seller's market

(b) Buyer's market

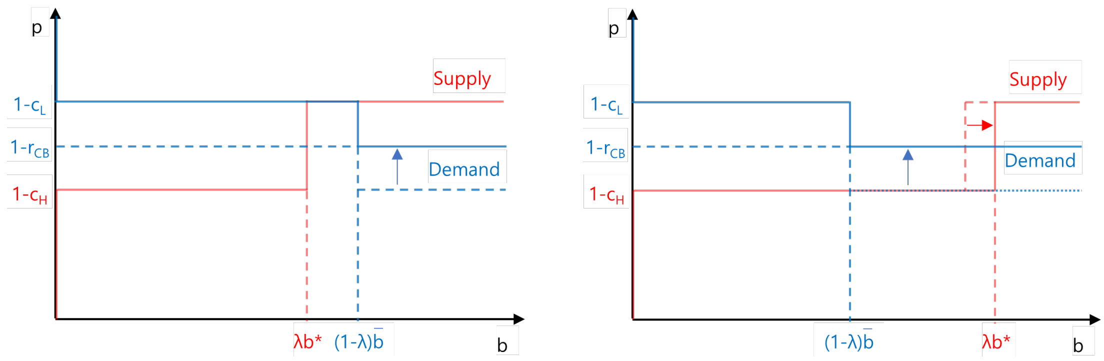
Figure 6: Walrasian secondary market

bank should put in will be different according to what type of shock causes the given size of the decrease in the asset price.

3.6 Walrasian market case

This section considers the case where the secondary market is Walrasian. To make it nontrivial and comparable to the benchmark OTC market case, I assume that any investor can purchase up to $\bar{b}$ units of bonds in the secondary market but can sell any amount of bonds. This restriction could be understood as a buyer's liquidity constraint due to the limited endowment m in the model or a balance sheet constraint that is not dealt with in the model. Note that it is implicit in the OTC market structure used in the benchmark model that the maximum amount of bonds a buyer can acquire in the secondary market is equal to the amount of bonds held by its counterparty, which is b^* in the symmetric equilibrium. That is, the benchmark OTC and the Walrasian markets are similar in that there exist upper bounds in purchasing the bonds but only different in the degree of matching and bargaining frictions.

Figure 6 shows the two possible cases in the secondary market. First, when the demand is strong, *i.e.* $(1-\lambda)\bar{b} > \lambda b^*$, the equilibrium in the secondary market is described by figure 6a. In such a case, every investor having the high cost c_H can sell all their bonds at the price $1 - c_L$ in the secondary market. Therefore, the equilibrium price and issuance of the bond in the primary market will be $p^* = 1 - c_L$ and $b^* = (\alpha A(1-c_L)^{\alpha})^{\frac{1}{1-\alpha}}$, respectively. In other words, changes in c_H do not have any



(a) Seller's market

(b) Buyer's market

Figure 7: The effect of central bank asset purchases in a Walrasian secondary market

effect at all when the asset markets are frictionless and the demand for the asset is strong. This is not the case in the benchmark model with a secondary OTC market. Even if the demand is strong, *i.e.* λ is small so that there are many buyers in the secondary market, a decrease in c_H leads to an asset price decrease in the secondary OTC market because the sellers face frictions: when a seller is matched with a buyer, it is common sense that the matching is the last chance for secondary trading for both parties and thus the seller cannot insist on the maximum price $1 - c_L$.

On the other hand, if the asset demand is deficient as in figure 6b, the secondary market price settles at $1 - c_H$, the valuation of the H-type investors for the bond. Now, H-type investors cannot benefit from selling their bonds in the secondary market. That is, if an investor is hit by the high-cost shock c_H, there is no way to avoid or reduce the cost through the secondary market. Accordingly, the primary market price and the bond issuance will be $p^* = 1 - \lambda c_L - (1 - \lambda)c_H$ and $b^* = (\alpha A(1 - \lambda c_L - (1 - \lambda)c_H)^\alpha)^{\frac{1}{1-\alpha}}$.

