Looking for Risk: Volatility Bounds in Macro*

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Abstract

We characterize the gap between the equity risk premium (ERP) and its SVIX-implied lower bound as an equilibrium object, increasing in the correlation of valuations and returns, their relative volatility, and risk aversion. Higher risk premia need not be reflected in options-implied volatility. Yet, models resolving the equity premium puzzle through high risk aversion will tend to understate the lower bound and risk-neutral variance of returns. Applying our findings to a RBC economy with disasters, we consider an increase in their probability leading to a 1 p.p. rise in the ERP, but a negligible rise in the SVIX-implied bound (10 b.p.).

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1. Introduction

Prominent contributions in macroeconomics identify a secular increase in the real value of U.S. corporate equity in recent decades and investigate how this relates to both business cycle dynamics and trends in macro aggregates (Farhi and Gourio, 2018; Greenwald, Lettau and Ludvigson, 2025; Corsetti, Lloyd, Marin and Ostry, 2025, amongst others). Yet, direct evidence of an increase in *expected* excess return or the equity risk premium (ERP) has proved elusive. A long tradition in finance therefore relies on asset-pricing bounds to discipline the unobserved ERP.[1] Hansen and Jagannathan (1991) derive an upper limit on the ERP given by the volatility of the stochastic discount factor, but operationalizing this bound requires strong assumptions on both preferences and the relevant consumption measure. In an important contribution, Martin (2017) establishes a lower bound on the ERP for an unconstrained investor with log preferences, given by the risk-neutral variance of returns, and shows this is measurable using traded index option prices. The associated volatility index is defined as the SVIX and is closely related to the VIX. This has rightly spurred the use of option-based volatility indices to discipline macro-finance models, but the behaviour of the implied lower bound is poorly captured and understood within these frameworks.

In this paper, we analytically derive the ERP alongside its upper and lower bounds, within a canonical equilibrium asset pricing framework with habits (Abel, 1990, 1999) and allowing for an imperfect correlation of consumption growth and equity returns. This deviation from the standard Lucas (1978) tree economy is a salient feature of the data, even though there is no consensus in the literature.[2] Analytically, for the case of short-lived equity, we show that the gap between the lower bound (which is monotonically increasing in the SVIX) and the ERP rises with the correlation between valuations and returns and the relative volatility of valuations, and the slope of this relationship is increasing in the coefficient of risk aversion. For short-lived equity,

[1] Campbell (2008); Duarte and Rosa (2015), amongst others, argue that realized equity returns are a poor proxy for expected returns as even very long samples have little power to reject an equity premium of zero.

[2] The correlation between net earnings growth in the data and measures of consumption growth never exceed 30% and are lower for measures of asset-holder consumption from the CEX, see Table 3 columns 1-3 in Appendix A report. Estimates in the literature vary substantially. Campbell and Cochrane (1999) choose a correlation around 0.2 as a benchmark but argue this is likely conservative. Bansal and Yaron (2004) estimate a value of 0.55 close to our constructed series.

the lower bound depends *only* on the volatility of (leveraged) payoffs, and so is independent of the correlation and other preference parameters. As such, through the lens of the model, the gap is an equilibrium object which can vary over time as these moments change.

Our results generalize to alternative preferences and shock processes. For the case of CRRA preferences, whereas allowing for habits implies that both the ERP and the bounds are affected by the emergence of term premia in the case of long-lived equity, these do not arise in the CRRA case. For the case of recursive utility (Epstein and Zin, 1989, 1991), the distance between the ERP and the lower bound has the same functional form, but is specifically driven by the risk aversion coefficient, while the elasticity of intertemporal substitution does not matter because of our maintained assumption that valuations and payoffs are i.i.d. With regards to alternative shock processes, we generalize our results to additionally allow for disaster risk (Barro, 2009; Gourio, 2012). We consider the possibility of a common crash occurring in both consumption growth and payoffs with a low probability, as well as the possibility of an offsetting "bonanza". We show that disaster risk introduces an additive second term to the gap between the ERP and the lower bound, which is increasing in their probability, under mild assumptions on risk aversion.

To investigate our results quantitatively, we proxy valuations and returns using series of net value added and the profit share from the National Income and Product Accounts (NIPA) over the period 1960-2023. We interpret these through the lens of a heterogeneous agent environment, building on Greenwald et al. (2025), who provide evidence that a rise in the share of GDP going to equity holders has been an important driver of higher stock *prices*. Relative to their framework, we allow equity holders to additionally earn non-financial income so that when the aggregate profit share rises, the correlation between equity holder valuations and payouts also rises, additionally leading to higher stock *returns*.[3]

We first conduct comparative statics. As an illustration, we calculate the effect of a one-off change in the profit share from 11.5% (1970-2000 average) to 19.2% (2001-2023) and show that, ceteris-paribus, this corresponds to a rise in the return-valuation

[3]This could be any source of income not due to profits, such as labour income, or can even reflect idiosyncratic risk that is uninsurable amongst equity holders, or a deviation from the representative firm assumption.

correlation for equity holders from 56% to 66%. The model is calibrated to match asset pricing moments in the data, including an unconditional ERP of 6.72% and a lower bound corresponding to an average SVIX of 20%. The 10 p.p. increase in correlation corresponds to an increase in the ERP from 6.26% to 7.11%, whereas the SVIX-implied lower bound rises only by 0.15 p.p. We conduct similar exercises for the implied changes in volatilities of valuations and payoffs. While the volatility of valuations changes only by 10 b.p., the volatility of payoffs falls by 5 p.p., leading to a substantial decrease in the lower bound, all else equal. We also disentangle the contribution of the term premium to our results, emphasizing that the short-term risk premium drives 78% of the ERP and 82% of the lower bound.

To investigate the evolution of the (observable) SVIX-implied lower bound and the (unobserved) ERP over time, we estimate a series of static models, matching the lower bound alongside the level and volatility of the risk-free rate, the term premium on 5-year TIPS, and the dividend yield. Focusing on the period around the GFC (2005–2012), we find that the gap between the ERP and the lower bound is highly volatile, ranging from about 2 p.p. to as much as 10 p.p., and rises post-GFC from an average of about 4 p.p. to about 7 p.p. During the GFC, the SVIX-implied lower bound (over 20% in 2008Q4) exceeds the model-implied ERP (15%), temporarily violating the condition laid out in Martin (2017), and we explain structurally why this occurs. While the model-implied ERP is large (10–15%), this reflects the lens of standard macro-finance models rather than a claim about the true level of expected returns.[4] Our main takeaway is that the gap is positive outside of crises and varies substantially over time.

Finally, we apply our findings to the real business cycle (RBC) model with disasters presented in Farhi and Gourio (2018), henceforth FG, which, in turn, builds on Barro (2009); Gourio (2012). We replicate the FG economy, which features capital accumulation, markups, and other desirable features of macro models, and focuses on two risky steady states calibrated to 1980-2000 and 2001-2016 averages. Whereas the ERP rises by 2.09 p.p., the SVIX-implied lower bound (*in the model*) only rises by 19 b.p. FG specifically pointed out that the non-trending SVIX goes against their finding that, through the lens of the model, the probability of disasters must have risen over time, unless the

[4]The SVIX-implied bound has several advantages which the theory-informed gap forfeits. Namely, the SVIX is constructed as a portfolio relying only on the assumption of no arbitrage. Instead the gap requires parameters estimation and relies on macro data which is both mismeasured and infrequently updated.

bias between the ERP and the lower bound is time-varying. We confirm this point both algebraically and quantitatively. However, the RBC economy underestimates the level of the lower bound almost by a factor of 10 (about 0.6% vs. about 4%). Through the lens of our own framework, we show this can be largely attributed to an insufficient volatility of payoffs, consistent with our theoretical results.

We discuss two main implications of our results. The first pertains to secular trends. While the average (S)VIX has not substantially changed over time, our framework suggests this does not immediately rule out an increasing ERP. Indeed, there is a wide variety of measures suggesting that uncertainty has risen (Bloom, 2009; Baker, Bloom, Canes-Wrone, Davis and Rodden, 2014), as well as measures of the ERP based on valuation ratios (Duarte and Rosa, 2015; Farhi and Gourio, 2018; Corsetti et al., 2025). We present these measures, alongside survey measures, in Figure 8. On the other hand, we can also use volatility bounds as a diagnostic test for macro models calibrated to match unconditional averages of realized equity returns. Our analytical results show that models which resolve the equity premium puzzle (Mehra and Prescott 1985) through high risk aversion, do so at the cost of understating the risk-neutral variance of returns and, with it, the level of the SVIX. If these models also overestimate the correlation between valuations and returns, as is common in representative agent economies where the correlation is assumed to be unity, they will i) underestimate the required degree of risk aversion and ii) still deliver the largest gap between the ERP and the lower bound.[5]

Related literature. Our paper bridges the gap between a large finance literature estimating risk premia and asset pricing bounds and a macroeconomics literature increasingly using asset prices as calibration targets. Caballero, Farhi and Gourinchas (2017); Kozlowski, Veldkamp and Venkateswaran (2019, 2020); Farhi and Gourio (2018), amongst others, show that risk and risk premia in the economy may have risen over time, through the lens of structural macro models. Relatedly, Baker et al. (2014) document government spending and political polarization as underlying drivers of

[5]Di Tella, Hébert, Kurlat and Wang (2023), building on Black (1972), Lopez-Lira and Roussanov (2020) and others, suggest that the security market line (SML) may be quite flat. If this is indeed the case, then even if missing the correlation, models may not substantially fail in matching the equity premium. However, the vast majority of models (especially in macro) predict a relatively steep SML.

uncertainty and risk. We contribute to this literature by showing that a non-trending SVIX is not, in fact, contradictory to rising risk premia, *through the lens of macro-finance models*. The gap between the ERP and its SVIX-implied lower bound is an equilibrium object which varies over time, and we characterize the drivers of this variation. This extends results in Martin (2017), who largely focuses on an investor with log preferences, but also shows that many prominent asset pricing frameworks underestimate the level of the SVIX and VIX, (e.g. Campbell and Cochrane (1999); Bansal, Kiku and Yaron (2012)). We further confirm this within the RBC framework of Farhi and Gourio (2018) and diagnose its causes. Greenwald et al. (2025), henceforth GLL, is one example of a framework where the ERP is calibrated to the SVIX-implied bound, although they allow for a time-invariant gap as robustness.

Our work draws heavily on the literature on heterogeneous agent asset pricing (Constantinides and Duffie, 1996; Lettau, 2002; Cogley, 2002; Kocherlakota and Pistaferri, 2009; Kozliakov, Marin and Singh, 2026), which breaks the one-to-one correspondence between returns, valuations and aggregate consumption. To motivate a change in correlation over time, we use data from national accounts to identify changing trends in corporate action and financial variables, as in GLL, Atkeson, Heathcote and Perri (2024) and others.

Finally, we draw insights from Hansen and Jagannathan (1991); Alvarez and Jermann (2005) and other contributions using asset prices to bound the volatility of the SDF from below.[6] Relatedly, a large literature uses present discount value models to estimate expected returns such as Goyal and Welch (2008); Campbell and Thompson (2008); Van Binsbergen and Koijen (2010) and others.

The remainder of the paper is organized as follows. Section 2 presents a primer on asset pricing bounds in a preference-free environment. Section 3 details an asset pricing framework with habits, explaining the role for the correlation between dividends and valuations. Section 4 details our macroeconomic environment and Sections 4.1-4.2 quantifies our results. Section 5 applies our findings to an RBC model with disasters and Section 6 concludes.

[6]See also Cochrane and Saa-Requejo (2000); Bakshi and Chabi-Yo (2012); Ghosh, Julliard and Taylor (2017); Chabi-Yo and Colacito (2019).

2. A PRIMER ON ASSET PRICING BOUNDS

The object of interest in a large number of macro-finance models is the equity risk premium (ERP), defined as a log *expected* excess return $ERP_t \equiv \ln\left(\frac{\mathbb{E}_t(R_{t+1})}{R_{f,t}}\right)$, but is both unobservable and often poorly proxied either by using realized equity returns R_{t+1} or by regressing them on financial variables such as valuation ratios. Absent arbitrage, returns are determined by the Euler equations:

$$\mathbb{E}_t(M_{t+1}R_{t+1}) = 1, \quad \mathbb{E}_t(M_{t+1})\, R_{f,t} = 1 \tag{1}$$

where $M_{t+1} \geq 0$ is the stochastic discount factor (SDF), R_{t+1} is a risky asset return, and $R_{f,t}$ is the risk-free rate.

Within macro-finance, the ERP is frequently used as a calibration target in models, aiming to capture the exposure of households to risk. This is because the ERP can be shown to be bounded above by the volatility of the stochastic discount factor (SDF):

$$\begin{aligned}\ln\left(\frac{\mathbb{E}_t(R_{t+1})}{R_{f,t}}\right) &= \ln\left(1 - Cov_t(M_{t+1}, R_{t+1})\right) \\ &\leq \ln\left(\sigma_t(M_{t+1})\,\sigma_t(R_{t+1}) + 1\right) \equiv \mathcal{U}_t^b,\end{aligned} \tag{2}$$

where the inequality binds when the correlation between equity returns and SDF is perfectly negative. The risky return R_{t+1} is often proxied by the market portfolio, the riskiest asset in the economy (Duffie (2010) Ch. 1). Condition (2) can be rearranged such that the volatility of the SDF bounds the Sharpe ratio on equity, as in Hansen and Jagannathan (1991), henceforth HJ:

$$\frac{\mathbb{E}_t(R_{t+1}) - R_{f,t}}{\sigma_t(R_{t+1})} \leq \frac{\sigma_t(M_{t+1})}{\mathbb{E}_t(M_{t+1})}. \tag{3}$$

Assuming, for illustration, power utility over consumption $\sigma_t(M_{t+1}) = A\sigma_t(\frac{C_{t+1}}{C_t})$, the Sharpe ratio on equity implies either that consumption growth is sufficiently volatile or risk aversion (A) is very high. Classical contributions often struggle to generate sufficient volatility or risk, leading to the excess volatility puzzle (Shiller, 1981; see also Atkeson et al., 2024), or excessively volatile risk-free rates (Weil, 1989, Di Tella et al., 2023). However, without strict assumptions on preferences and on what the relevant measure of consumption is (e.g. representative agent vs. equityholder), the $\mathcal{U}^b$ is unobservable.

Martin (2017) shows that no arbitrage implies the equity premium is also bounded below by the risk-neutral variance of returns. Under risk-neutral pricing, the Euler equation (1) can be re-written as:

$$\frac{1}{R_{f,t}}\,\mathbb{E}_t^*[R_{t+1}] = 1, \tag{4}$$

where $\mathbb{E}_t^*[X_{t+1}]$ is the risk-neutral expectation for some variable X_{t+1}. Analogously, $Var_t^*(X_{t+1}) = \mathbb{E}_t^*[X_{t+1}^2] - (\mathbb{E}_t^*[X_{t+1}])^2$. The lower bound for ERP (in levels) can be constructed as follows:

$$\begin{aligned}
\mathbb{E}_t(R_{t+1}) - R_{f,t} &= \frac{1}{R_{f,t}}\Big(\mathbb{E}_t^*(R_{t+1}^2) - R_{f,t}^2\Big) \\
&\quad - \Big(\mathbb{E}_t(M_{t+1}R_{t+1}^2) - \mathbb{E}_t(M_{t+1}R_{t+1})\,\mathbb{E}_t(R_{t+1})\Big) \\
&= \frac{1}{R_{f,t}}\,Var_t^*(R_{t+1}) - Cov_t(M_{t+1}R_{t+1},\,R_{t+1}) \\
&\geq \frac{1}{R_{f,t}}\,Var_t^*(R_{t+1}),
\end{aligned} \tag{5}$$

The second equality shows that the equity excess return can be decomposed into a risk-neutral variance term and a covariance term between returns and utility weighted returns. The final inequality arises by assuming $Cov_t(M_{t+1}R_{t+1}, R_{t+1}) \leq 0$, defined as the Negative Correlation Condition (NCC). We provide structural conditions under which the NCC can fail in Section 3.

Importantly, Martin (2017) proves that:

$$SVIX_{t,t+1}^2 = Var_t^*\Big(\frac{R_{t+1}}{R_{f,t}}\Big), \tag{6}$$

where the $SVIX_{t,t+1}^2$, under no arbitrage, is measurable using options prices providing us with an observable lower bound for the ERP at every moment in time.[7] The SVIX is

[7]To be specific, $SVIX_{t,t+1}^2 = \dfrac{2}{R_{f,t}\,S_t^2}\Big[\int_0^{F_{t,t+1}} put_{t,t+1}(K)\,dK + \int_{F_{t,t+1}}^{\infty} call_{t,t+1}(K)\,dK\Big]$, where S_t is a payoff or price of the stock, $F_{t,t+1}$ is a forward price, and K is a strike of options. In turn, $put_{t,t+1}(K)$ and $call_{t,t+1}(K)$ are time t prices of puts and calls expiring at time T with strike K. The forward price satisfies $F_{t,t+1} = \mathbb{E}_t^*(S_{t+1})$ in definition and it is the unique solution satisfying $call_{t,t+1}(F_{t,t+1}) = put_{t,t+1}(F_{t,t+1})$.

closely related to the VIX index, which instead measures the risk-neutral variance of log returns.[8]

Combining the bounds, the expected risk premium is constrained as follows:

$$\mathcal{L}_t^b \equiv \ln\Big(Var_t^*\Big(\frac{R_{t+1}}{R_{f,t}}\Big)+1\Big) \leq \ln\Big(\frac{\mathbb{E}_t(R_{t+1})}{R_{f,t}}\Big) \leq \ln\big(\sigma_t(M_{t+1})\,\sigma_t(R_{t+1})+1\big) \equiv \mathcal{U}_t^b, \quad (7)$$

but it remains an open question how *tight* these bounds are, and if the gaps vary over time and the business cycle. Condition (7) can be expressed in terms of gaps as follows:

$$\mathcal{U}_t^b - ERP_t = \ln\Big(\frac{\sigma_t(M_{t+1})\,\sigma_t(R_{t+1})+1}{-\rho_t(M_{t+1},R_{t+1})\,\sigma_t(M_{t+1})\,\sigma_t(R_{t+1})+1}\Big), \quad (8)$$

$$ERP_t - \mathcal{L}_t^b = \ln\Big(\frac{-Cov_t(M_{t+1}R_{t+1},R_{t+1})/R_{f,t}+SVIX_{t,t+1}^2+1}{SVIX_{t,t+1}^2+1}\Big). \quad (9)$$

Starting with the upper bound, it is easy to see that it binds exactly as the correlation $\rho_t(M_{t+1},R_{t+1}) \to -1$. The HJ bound thus implies that the volatility of the SDF in the economy exactly reflects the riskiest available asset.

The gap to the lower bound is non-zero only if the NCC covariance is non-zero, else the ERP is entirely captured by the risk-neutral variance. For illustration, consider a representative agent with log utility $(A=1)$ who consumes the aggregate dividend, corresponding to Example 3a in Martin (2017). In this case, $M_{t,t+1} = C_t/C_{t+1}$ and suppose $R_{t+1} = D_{t+1}$. In the Lucas (1978) economy where $C_{t+1} = D_{t+1}$, $M_{t+1}R_{t+1} = 1$ and, as such, the lower bound exactly holds. We investigate the ERP, its bounds, and the NCC covariance further within an equilibrium asset pricing framework below.

3. A SIMPLE ASSET PRICING FRAMEWORK

We consider a discrete time, infinite horizon environment $t \in \{0,1,...\}$ with a representative investor. The investor derives utility from their current and past consumption as follows:

$$U_t = \mathbb{E}_t\Big[\sum_{j=0}^{\infty}\Big(\frac{1}{1+\delta}\Big)^j u(C_{t+j},\nu_{t+j})\Big],$$

[8] As a result, the SVIX is consistently lower than the VIX, capturing higher order moments. However, within any log normal framework such as ours, but also Campbell and Cochrane (1999); Bansal and Yaron (2004) and others, the two are indistinguishable. Outside of crises, Figure 8 shows the difference is modest.

where ν_t is an exogenous benchmark level of consumption and δ is the constant rate of time preference. We consider a specification with habits in order to account for both the risk premium and the term premium contained within equity returns (the latter because equity is a long-lived asset). To derive closed form results, we follow Abel (1999) and assume:

$$u(C_t, \nu_t) = \frac{\left(C_t/\nu_t\right)^{1-\sigma}}{1-\sigma}, \quad \nu_t \equiv (C_t^a)^{\gamma_0}(C_{t-1}^a)^{\gamma_1}(G^t)^{\gamma_2},$$

where C_t^a is aggregate consumption, $G \geq 1$ captures a trend and $\gamma_i \in [0,1]$ capture the details of habit formation.[9] The SDF between period t and $t+1$ is given by:

$$M_{t+1} = \beta\, x_{t+1}^{-A}\, x_t^{\theta}, \tag{10}$$

where $x_{t+1} = \frac{C_{t+1}}{C_t}$, $A \equiv \sigma - \gamma_0\,(\sigma - 1) > 0$, $\beta \equiv \left(\frac{1}{1+\delta}\right) G^{\gamma_2(\sigma-1)} > 0$, and $\theta \equiv \gamma_1\,(\sigma - 1)$. The simple CRRA case is nested by $\theta = 0$. We consider a general class of assets which pay $a_j D_{t+n-j}^{\lambda}$ in j periods before the terminal period where $D_{t+n-j} > 0$ is a random variable and $a_j \geq 0$ (> 0 if $j = 0$) and λ are constants. The variable λ captures leverage, tying the underlying returns process to the investor's payoffs. Define $z_{t+1} \equiv D_{t+1}/D_t$ as the growth rate of D_t.