Now, let's see what happens with the central bank's asset purchase policy. For comparison, I consider the same type of policy described in section 3.4, where the central bank purchases any amount of bonds in the secondary market at the price $1 - r_{CB}$, where $c_L < r_{CB} < c_H$. Figure 7 shows its effects in two different cases. First, in the seller's market in figure 7a, nothing happens at all. Although the demand curve shifts upward from $1 - c_H$ to $1 - r_{CB}$ beyond $(1 - \lambda)\bar{b}$, it does not have any effect on the equilibrium. Nobody will trade with the central bank because the central bank

Table 1: Asset purchase program's effectiveness

Cases	OTC	Walrasian
Many sellers & Less buyers	Effective with $b_{CB} > 0$	Effective with $b_{CB} > 0$
Many buyers & Less sellers	Effective with $b_{CB} = 0$	No Effect

Notes: $M(x, y) = min\{x, y\}$ for all $x, y \in [0, \infty)$. is assumed in the OTC market case.

bids a price lower than the market equilibrium price. Therefore, the primary market equilibrium is the same as before and so are the firm's investment and output.

The central bank's asset purchase policy does increase the secondary market price in the buyer's market from $1-c_H$ to $1-r_{CB}$ as described in figure 7b. Accordingly, the primary market bond price and issuance will increase to $p^* = 1 - \lambda c_L - (1-\lambda)r_{CB}$ and $b^* = (\alpha A(1 - \lambda c_L - (1-\lambda)r_{CB})^\alpha)^{\frac{1}{1-\alpha}}$, making the firm invest and produce more. Similar to the OTC market case with $\lambda > 0.5$, the central bank should purchase at least $\lambda b^* - (1\lambda)\bar{b}$.

Table 1 compares the effectiveness of the central bank's asset purchase policy in two different market structures with different buyer-to-seller ratios in the secondary market. In the table, the first "Many sellers & Less buyers" corresponds to the cases where $\lambda > 0.5$ in the OTC market and $\lambda b^* > (1-\lambda)\bar{b}$ in the Walrasian market. In this environment with many sellers, the effects of the policy are qualitatively the same under both market structures. The asset purchase policy increases the secondary market bond price and thereby the investment and output. At the same time, the central bank balance sheet expands. In the OTC market, $2\lambda - 1$ measure of unmatched sellers sell their bonds to the central bank, and the central bank should absorb the excess supply at the price of $1 - r_{CB}$ in the Walrasian market.

The difference arises in the second case, "Many buyers & Less sellers", which implies $\lambda < 0.5$ in the OTC market and $\lambda b^* < (1-\lambda)\bar{b}$ in the Walrasian market. In the Walrasian market, the price is determined by the buyer's valuation for the bond, $1 - c_L$, if the demand is strong. In other words, the seller's holding cost of the bond c_H does not matter at all. Therefore, the central bank does not need to respond to a decrease in c_H from the first, and even if it does, nothing really happens. On the other hand, even if there are many buyers, a seller cannot visit all of them in the secondary OTC market. Of course, a seller can benefit from a higher matching probability with a larger number of buyers but once the seller is matched with one, she cannot insist on a higher price because this is her last chance to sell the bonds in the secondary market. Therefore, c_H does matter. It is in this environment that the central bank's asset purchase policy becomes extra effective. Since now the problem is that sellers' outside option values are too low, not that there are too many sellers in the market, all it needs to do is to increase their outside option values by announcing asset purchases without massively expanding its balance sheet.

4 Discussion: A tale of two crises

Going back to the motivation, how can the model shed light on the differences between the two Fed's asset purchase programs during the GFC and the COVID crisis summarized in figure 1? Here I argue that it is the nature of shocks that differentiates those two episodes. Specifically, the MBS market during the GFC could be mainly captured by an increase in λ, while the corporate bonds market during the COVID crisis could be well characterized by an increase in c_H with a low λ in the model.

Let's examine the GFC episode first. Although the benchmark model considers the corporate bonds market, it can be readily modified to the MBS market by replacing the representative firm that issues corporate bonds in the model by a household that issues mortgage loans. Then, the secondary market could be naturally seen as the MBS market. Given the severity of the GFC, it would be proper for the model to capture the crisis by adverse shocks in all parameters, c_L, c_H, and λ, that govern the asset price. In terms of the comparison in the actual sizes of the Fed asset purchases in the two crises, however, the most important feature is the increase in λ.