The holding period return for an $n-$period asset is:

$$R_{t,t+1}(n,\lambda) = \frac{p_{t+1}(n-1,\lambda) + CF_{t+1}(n-1,\lambda)}{p_t(n,\lambda)} \tag{11}$$

where $p_t(n,\lambda)$ is the price of n-period asset after the cash flow in period t, λ is defined as leverage and captures the volatility of payouts relative to output, and $CF_{t+1}(n-1,\lambda)$ are cash flows of the asset in any period t.

Following Abel (1999), we assume that x_{t+1} and z_{t+1} are i.i.d processes. The price of any asset can be written as a linear form of growth rates of consumption and stochastic

[9] E.g. the special case of $\gamma_0 = \gamma_2 = 0$ reflects catching up with the Jones' in Abel (1990) whereas $\gamma_1 = \gamma_2 = 0$ reflects consumption externalities in Gali (1994). Relative to Campbell and Cochrane (1999); Campbell, Pflueger and Viceira (2020), our habit formulation is multiplicative rather than additive, allowing for closed form solutions.

component of payout $p_t(n, \lambda) = \omega(n, \lambda)\, x_t^{\theta}\, D_t^{\lambda}$, so we can rewrite (11) as:

$$R_{t+1}(n, \lambda) = \frac{\omega(n-1, \lambda)\, x_{t+1}^{\theta} + a_{n-1}}{\omega(n, \lambda)\, x_t^{\theta}} z_{t+1}^{\lambda}. \tag{12}$$

Substituting returns (12), the SDF (10), into the risky asset Euler (1), and using the fact that growth rates are i.i.d., $\omega(n, \lambda)$ follows a difference equation:

$$\omega(n, \lambda) = \kappa(\lambda)\, \omega(n-1, \lambda) + \beta\, a_{n-1}\, \mathbb{E}(x_{t+1}^{-A} z_{t+1}^{\lambda}),$$

where $\kappa(\lambda) \equiv \beta\, \mathbb{E}(x^{\theta - A} z^{\lambda})$ is a maturity factor, and we require $\kappa(\lambda) \in [0, 1]$ to guarantee a convergence of the difference equation as $n \to \infty$.

Risk-free Rate The one-period risk-free bond is characterized by $n = 1$ and $\lambda = 0$. Then, since $\omega(1, 0) = \beta\, a_0\, \mathbb{E}(x^{-A})$, the rate of return on the risk-free asset is:

$$R_{f,t} \equiv R_{t+1}(1, 0) = \frac{1}{\beta\, \mathbb{E}(x^{-A})\, x_t^{\theta}}.$$

Equity We define equity as a levered claim on z_{t+1}, with $a_j = 1$. It is useful for our analysis to delineate between short-term equity $R_{t+1}(1, \lambda)$, and infinitely lived equity $R_{t+1}(\infty, \lambda)$, which we adopt for our quantitative exercise. Solving the difference equation, realized equity returns are given by:

$$R_{t+1}(\infty, \lambda) = \left[x_{t+1}^{\theta} z_{t+1}^{\lambda} + \frac{1 - \beta\, \mathbb{E}(x^{\theta - A} z^{\lambda})}{\beta\, \mathbb{E}(x^{-A} z^{\lambda})} z_{t+1}^{\lambda} \right] x_t^{-\theta} \tag{13}$$

The lemma below characterizes the general form of the expected log equity premium and the asset pricing bounds, assuming only that the growth of consumption and cash flows are i.i.d.

Lemma 1 (ERP and Bounds) *In the economy with preferences given by (10), for infinitely-lived equity ($n \to \infty$), (i) the expected risk premium is given by:*

$$ERP_t = \underbrace{\ln\left(\frac{\beta\big(Cov(x^{\theta - A} z^{\lambda}, z^{\lambda}) - Cov(x^{-A} z^{\lambda}, x^{\theta} z^{\lambda})\big)}{\mathbb{E}(z^{\lambda})} + 1 \right)}_{\text{Term premium}} + \underbrace{\ln\left(\frac{-Cov(x^{-A}, z^{\lambda})}{\mathbb{E}(x^{-A} z^{\lambda})} + 1 \right)}_{\text{Short-term risk premium}} \tag{14}$$

(ii) *The upper bound (Hansen and Jagannathan, 1991) is given by:*

$$\mathcal{U}_t^b = \ln\left(\beta\,\sigma(x^{-A})\,\sigma\left(x^{\theta} z^{\lambda} + \tfrac{1-\beta\,\mathbb{E}(x^{\theta-A}z^{\lambda})}{\beta\,\mathbb{E}(x^{-A}z^{\lambda})} z^{\lambda}\right) + 1\right). \tag{15}$$

iii) *The lower bound (Martin, 2017) is given by:*

$$\mathcal{L}_t^b = \underbrace{\ln\left(\frac{\Omega(\lambda)}{\mathbb{E}(x^{-A}z^{2\lambda})} + 1\right)}_{\text{Term premium}} + \underbrace{\ln\left(\frac{Var(x^{-A}z^{\lambda}) - Cov(x^{-A}, x^{-A}z^{2\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})^2} + 1\right)}_{\text{Short-term risk premium}} \tag{16}$$

where $\Omega(\lambda) \equiv \begin{bmatrix} \beta^2\,\mathbb{E}(x^{-A}z^{\lambda})\Big\{Cov(x^{\theta-A}z^{\lambda}, x^{\theta-A}z^{2\lambda}) - Cov(x^{-A}z^{\lambda}, x^{2\theta-A}z^{2\lambda})\Big\} \\ +\big(\beta^2\,\mathbb{E}(x^{\theta-A}z^{\lambda}) - 2\beta\big)\Big\{Cov(x^{-A}z^{\lambda}, x^{\theta-A}z^{2\lambda}) - Cov(x^{-A}z^{2\lambda}, x^{\theta-A}z^{\lambda})\Big\} \end{bmatrix}.$

Proof. See Appendix B.1. □

The ERP consists of a term premium and a conventional risk premium, which reflects the covariance of valuations and returns, accounting for leverage λ. Given a positive $\rho_{x,z}$, the risk premium is always positive. The term premium captures the interplay of habits with the fact that equity is a long-lived asset. This term is zero in the limit $\theta \to 0$ (no habits) or $n = 1$ (short-term equity), and we investigate its contribution in our quantitative section.[10] Parts (ii) and (iii) detail the associated upper and lower bounds, which both admit a similar decomposition.

To simplify these expressions further, we assume x_t and z_t are not only i.i.d but also log-normally distributed:

$$\ln\begin{bmatrix} x_{t+1} \\ z_{t+1} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_x \\ \mu_z \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \sigma_{xz} \\ \sigma_{xz} & \sigma_z^2 \end{bmatrix}\right),$$

where μ_i for $i = \{x, z\}$ denotes the mean and $\sigma_{i,j}$ denotes the volatility matrix. The expressions for the ERP and the bounds in the special case of short-lived equity $(n = 1)$ are given as follows:

[10] To wit, when x and z are lognormally distributed, the term premium becomes $\kappa(\lambda)\,\big(\exp(\theta A\sigma_x^2) - 1\big)$, where $\kappa(\lambda)$ is a maturity factor which ensures the yield curve flattens as $n \to \infty$ and the remaining term is a scale factor which depends on parameters and the volatility of consumption.

Proposition 1 (Short-term ERP and Bounds) *Consider the case of short-lived equity* $(n = 1)$*. In the economy with preferences given by* (10)*, the ERP and lower bound are given as follows:*

$$
\begin{aligned}
ERP &= A\lambda\sigma_{xz}, \\
\mathcal{U}^b &= A\lambda\sigma_{xz} + \ln\left(\sqrt{e^{A^2\sigma_x^2}-1}\sqrt{e^{\lambda^2\sigma_z^2}-1} + e^{-A\lambda\sigma_{xz}}\right), \\
\mathcal{L}^b &= \lambda^2\sigma_z^2
\end{aligned}
$$

Proof. See Appendix B.2. □

First, we recover a familiar result: the ERP is determined by the covariance of valuations and returns, scaled by the effective risk aversion A and leverage λ. Excess returns are compensation for poor payoffs in states when the marginal investor values returns highly. As such, to target a given ERP, but also correctly account for the correlation $\rho_{x,z} = \sigma_{xz}/\sigma_x\sigma_z < 1$, the framework requires either higher risk aversion, higher leverage, or additional risk. The HJ upper bound $\mathcal{U}^b$ also rises with the covariance σ_{xz}, since the first term is identical to the ERP, but the gap $\mathcal{U}^b - ERP$ is decreasing in the covariance. Next, we turn to our object of interest, the lower bound $\mathcal{L}^b$. In contrast, this only depends on the volatility of leveraged payout growth $\lambda^2\sigma_z^2$ – and importantly is independent of risk aversion.

Consider the limit $A = 1, \lambda = 1, \rho_{x,z} \to 1$ corresponding to a Lucas (1978) economy. In this case, $ERP = \sigma_x\sigma_z$ and the gap to the upper bound is strictly positive. If $\sigma_x = \sigma_z$, the lower bound exactly coincides with the ERP and the NCC condition $(cov(MR, R))$ binds exactly– corresponding to Example 2 in Martin (2017). To generalize beyond a correlation of 1 $(\rho_{x,z} < 1)$, yet ensure the NCC condition is met, we allow $\sigma_z < \sigma_x$. Intuitively, consumption growth must become more volatile, since the return can only be as risky as consumption growth. This also generates a Sharpe ratio at least as high as the volatility of returns. Lowering payout volatility shifts the $\mathcal{L}^b$ downward, which will now intersect the ERP at the new $cov(MR, R) = 0$ point.

Away from this limit, as $\sigma_{x,z}$ rises, both the ERP and $\mathcal{U}^b$ rise and it can be shown that $\frac{d\mathcal{U}^b}{d\sigma_{x,z}} < \frac{dERP}{d\sigma_{x,z}}$. In contrast, the lower bound $\mathcal{L}^b$ (and the implied $SVIX$) is flat. Figure 1 (LHS panel) illustrates how the ERP and its bounds $\{\mathcal{U}^b, \mathcal{L}^b\}$ vary with the correlation between valuations and returns.

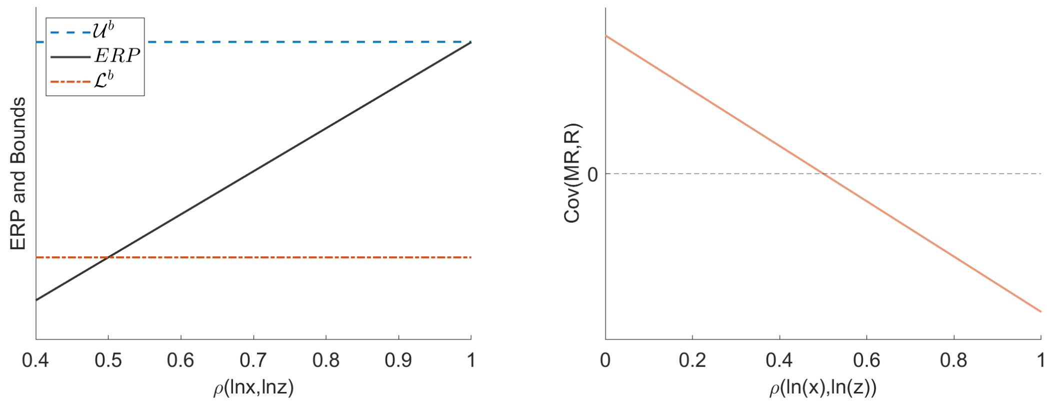


Figure 1: *Stylized example for illustration*

Log utility without leverage, i.e. $\theta = 0$, $\lambda = 1$, $A = 1$, *and* $\beta < 1$, *with* $\sigma_z = 0.5\,\sigma_x$.

If the correlation (or other moments) are time-varying, the ERP can evolve independently of $\mathcal{L}^b$, illustrated by moving along the x-axis. Additionally, having relaxed the assumption $\sigma_z = \sigma_x$, the gap is also increasing if the volatility of valuations σ_x grows faster than the volatility of payoffs σ_z. We investigate the role of the volatilities of dividend and valuation processes in Appendix C.2. Interestingly, the limit of $\rho_{x,z} = 1$ is the point at which the lower bound and the risk premium are furthest – whereas the upper bound is tightest. Figure 1 (RHS panel) shows that as the correlation rises, $cov(MR, R)$ becomes increasingly negative. The corollary below highlights our main result for the case of short-lived equity.

Corollary (ERP and Lower Bound) *The distance between the ERP for short-term equity* $(n = 1)$ *and* $\mathcal{L}^b$ *can be expressed as:*

$$ERP_t^{n=1} - \mathcal{L}_t^{b,n=1} = \lambda\sigma_z \times (A\sigma_x\rho_{x,z} - \lambda\sigma_z)$$

The gap $ERP_t^{n=1} - \mathcal{L}_t^{b,n=1} \geq 0$ *as long as the NCC condition* $\rho_{x,z} \geq \frac{\lambda\sigma_z}{A\sigma_x}$ *is met.*

The rate at which the gap between the ERP and $\mathcal{L}^b$ increases with the correlation $\rho_{x,z}$, is itself rising in the effective risk aversion A. As such, models which rely on high levels of risk aversion to resolve the equity premium puzzle (Mehra and Prescott,

1985) will necessarily "miss" the lower bound, as long as the NCC condition is met. Put differently, a model that matches the ERP using a high degree of risk aversion, necessarily underestimates the contribution of the risk-neutral variance. This is an observable counterfactual implication in the data, as the level of the $SVIX$ already implies an excess return of at least about 4%. In contrast, λ and σ_z enter both the slope and the intercept of the gap, and also raise the level of the lower bound. As such, to simultaneously deliver a realistic ERP and SVIX, models will require sufficient cash-flow volatility.

3.1. Alternative Preferences and Disaster Risk

We next discuss how our results change when i) preferences are CRRA (e.g. Mehra and Prescott (1985)), ii) utility is recursive which allows us to distinguish between risk aversion and the elasticity of intertemporal substitution EIS, iii) the introduction of disaster risk.[11]

CRRA Preferences (No Habits) The case of CRRA utility is nested in (10) by setting $\theta = 0$. Proposition 1 and the Corollary are completely unchanged since they focus on short-lived equity ($n = 1$). However, the case of $n > 1$ differs: habits introduce a term premium, which is absent if utility is CRRA for the case of longer-lived equity ($n > 1$). We discuss the implications of the term premium quantitatively in Section 4.1.

Epstein and Zin (1991) preferences The equity holder's utility is

$$U_t = \left((1-\beta)\, C_t^{1-\sigma} + \beta\, \mathbb{E}_t(U_{t+1}^{1-A})^{\frac{1-\sigma}{1-A}}\right)^{\frac{1}{1-\sigma}},$$

where β is the discount factor, A is now the risk-aversion parameter, and σ is the inverse of the elasticity of intertemporal substitution (EIS). Then, the stochastic discount factor (SDF) can be derived as

$$M_{t+1} = \beta^{\frac{1-A}{1-\sigma}}\, x_{t+1}^{\frac{-\sigma(1-A)}{1-\sigma}}\, (R_{t+1}^a)^{\frac{\sigma-A}{1-\sigma}}, \tag{17}$$

[11] Detailed derivations for this Section are relegated to Appendix B.

where R^a_{t+1} is the gross return on a unlevered wealth portfolio that perpetually delivers *aggregate* consumption as its payout. We derive R^a_{t+1} and then, substituting this back into the SDF, we re-derive the ERP and its bounds following the same steps.

With recursive preferences, the corollary is unchanged, although A is now specifically the coefficient of risk aversion. The EIS does not affect our results only because of our maintained assumption that consumption growth x_t and returns z_t are i.i.d.

Disaster Risk (Barro, 2009; Gourio, 2012) Consider an i.i.d. disaster shock χ_{t+1} which follows a three-point distribution:

$$\chi_{t+1} = \begin{cases} 0 & \text{with prob. } 1-2p \\ \ln(1+b_H) & \text{with prob. } p \\ \ln(1-b) & \text{with prob. } p, \end{cases} \tag{18}$$

where $b \in [0,1]$ captures how much growth rates decline in a disaster episode and p captures the probability of disaster. We define b_H as the growth rate increase during a "bonanza".[12] We assume disasters affect both the consumption process and payout (dividend) process equally. Specifically, we define an adjusted dividend growth process $\frac{\tilde{D}_{t+1}}{\tilde{D}_t} \equiv z_{t+1}e^{\chi_{t+1}}$ and an adjusted consumption growth process $\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \equiv x_{t+1}e^{\chi_{t+1}}$. As such, the defined moments on x_t and z_t, σ_x and σ_z, respectively, are unchanged by the introduction of disasters. The ERP and bounds still satisfy Lemma 1, but evaluating the expectations now requires additional terms.

Proposition 2 (Short-term ERP and Bounds with disasters) *Consider the case of short-lived equity* $(n=1)$. *In the economy with preferences given by* (10) *and disaster risk characterized by* χ_t, *which occurs with probability* p *and follows a three-point distribution given by* (18), *the ERP and lower bound are given as follows:*

$$ERP^{n=1} = A\lambda\sigma_{xz} + \ln\left(\frac{D_{-A}(p)\,D_{\lambda}(p)}{D_{\lambda-A}(p)}\right),$$

$$\mathcal{L}^{b,n=1} = \lambda^2\sigma_z^2 + \ln\left(\frac{D_{-A}(p)\,D_{2\lambda-A}(p)}{D_{\lambda-A}(p)^2}\right),$$

[12] The quantity b_H can be set to zero and does not affect the expressions for the ERP and the lower bound. However $b = b_H$ corresponds to the special case where $\mathbb{E}[e^{\chi}_{t+1}] = 1$.

where $D_k(p) \equiv 1 - 2p + p(1+b_H)^k + p(1-b)^k$. Assuming $b = b_H < b^{max}$ for some b^{max}, the gap $ERP^{n=1} - \mathcal{L}^{b,n=1}$ is increasing in p as long as $A > \lambda$.

Proof: See Appendix B.3. □

The presence of disasters does not alter the economics behind the ERP or the bounds, which are still described by Lemma 1, but it does change how the expectations are evaluated. Conveniently, because of the assumption that quantities are log-normally distributed and disasters enter multiplicatively, the terms enter additively in the ERP and bounds. During a disaster, payoff growth is multiplied by a factor $(1-b)^\lambda$ whereas the marginal utility is multiplied by a factor $(1-b)^{-A}$, leading to a higher required ERP by investors beforehand. The lower bound also rises because payoff volatility increases in the presence of disasters. As such, the gap as a function of p can be expressed as:

$$ERP^{n=1}(p) - \mathcal{L}^{b,n=1}(p) = \lambda\sigma_z\big(A\sigma_x\rho_{x,z} - \lambda\sigma_z\big) + \ln\!\big(D_\lambda(p)\, D_{\lambda-A}(p)\big) - \ln\!\big(D_{2\lambda-A}(p)\big) \tag{19}$$

The gap is increasing in the probability of disaster p as long as investors' valuations respond sufficiently strongly to disasters, relative to the payoff exposure. In this case, worsening tail risk raises priced risk more than it raises the risk-neutral variance of returns, as reflected in the condition $A > \lambda$.[13]

Section 4 describes how we calibrate payouts and valuations from national accounts data, and then Sections 4.1-4.2 quantitatively explore the ERP and its bounds beyond the case of short-term equity, also allowing for disasters. Section 5 then applies our findings to an RBC model with disasters.

4. Macro Data

Our asset pricing model points to $\rho_{x,z}$ as an important driver of asset returns, with very differentiated implications for the asset pricing bounds. We turn to a general production framework, building on Greenwald et al. (2025), henceforth GLL, to investigate

[13] If $A > 2\lambda$, a sufficient but not necessary condition, the gap is increasing in p for any $b \in [0, 1]$. This condition is easily satisfied in our calibrations where A is over 13 on average, whereas λ is about 1.

this further. The model captures three key features of the economy: i) financial wealth is concentrated among few households, ii) the distribution of profits across households varies over time, iii) the correlation of equity holders' consumption with equity returns is not unity.