Getting into more detail, it is the system-wide lack of liquidity during the GFC that can be captured by an increase in λ, *i.e.* an increase in the share of MBS sellers in the secondary market. Informational frictions and high leverage ratios of financial intermediaries were the two main driving forces for such a liquidity crisis. First, many factors contributed to exacerbating informational frictions during the GFC. Although the main toxic asset, subprime mortgage loans, occupied only 3% of total nongovernment U.S. debt in 2008 (Acharya et al., 2009), complex structured derivatives like MBS and Collateralized Debt Obligations had been built over and over the loans, exposing more and more financial intermediaries to the relevant risks. To make matters worse, many financial institutions held those assets in off-balance-sheet vehicles worsening informational frictions. On top of that, credit ratings on securitized assets backed by mortgage loans had errors and hence could not help reduce the friction (Ashcraft and Schuermann, 2008). Finally, nobody knew where the risks were, and everybody stopped lending to each other drying up liquidity (Gorton, 2008).

Second, the financial intermediaries were highly leveraged at the onset of the GFC. According to Blank et al. (2020), the average ratio of Common Equity Tier 1 to Risk-Weighted Assets (CET1 ratio) of U.S. Bank Holding Companies (BHC) was only 5.8% in the first quarter of 2009 while it was 11.7% in the fourth quarter of 2019, right before the first case of the COIVD in the US. With increased informational frictions and counterparty risks, the haircuts in the repo market rose sharply, forcing financial intermediaries to deleverage. Since the repo market is one of the most important places where financial intermediaries seek liquidity by allowing them to borrow in the short term, the freeze in the repo market dried up liquidity in the whole financial system. This caused a further decrease in the value of the collateral used in repo contracts, as the repo buyer was not likely to sell the collateral in the secondary market in the case of the repo seller's default.[2]

Note that here I am not arguing that the agency MBS *per se*, the targeted asset of the Fed's QE1, had suffered from informational frictions during the GFC. They had been directly or indirectly insured by the U.S. government and hence maintained their status as information-insensitive safe assets. The problem was that, because of the widespread lack of liquidity caused by increased information frictions and deleveraging of highly leveraged financial intermediaries, there were not many players who were willing to purchase the agency MBS in the secondary market.[3]

This is the feature exactly captured by an increase in λ in the model. In terms of the central bank's asset purchase, the model relates a higher λ to a larger scale of purchases by the central bank: as λ gets higher, the number of sellers left unmatched in the secondary market increases and then the central bank should absorb their assets. This is indeed a good description of the actual implementation of the QE1: the Fed had to purchase more than one trillion dollars of agency MBS to raise the price and stabilize the mortgage rate.

[2]For more details, see Gorton (2009); Krishnamurthy (2010).

[3]This view is also supported by the increase in haircuts for the agency MBS during the GFC documented in Krishnamurthy (2010).

In contrast, the corporate bonds market turbulence during the COVID crisis initiated by fixed-income mutual funds. With increasing COVID cases and the stock market crash, they faced a massive redemption in March 2020, forcing them to sell corporate bonds held in their portfolios to meet the redemption (Falato et al., 2021; O'Hara and Zhou, 2021; Ma et al., 2022; Liang, 2020). In this respect, the costs of holding the bonds rose sharply for such mutual funds because failing to sell the bonds would result in default. Therefore, we can interpret this event as an increase in c_H in the model.

The shock was however restricted to fixed-income mutual funds but not contagious to other financial institutions. To put it in the model's framework, λ, the share of investors who get the high-cost c_H, remained low during the COVID crisis. This was because the two main problems, informational frictions and high leverage ratios, that had caused the system-wide lack of liquidity during the GFC were less severe in 2020 than in 2008. First, while the opaque structures of the newly developed financial derivatives were at the center of the GFC propagating the negative impacts of the drop in housing prices to the whole financial system, there were no such assets that could bring about similar situations during the COVID crisis. In other words, it was relatively easier to identify and locate risks during the COVID crisis than during the GFC. It reduced counterparty risks for financial intermediaries making them more willing to purchase assets in the secondary market compared to the GFC. Second, financial intermediaries had relatively low leverage ratios, and as such deleveraging pressure was not as strong as during the GFC. As mentioned above, the U.S. BHCs' CET1 ratio was 11.7% in the fourth quarter of 2019, about twice higher than 5.8% in the first quarter of 2009 (Blank et al., 2020). A number of studies attributed the high capitalization of U.S. banks at the beginning of the COVID crisis to reinforced banking regulations during and after the GFC: overall capital requirements were raised, especially global systemically important banks (G-SIB) were required additional capital buffers, liquidity regulations were introduced, and stress tests have been conducted (Berger and Demirgüç-Kunt, 2021; Giese and Haldane, 2020; Duncan et al., 2022).