The economy consists of a representative firm producing aggregate output according to constant returns to scale technology:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{20}$$

where A_t represents total factor productivity (TFP), K_t is input of capital and L_t is the aggregate labor endowment. Workers inelastically supply labor to produce output. Following GLL and Jermann (1998), we assume a constant fraction of output ω is invested every period.

Where we deviate from GLL is in allowing the consumption of equity holders to be imperfectly correlated with returns to equity (point (iii) above). This can arise for various reasons: e.g. equity holders also earn labour income, there is uninsurable idiosyncratic risk, or there is foreign income. All these cases can be captured by:

$$C_{k,t} = \underbrace{E_t - \omega Y_t}_{D_t} + \tilde{C}_{k,t} \tag{21}$$

where E_t are earnings of shareholders defined as $E_t = S_t Y_t$ and $\tilde{C}_k$ is a deviation of consumption from dividends D_t.[14] The profit share is a ratio of total earnings over net value added where the total earnings are defined as a domestic after-tax profit to net value added plus foreign earnings to net value added ratios:

$$\begin{aligned} S_t &= E_t^D + F_t \\ &= (1 - \tau_t)\frac{ATP_t}{LC_t + ATP_t} + F_t, \end{aligned}$$

where LC_t is a labor compensation, ATP_t is a profit after-tax with inventory valuation and capital consumption adjustments, and τ_t is sum of taxes on production and imports

[14]This does not imply that individual shareholders are hand-to-mouth households. As in GLL (also Constantinides and Duffie (1996)), we may assume they trade an arbitrary set of assets with each other and this is the post-trade equilibrium.

less subsidies, net interest and miscellaneous payments, net business transfer payments, and taxes on corporate income over the net value added, which are from BEA NIPA Table 1.14. The ratio F_t is calculated by subtracting the corporate profit from corporate profits from BEA NIPA Table 1.12, over the net value added, and corresponds to less than 5%.

In our exercise, we assume $\tilde{C}_k$ arises from labour income such that $\tilde{C}_{k,t} = \zeta w_t L_t$ where $w_t L_t = (1-S_t)Y_t$. We calibrate ζ to match the share of equityholder consumption to total output, and also generate a realistic correlation between valuation and returns. Consumption of equity holders and dividends are given by:

$$C_{k,t} = (\zeta - \omega + (1-\zeta)S_t)Y_t, \tag{22}$$

$$D_t = (S_t - \omega)Y_t, \tag{23}$$

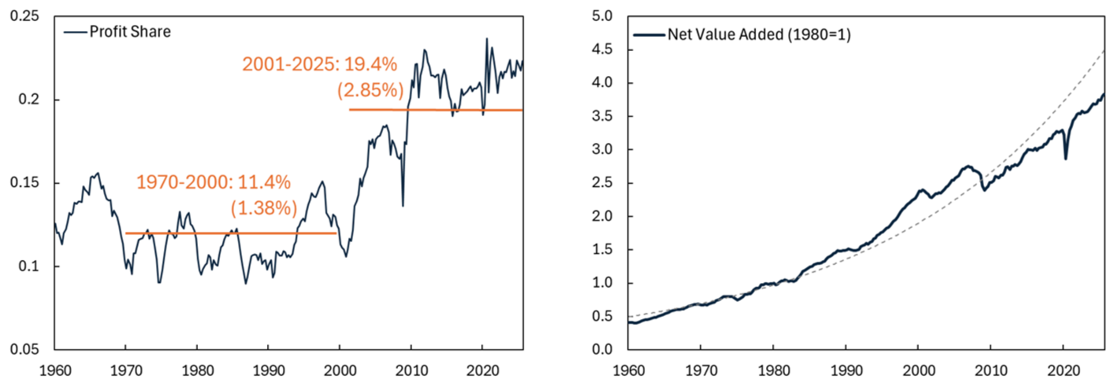
We use the growth of investors' consumption $C_{k,t}$ and equity payouts D_t to proxy for x_t and z_t within our asset pricing framework. We update habit formation accordingly, investors care only about their own past consumption. The key moments for pricing equity become $\rho_{x,z} = \rho(\Delta c_k, \Delta d)$, as well as $\{\sigma_{\Delta c_k}, \sigma_{\Delta d}\}$. Allowing for equity holders to earn non-equity earnings has two key implications. First, the correlation between their valuation ($\ln x = \Delta c_k$) and the dividend process ($\ln z = \Delta d$) becomes imperfect ($\rho_{x,z} < 1$). Second, as the profit-share to GDP rises in the aggregate economy, the share of equity earnings to total income of equity holders also rises ($\rho_{x,z} \uparrow$).

Figure 2 plots the two data processes underlying consumption and dividends, namely, the profit share S_t and our measure of output Y_t. There are significant data challenges to shift focus from aggregate consumption to more appropriate measures such as equity-holder or asset-holder consumption. Rather than attempting to measure C_k in the data, we use our model-implied measure which is entirely driven by the processes of S_t and Y_t in the data.[15]

We conduct two sets of exercises. In Section 4.1, we calibrate the model to average moments of the data over a long sample (1960Q1-2023Q4), and conduct counterfactual exercises where we manipulate a single moment of x or z keeping all else constant. In

[15] We discuss data challenges pertaining to both measurement of the profit share and equity holders consumption to Section C.3.

Figure 2: *Profit Share and Output*



Note: The profit share is a ratio of after-tax profits to output, adjusted for foreign earnings, and the output is a real net value added for the corporate sector normalized to 1980 value, see Appendix A. The values in parentheses indicate standard deviations of sub-periods. Source: National Income and Product Accounts (NIPA) data over the period 1960:Q1-2025:Q3.

Section 4.2, we estimate a sequence of static models to match the evolution of the lower bound and a number of observable asset pricing moments quarter by quarter, and we derive the model-implied ERP series.

4.1. Comparative Statics

We consider the baseline model in Section 3, with preferences given by (10), calibrated using annualized moments of the model-implied processes for log equity holder consumption and dividend growth, given by (22) and (23), for x and z respectively over the period 1960Q1-2023Q4. We calculate innovations to dividend growth by regressing it on its lag and taking the residual, and we use the volatility of these residuals for the volatility of dividend growth rate in the model. We assume an investment share $\omega = 0.0558$, matching the average net investment to net value added ratio. Moreover, we choose $\zeta = 0.71$, which implies that equity-holder consumption accounts for ≈70% of total output.[16] Critically, the resulting correlation between investors' consumption and dividend growth is 58%, comparable to Bansal and Yaron (2004).[17]

[16]It follows from (22) that $\zeta = \frac{\overline{C_k/Y} - (\overline{S} - \omega)}{1 - \overline{S}}$. While equity ownership is highly concentrated, stock market participation including indirect holdings through retirement accounts extends to 58% of U.S. households (2022 Survey of Consumer Finances). In turn, the top three income quintiles account for approximately 79% of aggregate consumer spending (BLS Consumer Expenditure Surveys, 2024), so the implied C_k/Y is broadly consistent with equity holders spanning the upper half to two-thirds of the income distribution.

[17]Our robustness analysis in Figure 10 shows that lowering ζ to 0.48 increases the baseline correlation only modestly, from 58% to approximately 63%, and our main qualitative results are unchanged.

We calibrate the preference and leverage parameters $\{\theta, A, \beta, \lambda\}$ to match the unconditional means of the equity return, the risk-free rate, the lower bound and the dividend yield over our sample, and we assume the probability of disasters is zero $(p = 0)$. We construct unconditional averages using the CRSP quarterly value-weighted return as the equity return and the three-month U.S. Treasury bill rate as the risk-free rate. In order to convert to real rates, we calculate an expected inflation by regressing the quarterly GDP deflator inflation on year-lags of inflation. The lower bound is constructed using a quarterly average of the 3-month SVIX, but we truncate the top decile to remove the most extreme realizations of the GFC. Still, during the GFC, the lower bound hikes to 20% which is likely to violate the NCC, a point we discuss further in 4.2. Table 1 summarizes our calibration.[18]

Table 1: *Baseline Calibration and Moments*

Parameter Values:		Shock Process (p.a.):	
ζ	0.7141	μ_x	0.0353
ω	0.0558	σ_x	0.0347
$\hat{\theta}$	0.8702	μ_z	0.0497
$\hat{A}$	13.7935	σ_z	0.1861
$\hat{\beta}$	1.3916	$\rho_{x,z}$	0.5808
$\hat{\lambda}$	0.9686		
Unconditional Moments (p.a.):			
Targeted	Data	Model	
$E(R)$	7.865%	7.865%	
$E(R_f)$	1.142%	1.142%	
$E(\mathcal{L}^b)$	3.964%	3.964%	
$E(D^a/P)$	2.777%	2.777%	
Non-targeted			
$\sigma(R)$	17.023%	21.944%	
$\sigma(R_f)$	1.214%	3.051%	

Notes: (i) Following Abel (1999), we assume additional restrictions: $\gamma_0 = 0$, $\gamma_0 + \gamma_1 + \gamma_2 = 1$ and $G = 1 + \mu_x$. (ii) The dividend yield D^a/P considers the sum of dividends paid over four quarters per share, divided by the share price. (iii) To construct σ_z, we calculate innovations to dividend growth and take their volatility (18.61%) but the unconditional volatility is very similar (18.62%). (iv) The sample for the SVIX is 1996Q1-2012Q1 due to data availability.

[18]The calibration must satisfy two stationarity conditions: a finite utility ($U_t < \infty$) and the asset pricing convergence ($\kappa(\lambda) \in [0, 1]$). The finite utility is equivalent to $\beta\,\mathbb{E}(x^{1-A+\theta}) < 1$, see footnote 3 in Abel (1999).

In our calibration, payouts to equity holders coincide with net corporate earnings which are very volatile in the data ($\sigma_z \approx 20\%$). Yet, dividend smoothing is a pronounced feature in practice, see e.g. Chen, Da and Priestley (2012); Eeckhout (2025). As a result, within our framework where payouts are given by z_t^λ, we estimate leverage $\lambda < 1$, suggesting that the priced cash-flow is less volatile than net earnings. This is in contrast to classical papers studying the equity premium which proxy dividends using aggregate consumption which is much less volatile, so they require λ to be significantly larger than 1. For the remaining parameters, we estimate a risk aversion parameter (A) close to 14.[19] The habits parameter $\theta \approx 0.9$ which may appear low, but habits are needed both to reconcile the volatility of the risk-free rate and to help match the mean of the lower bound. When $\theta \to 0$, because processes for both x_t and z_t are i.i.d, the risk-free rate would be constant. Moreover, we highlight that the lower bound falls by about 0.5 p.p. if we eliminate the term premium, see Table 4 in Appendix C.1. With regards to the discount factor, note that β is a composite parameter incorporating the trend growth, while the pure time preference parameter is given by $\delta = 0.92$.

To illustrate the effects of the shift in the profit share on the relevant asset pricing moments, we approximate consumption and dividend growth as follows:

Lemma 2. *A first-order Taylor approximation around $\overline{S}$ delivers:*

$$\Delta c_{k,t} \approx \Delta y_t + \underbrace{\frac{(1-\zeta)\overline{S}}{\zeta - \omega + (1-\zeta)\overline{S}}}_{\phi_C(\overline{S})} \Delta s_t, \tag{24}$$

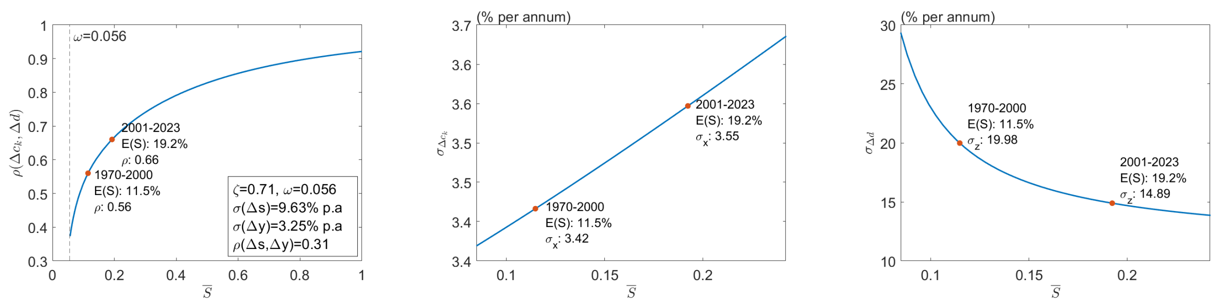
$$\Delta d_t \approx \Delta y_t + \underbrace{\frac{\overline{S}}{\overline{S} - \omega}}_{\phi_D(\overline{S})} \Delta s_t, \tag{25}$$

The correlation between consumption growth and the returns to equity $\rho_{x,z}$ is strictly increasing in $\overline{S}$ as long as $\zeta - \omega > 0$. In contrast, the volatility of dividends σ_z is decreasing in $\overline{S}$, while the volatility of valuations σ_x is rising. Figure 3 summarizes the relationship between asset pricing moments $\{\rho_{x,z}, \sigma_x, \sigma_z\}$ and $\overline{S}$ and illustrates how these change moving from a 'steady-state' profit share of 11.5% to one of 19.2%, coinciding with averages during the periods 1970Q1-2000Q4 and 2001Q1-2023Q4 respectively. For the case of the correlation of log consumption growth and log dividend

[19] Because of the high risk aversion, the NCC holds well for reasonable values of $\rho_{x,z}$.

growth, this rises from 56% to 66%. Interestingly, the correlation is sensitive to changes in the profit share because of leverage effects discussed in GLL, arising due to the presence of the investment share ω in the denominator of both ϕ_C and ϕ_D.

Figure 3: *Volatilities and Correlation of Log of Consumption and Dividend Growth*



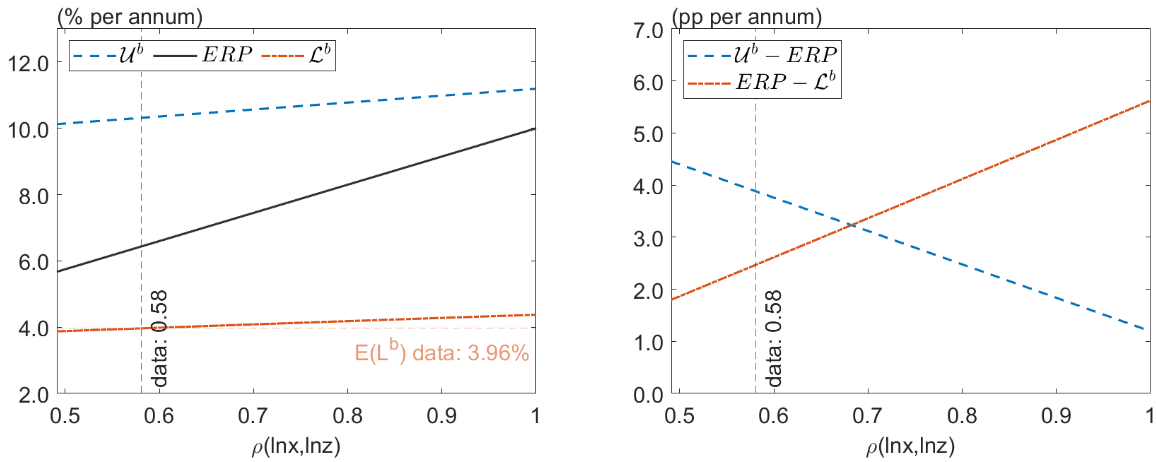
Note: Comparative statics for moments of log consumption growth (24) and log dividend growth (25), as functions of the average profit $\bar{S}$.

Figure 4 (LHS) plots the ERP and its bounds for infinitely-lived equity, corresponding to (14)-(16) from Lemma 1, using our estimated parameters and moments. The x-axis $\rho_{x,z}$ now corresponds to the correlation between the equity-holders' valuation and dividend (net earnings) growth. At the correlation of 0.58 (from the data), we exactly match the means of equity return, risk-free rate, dividend yield, and the lower bound in the data. The average lower bound in our calibration is 3.96%, implying an SVIX of 20%, which rises modestly to just under 21% as the correlation approaches 1.[20]

Accounting for the change in the profit share, ceteris paribus, the correlation ($\rho_{x,z} = 56\%$) rises from 56% to 66%. As a consequence, the ERP increases from 6.26% to 7.11%, in line with compensation for risk, yet the upper and lower bounds rise modestly. The resulting gap between $\mathcal{L}^b$ and ERP increases from 2.32 p.p. to 3.06 p.p., a difference of 0.74 p.p. (Figure 4 RHS), out of the total increase in the ERP of 0.85 p.p. Based on the change in $\mathcal{L}^b$, using (6), we calculate that the corresponding rise in the SVIX^2 is only 11 b.p. (SVIX rises by 26 b.p.). This example illustrates that higher risk premia need not be reflected in the SVIX when the underlying forces also change the correlation of valuations and returns. Moreover, in the limit $\rho_{x,z} \to 1$, the model-implied gap between ERP and lower bound is largest at 5.62 p.p. while that of upper bound is 1.19 p.p.

[20] Given that our model is log-normal, the relationship between the VIX and SVIX is counterfactual. As a result, we report values of the SVIX everywhere, see Martin (2017) for a comparison of the two measures.

Figure 4: *Matching the ERP and Bounds with Correlation*



Note: The panel on the left plots the ERP and its bounds for $n = \infty$, annualized. The model is calibrated to match unconditional moments of stock return and risk-free rate based on a 58% correlation between $\ln x$ and $\ln z$. The right panel shows the gap between ERP and upper (lower) bound decreases (increases) in the correlation.

We decompose the ERP and its bounds into a term premium and a short-term risk premium. In our calibration, the risk premium accounts for 78% of the total ERP – so the gap between the ERP and the lower bound predominantly arises due to the forces analytically detailed in Corollary 1. However, whilst the lower bound on short-term equity is flat, the contribution of the term premium to the lower bound (82% for our baseline) does increase modestly with the correlation. As such, habits $\theta > 0$ contribute to explaining the level and slope of the lower bound. Figure 9 in Appendix C.1 illustrates the decomposition.

We also investigate how the ERP and its bounds change with the volatilities of payouts and valuations, $\{\sigma_z, \sigma_x\}$ respectively. Consistent with our analytical results, the gap between the ERP and its lower bound is rising with the volatility of valuations (assuming the NCC is satisfied), whereas it falls as the volatility of payoffs rises, as illustrated in Figure 10 Appendix C.2. The rise in the profit share implies a small increase in the volatility of valuations (from 3.4% to 3.5%) and a substantial decrease in the volatility of payoffs (from 20% to 14.9%), both of which lead to a widening to the gap between the ERP and the lower bound, reported in Table 5.

4.2. Matching the Lower Bound over Time

We next estimate a series of static models, matching quarterly variation in the SVIX-implied lower bound, as well as the level and volatility of the risk-free rate, the term premium on the 5-year TIPS and the dividend yield, and we investigate the model-implied variation in the unobservable ERP. As well as parameters $\{\theta, A, \beta, \lambda\}$, we allow for a positive probability of disaster p, assuming this is a common shock χ_t to valuations and payoffs.[21] We set the losses during disasters b (and the size of the corresponding bonanza b_H) to 15%, as in Gourio (2012). We restrict our sample to 1996Q1-2012Q1 due to our data availability for the SVIX. We use a 10 year (backward) rolling average window to estimate proxies for the conditional means, variances and correlation of payoffs and valuations.[22] Despite the smoothing, these moments jump significantly during the GFC. We report data and asset pricing moments, as well as the time series for $\{\mu_x, \mu_z, \sigma_x, \sigma_z, \rho_{x,z}\}$ in Appendix C.4.