It is worth noting that a series of Fed's facilities during the COVID crisis, the Primary Dealer Credit Facility (PDCF), PMCCF, and SMCCF, were also effective in reducing λ *per se*, whereas the model presented here is mute on such an effect. For example, O'Hara and Zhou (2021) document that as dealers had gotten close to their balance sheet constraints, they were reluctant to absorb the selling pressures in the corporate bonds market in early March 2020, and it was the Fed's announcements of the facilities that made the dealers restart to provide liquidity. Kargar et al. (2021) show that the facilities not only increased the willingness of dealers to accumulate more corporate bonds but also reduced the "dash for cash" behavior from bondholders.

This study is complementary to theirs by adding one more source of the announcement effect of the facilities. That is, such facilities could have two stages of announcement effect. First, as documented in O'Hara and Zhou (2021); Kargar et al. (2021), the announcement itself could have motivated dealers to accumulate inventories and reduced selling pressures from mutual funds, which materializes by the decrease in λ in the model. Second, under such an environment where the share of asset sellers in the secondary market got smaller, the announcement effect would have been amplified through the outside option channel suggested here. It is likely that both mechanisms worked together and contributed to decreasing the size of the actual purchase: SMCCF had purchased \$14 billion of corporate bonds, about 1.86% out of the maximum \$750 billion until the end of the program.

5 Conclusions

This paper suggests the outside option channel of asset purchase policies. With financial turmoils under low-interest rate environments, central banks worldwide have started to run asset purchase programs in frictional OTC markets, where the asset price is determined through bargaining. In this circumstance, the announcement of the program would raise the outside option value of the asset seller and therefore the asset price.

Through a two-period model, I show that the effect of the outside option channel is stronger as the share of the asset seller gets smaller in the secondary OTC market. That is, the magnitude of the purchase required for the central bank to increase the asset price by any given amount is smaller when there is a lower share of asset sellers. This is because only the unmatched asset sellers actually sell their assets to the central bank, since the matched sellers just use the announcement of the asset purchase to increase the asset price in the bargaining. Obviously, fewer sellers lead to fewer unmatched sellers.

This could provide an explanation for why the Fed had to purchase more than one trillion dollars of MBS during the GFC, while it succeeded in stabilizing the corporate bonds market turbulence during the COVID crisis with only $14 billion of corporate bonds purchases out of the maximum $750 billion of purchasement. The GFC was a systemic crisis, as is clear in the fact that it was triggered by one of the most systemically important banks, Lehman Brothers. It gave rise to a systemwide lack of liquidity, which led to a sharp increase in the ratio of asset sellers to buyers in MBS markets. On the contrary, the trouble in the corporate bonds market in March 2020 was caused by redemptions from fixed-income mutual funds. This raised the costs of holding the bonds for the mutual funds facing large redemptions and therefore decreased their outside option values in the bargaining. They were, however, relatively a small part of the whole financial system. Therefore, it was enough for the Fed to recover their outside option values by the announcement of the SMCCF.

A A proof of proposition 1

Let's start from the initial equilibrium bond price in equation 6. Consider $\Delta c_H > 0$ and $\Delta\lambda > 0$ that respectively cause a same amount of decrease in the bond price, $-\Delta p < 0$. It is formally presented in equation 9.

$$\begin{aligned} p - \Delta p &= 1 - (1-\lambda)c_L - \lambda\left(\frac{M(\lambda, 1-\lambda)}{\lambda}(1-\theta)c_L + \left(1 - (1-\theta)\frac{M(\lambda, 1-\lambda)}{\lambda}\right)(c_H + \Delta c_H)\right) \\ &= 1 - (1-\lambda-\Delta\lambda)c_L - (\lambda+\Delta\lambda)\left(\frac{M(\lambda+\Delta\lambda, 1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}(1-\theta)c_L + \left(1 - (1-\theta)\frac{M(\lambda+\Delta\lambda, 1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)c_H\right) \end{aligned} \tag{9}$$

From equations 6 and 9, we have the following equations.

$$\begin{aligned} \Delta p &= \lambda\left(1 - (1-\theta)\frac{M(\lambda, 1-\lambda)}{\lambda}\right)\Delta c_H \\ &= -\Delta\lambda c_L + (\lambda+\Delta\lambda)\left(\frac{M(\lambda+\Delta\lambda, 1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}(1-\theta)c_L + \left(1 - (1-\theta)\frac{M(\lambda+\Delta\lambda, 1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)c_H\right) \\ &\quad - \lambda\left(\frac{M(\lambda, 1-\lambda)}{\lambda}(1-\theta)c_L + \left(1 - (1-\theta)\frac{M(\lambda, 1-\lambda)}{\lambda}\right)c_H\right) \end{aligned} \tag{10}$$

Next, assume that the central bank tries to increase the bond price by $\Delta_{CB}p \in (0, \Delta p]$ in response to each shock. Consider the same practice described in section 3.4: the central bank announces the price at which it would purchase the bonds in the secondary market. Let $p^1_{CB} \equiv 1 - r^1_{CB}$ and $p^2_{CB} \equiv 1 - r^2_{CB}$ the prices that are needed to be suggested by the central bank to achieve the same amount of increase in the bond price by $\Delta_{CB}p$, where $r^1_{CB} \in (c_L, c_H + \Delta c_H]$ and $r^2_{CB} \in (c_L, c_H]$. Then, the relationship between $\Delta_{CB}p$, r^1_{CB}, and r^2_{CB} is given by equation 11.

$$\begin{aligned} \Delta_{CB}p &= \lambda\left(1-(1-\theta)\frac{M(\lambda,1-\lambda)}{\lambda}\right)(c_H+\Delta c_H - r^1_{CB}) \\ &= (\lambda+\Delta\lambda)\left(1-(1-\theta)\frac{M(\lambda+\Delta\lambda,1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)(c_H - r^2_{CB}) \end{aligned} \tag{11}$$

In each scenario of the increases in c_H and λ, let $\tilde{m}^1_{CB}$ and $\tilde{m}^2_{CB}$ be the amount of final goods that the central bank should spend in the secondary market to purchase the bonds. They are given by equations 12. Assuming $\lambda_1 \geq \lambda_2$ iff $\frac{M(\lambda_1,1-\lambda_1)}{\lambda_1} \leq \frac{M(\lambda_2,1-\lambda_2)}{\lambda_2}$, it is sufficient to show that $r^1_{CB} \geq r^2_{CB}$ to prove that $\tilde{m}^2_{CB} > \tilde{m}^1_{CB}$.

$$\begin{aligned} \tilde{m}^1_{CB} &\equiv (1-r^1_{CB})\lambda\left(1-\frac{M(\lambda,1-\lambda)}{\lambda}\right)(\alpha A(p-\Delta p+\Delta_{CB}p)^{\alpha})^{\frac{1}{1-\alpha}} \\ \tilde{m}^2_{CB} &\equiv (1-r^2_{CB})(\lambda+\Delta\lambda)\left(1-\frac{M(\lambda+\Delta\lambda,1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)(\alpha A(p-\Delta p+\Delta_{CB}p)^{\alpha})^{\frac{1}{1-\alpha}}. \end{aligned} \tag{12}$$

From equations 10 and 11, we have

$$\begin{aligned} &\lambda\left(1-(1-\theta)\frac{M(\lambda,1-\lambda)}{\lambda}\right)(c_H - r^1_{CB}) + \Delta p = (\lambda+\Delta\lambda)\left(1-(1-\theta)\frac{M(\lambda+\Delta\lambda,1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)(c_H - r^2_{CB}) \\ &\Leftrightarrow \lambda\left(1-(1-\theta)\frac{M(\lambda,1-\lambda)}{\lambda}\right)r^1_{CB} + \Delta c_L\left(1-(1-\theta)\frac{M(\lambda+\Delta\lambda,1-\lambda-\Delta\lambda)-M(\lambda,1-\lambda)}{\lambda}\right) \\ &= (\lambda+\Delta\lambda)\left(1-(1-\theta)\frac{M(\lambda+\Delta\lambda,1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)r^2_{CB} \\ &\Leftrightarrow \lambda\left(1-(1-\theta)\frac{M(\lambda,1-\lambda)}{\lambda}\right)(r^1_{CB}-c_L) = (\lambda+\Delta\lambda)\left(1-(1-\theta)\frac{M(\lambda+\Delta\lambda,1-\lambda-\Delta\lambda)}{\lambda+\Delta\lambda}\right)(r^2_{CB}-c_L). \end{aligned} \tag{13}$$

Then $r_{CB}^2 > r_{CB}^1$ contradicts the last equation in 13. Therefore, $r_{CB}^2 \leq r_{CB}^1$ and hence $\tilde{m}_{CB}^2 > \tilde{m}_{CB}^1$.
$\square$

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