Figure 5 illustrates the results. The lower bound fluctuates around 1-6% pre-GFC, peaks at 20%, returning to between 3-9% post GFC, and the model is able to almost exactly match this quarter by quarter. The model predicted ERP is above the lower bound for all quarters with the exception of 2008Q4, where the NCC condition fails. Extrapolating from our analytical results for the case of short-lived equity, the failure can be largely attributed to a fall in the estimated coefficient of risk aversion and a rise in estimated leverage.[23] Our main takeaway from this exercise is that the gap is time-varying, can be large and is usually positive. In the pre-2008Q4 period the gap was around 3.95% on average, but rose to 6.78% post-GFC. It is as small as 0.39% in 2009Q1 and as large as 9.67% in 2010Q3. For comparison, we also plot a 10 year rolling average of the realized ERP and show it differs substantially from both the model-implied and the SVIX-implied ERP.[24]

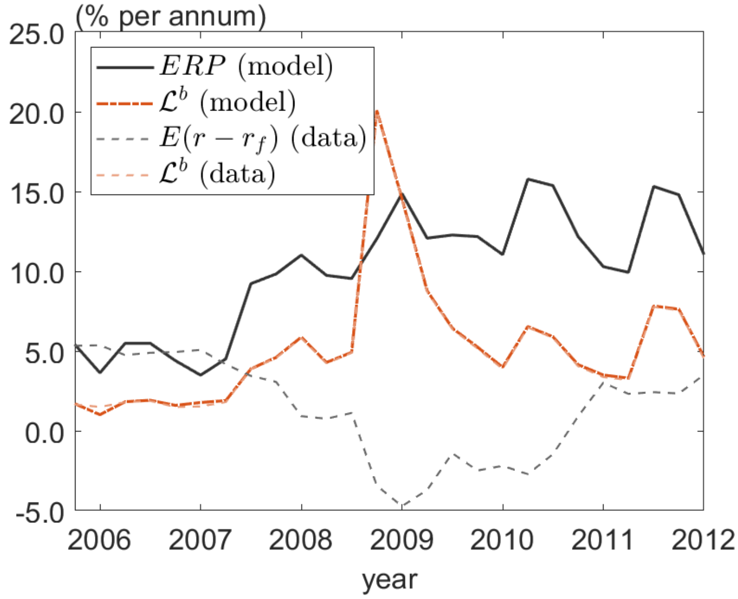
[21]We describe how disasters enter the economy and their process (18) in Section 3.1. The literature widely allows for movements in the discount factor β, and Greenwald et al. (2025) also estimate a process for the price of risk, corresponding to A. Time variation in habit formation (θ) can capture more general processes of habit formation (e.g. Campbell et al., 2020) while movements in λ reflect dividend smoothing behaviour.

[22]We use long windows to estimate conditional moments which are very sensitive to noise (see, e.g., Merton, 1980; Forbes and Rigobon, 2002). Additionally, this captures that investors may update information slowly over time (see, e.g., Kozlowski et al., 2020).

[23]In Appendix C.4, we re-estimate the model keeping the level of risk aversion fixed to its average $A = 13.79$ and the ERP is largely unchanged, but the NCC condition is never violated.

[24]Martin (2017) shows that a predictive regression of realized excess returns on the SVIX-based lower

Figure 5: *Rolling Estimation*



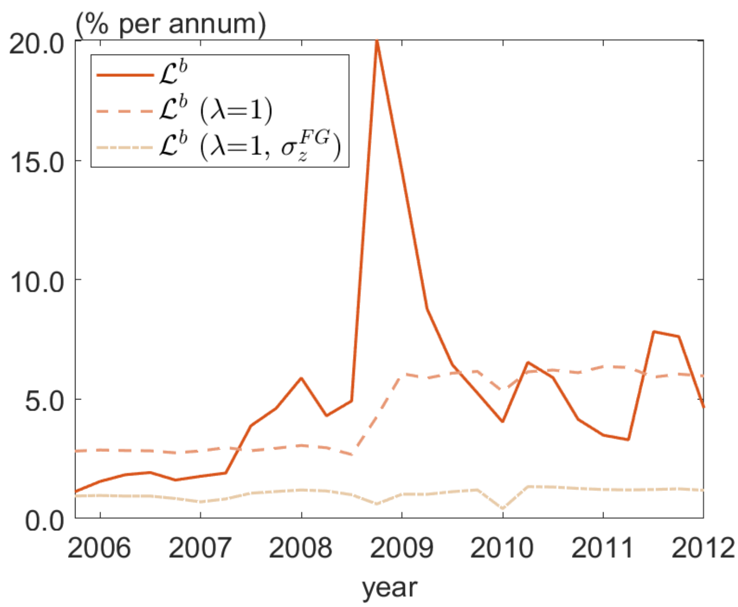
Note: For each quarter, the model is estimated to match the lower bound $\mathcal{L}^b$, constructed using the 3-month SVIX. The log excess return of equity from data is 10-year rolling average of realized excess return. The ERP is conditional expectation of log excess return implied by our model 3 months ahead.

We discuss some key features of the underlying moments of the data drive our results. Focusing on the lower bound, the spike during the GFC is driven by two forces: (i) a large increase in payoff volatility and (ii) an increase in our estimate for leverage, which doubles from $\lambda < 1$ to over 2. While in normal time, the low value for leverage captures dividend smoothing, during the crisis, as firms cut dividends and suspended buybacks, the priced cash flow becomes amplified relative to earnings, rather than dampened. While our estimated payout volatility remains elevated post GFC, the lower bound comes back down because the risk aversion coefficient A remains low, offset by a higher probability of disasters p. The disaster probability rises from close to 0 to almost 2%. Consistent with Proposition 2, a higher p, alongside a higher estimated correlation between valuations and returns, leads to a larger gap between the ERP and its lower bound. Our estimated parameters $\{\beta, \theta\}$ are decreasing over the sample, consistent with falling average interest rates, and to offset the effects of p on the volatility of interest rates and the term premium.

bound delivers an R^2 as high as almost 5%, substantially exceeding that from forecasts based on valuation ratios.

Figure 6 below decomposes the lower bound into some of its contributors. In particular, the dashed line illustrates the lower bound, keeping all parameters as in our baseline but setting leverage to 1. While the lower bound appears to rise post GFC, the spike is completely absent. More interestingly, the dash-dotted line plots the lower bound when leverage is fixed at 1, and payoff volatility is kept constant at a value of 5.63%, consistent with the calibration in Farhi and Gourio (2018). In this case, the lower bound, and the implied SVIX, are significantly underestimated, do not spike in the GFC and do not rise in the latter half of the sample.

Figure 6: *Decomposing the Lower Bound*



Note: Decomposition of the $\mathcal{L}^b$ in 5, (i) setting $\lambda = 1$, (ii) also setting $\sigma_z^{FG} = 5.63\%$ (as in Farhi and Gourio, 2018).

5. Revisiting Farhi and Gourio (2018)

Finally, we apply our analysis to a representative agent framework with capital accumulation and disasters, as in Gourio (2012); Farhi and Gourio (2018). The only source of uncertainty in the model is the disaster shock χ_{t+1} which follows the same three-point distribution (18), as in Section 3. Our analysis reveals two main findings, echoing our existing results. First, the gap between the ERP and the SVIX-implied lower bound is non-zero and rises across the two samples they consider (from 2.62%

for 1984-2000 to 4.52% for 2001-2015). Second, the model underestimates the level of the SVIX-implied lower bound by nearly a factor of 10. We trace this primarily to insufficient dividend volatility: disaster risk alone generates insufficient implied dividend volatility compared to our measured net earnings, so the contribution of the risk-neutral variance to the ERP is underestimated.

What changes relative to our baseline (Section 3) is that the processes D and C_k are driven by a representative agent model with Cobb-Douglas production, markups and capital accumulation. We focus on a risky balanced growth path (BGP) where the level variables grow at a constant rate but with stochastic shifts. For instance, output on the BGP is given by $Y_t = T_t\, \tilde{S}_t\, y^*$ where T_t and $\tilde{S}_t$ are deterministic and stochastic trends with $\tilde{S}_{t+1} = \tilde{S}_t e^{\chi_{t+1}}$, and y^* is a constant. The martingale shock process governs both (labor-augmenting) technology and capital destruction. We detail the full model in Appendix D and summarize key objects below.

Profits are the sum of rental payments to capital and pure monopoly rents. Consistent with Section 4, we assume dividends are given by net profits. Consumption consists of wage income and dividends. On the BGP:

$$
\begin{aligned}
D_t &= \Pi_t - I_t = \frac{\mu - 1 + \alpha}{\mu} Y_t - I_t = T_t \tilde{S}_t \Big(\frac{\mu - 1 + \alpha}{\mu} y^* - \iota^* \Big) \equiv T_t \tilde{S}_t d^* \\
C_{k,t} &= D_t + W_t N_t = T_t \tilde{S}_t \Big(d^* + \frac{1-\alpha}{\mu} y^* \Big) \equiv T_t \tilde{S}_t c^*,
\end{aligned}
$$

where μ is gross markup, α is the capital share, and ι^*, d^*, and c^* are constants governing key macroeconomic ratios on the BGP. Thus, BGP consumption growth is given by $\frac{C_{k,t+1}}{C_{k,t}} = x e^{\chi_{t+1}}$ where x would be the constant growth rate of deterministic trend T_t. Accordingly, dividend growth is given by $\frac{D_{t+1}}{D_t} = z e^{\chi_{t+1}}$, and $x = z$. Leverage is fixed to unity.

Investors have recursive preferences as in Epstein and Zin (1989), given by:

$$
V_{k,t} = \Big((1-\beta)\, L_{k,t}\, c_{k,t}^{1-\sigma} + \beta\, E_t (V_{k,t+1}^{1-A})^{\frac{1-\sigma}{1-A}} \Big)^{\frac{1}{1-\sigma}},
$$

where β is the discount factor, A is the risk-aversion parameter, and σ is the inverse of EIS. On the BGP, the SDF can be expressed as follows:

$$M_{t+1} = \beta\,(1+g_{pc})^{-\sigma}\,\mathbb{E}\big(e^{(1-A)\chi_{t+1}}\big)^{\frac{A-\sigma}{1-A}} e^{-A\chi_{t+1}}$$
$$\equiv \frac{\beta^{*} e^{-A\chi_{t+1}}}{\mathbb{E}\big(e^{(1-A)\chi_{t+1}}\big)}, \tag{26}$$

where $\beta^{*} \equiv \beta\,(1+g_{pc})^{-\sigma}\,\mathbb{E}\big(e^{(1-A)\chi_{t+1}}\big)^{\frac{1-\sigma}{1-A}}$ is a scaled discount factor (Gourio, 2012).

From the asset pricing equation $P_t = \mathbb{E}_t\big(M_{t+1}(P_{t+1}+D_{t+1})\big)$, where $P_t = T_t\tilde{S}_t p^{*}$ and $D_t = T_t\tilde{S}_t d^{*}$, the price-dividend ratio on the risky BGP is given by $\dfrac{p^{*}}{d^{*}} = \dfrac{\beta^{*}(1+g_T)}{1-\beta^{*}(1+g_T)}$. As a result, equity returns are given by:

$$R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{1}{\beta^{*}} e^{\chi_{t+1}}.$$

The Proposition below details the ERP and its bounds in this environment.

Proposition 3 (ERP and Lower Bound in Farhi and Gourio (2018)) *In the economy with SDF given by* (26) *and disaster* χ_t *, which follows a three-point distribution* (18) *and* $b = b_H$*, the ERP and lower bound are given as follows:*

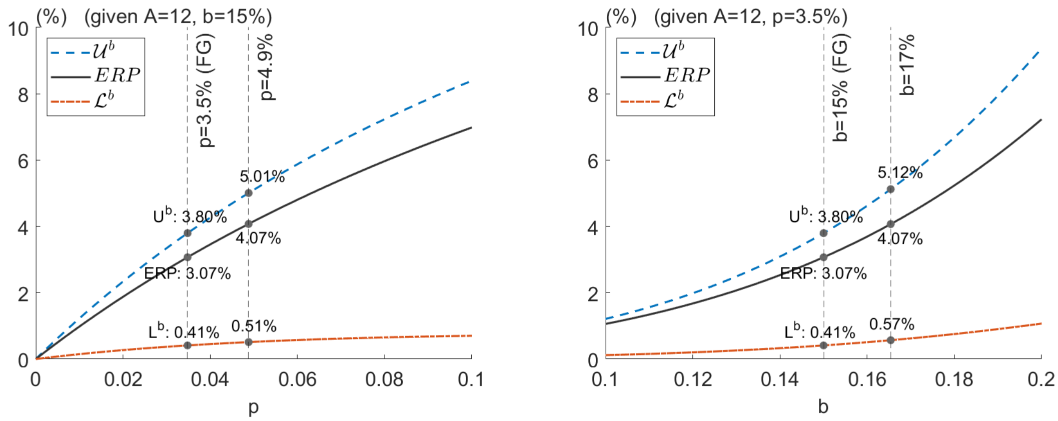
$$ERP = \ln\left(\frac{D_{-A}(p)}{D_{1-A}(p)}\right)$$
$$\mathcal{L}^{b} = \ln\left(\frac{D_{2-A}(p)\,D_{-A}(p)}{D_{1-A}(p)^{2}}\right),$$

where $D_k(p) \equiv 1-2p+p(1+b)^{k}+p(1-b)^{k}$.

Proof: See Appendix D.1.

The expressions correspond to those in Proposition 2, with $\lambda = 1$ and $b = b_H$. Figure 7 illustrates how the ERP and its bounds vary with the disaster probability p (LHS) and the size of the disaster b (RHS). Our headline finding here is that a 1 p.p. rise in the ERP corresponds to 1.4 p.p. increase in the probability of a disaster – but the lower bound rises only by 10 b.p., all else held equal. In this model, the correlation between valuations and returns is always 1, so the gap $ERP - \mathcal{L}^{b}$ instead increases due to changes in the relative volatilities of valuations and payoffs. Next, we investigate how the ERP has risen across two BGPs, calibrated to match changes in key macroeconomic ratios over time.

Figure 7: *ERP and Bounds in Farhi and Gourio (2018)*



Note: Left (Right) panel shows the ERP and its bounds as functions of p (b) with fixed $b = 0.15$ ($p = 0.035$). A 1.4 p.p. increase in p with fixed b, or a 2 p.p. increase in b with fixed p, both imply 1 p.p. increase in ERP, and the corresponding increases in $\mathcal{L}^b$ are 10 b.p. and 16 b.p., respectively.

Quantitative Exercise The authors conduct an estimation exercise to match nine key macroeconomic ratios, including the profit (capital) share and the price-dividend ratio. The authors identify a rise in the ERP from 3.02% (1980-2000) to 5.11% (2001-2016), due to a number of parameter changes, including an almost 3% increase in disaster probability (3.4% to 6.5%).[25]

First, we find that the gap (or bias) between the SVIX-implied lower bound and the average ERP is non-zero and varies over the two samples (BGPs), rising from 2.62% to 4.52%. This reflects a substantial increase in the ERP, entirely driven by a fall in the risk free rate, reported in Table 2 below. Second, however, the calibration strongly underestimates the implied lower bound, which rises from 0.41% to 0.60%. For comparison, the corresponding implied SVIX-based lower bound in the data would be 4.58% in the early period and 4.70% in the latter period. While the model is able to match equity returns and the ERP, this suggests the underlying risk neutral variance of returns is strongly underestimated, and this is exactly what is captured by the large (negative) implied NCC covariance.

Drawing on both the decomposition in Section 4.2 and our analytical results (Corollary 1), we attribute the low SVIX-implied lower bound in the FG model, predominantly, to an insufficient volatility of dividends. In the FG framework, the implied volatility of

[25]Table 6 in Appendix D details the calibration and moments in the Farhi and Gourio (2018) for the periods 1984-2000 and 2001-2016. The externally calibrated parameters are $A = 12$ and $\sigma = 0.5$ and $b = 15\%$.

Table 2: *ERP and Bounds in Farhi and Gourio (2018)*

Mean of Returns:		
	1984-2000	2001-2016
R_{t+1}	5.939%	4.878%
$R_{f,t}$	2.787%	-0.350%
ERP	3.020%	5.113%
$\mathcal{L}^b$	0.405%	0.596%
$ERP - \mathcal{L}^b$	2.615%	4.517%

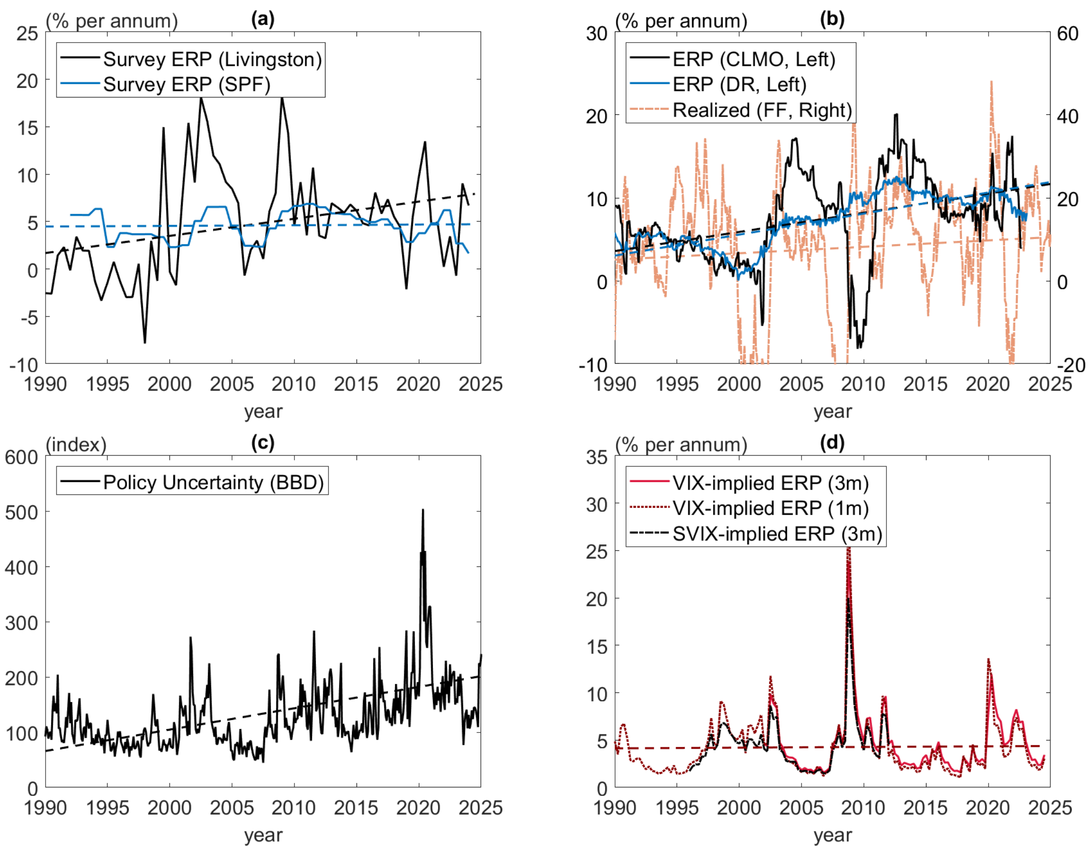
Note: The risk-free rate and price-dividend ratio are amongst the estimation targets for each sub-period, see Table 6 for a full list. R_{t+1}, ERP, and $\mathcal{L}^b$ are the untargetted, model-implied stock return, the ERP, and lower bound.

dividends is 3.96% (1984-2000) and 5.63% (2001-16), relative to our measured volatility of net earnings of 17.67% (1984-2000) and 18.64% (2001-16). The discrepancy arises for two reasons. First, disaster risk alone does not generate sufficient volatility, despite the large crash of 15%, because of their low probability. Consistent with this, Martin (2017) shows that the implied (6 month ahead) probability of a 20% stock price crash from the SVIX series is, on average, over 6%, and has a maximum over 15%, dwarfing the estimates in Table 6. Second, in our baseline exercise, we consider net earnings which are more volatile than measures of dividend payouts because of the possibility of dividend smoothing.

5.1. Comparing to other Measures of Risk

The fundamental problem motivating our paper is that the equity risk premium — the conditional expectation of excess returns — is unobservable. There are, however, many proxies available in the literature, which we plot in Figure 8. Panel (a) reports survey-based measures of the ERP from the Livingston Survey and the Survey of Professional Forecasters. Panel (b) reports model-implied expectations based on valuation ratios, following Duarte and Rosa (2015) and Corsetti et al. (2025), alongside the realized excess returns. Panel (c) plots the policy uncertainty index from Baker et al. (2016), and Panel (d) reports the ERPs implied by the annualized VIX (30-day) and (S)VIX (3-month horizon), respectively. Several features of the data are noteworthy. First, different estimates of the ERP differ substantially in their levels, volatilities, and

Figure 8: *Empirical ERPs and VIXs*



Note: (a) Authors' calculation based on current and expected one-year future S&P 500 index from the Livingston Survey. The ERP from the Survey of Professional Forecasters (SPF) is constructed from the median value of individual responses of expected returns on the S&P 500 over the next ten years. (b) Model-implied ERPs from Corsetti et al. (2025) (CLMO) and Duarte and Rosa (2015) (DR) (left axis) and the realized annual excess return from Kenneth French data library (right axis). (c) The policy uncertainty index is from Baker, Bloom and Davis (2016). (d) (S)VIX (3 month) and VIX (1 month)-implied ERPs constructed using (5)-(6). Samples differ due to data availability.

cyclical behavior. Survey-based and valuation-based measures both exhibit a mild upward trend over our sample, and spike during crises (2000, 2008, 2020). Second, realized excess returns (Panel b) are extremely noisy — consistent with Campbell and Thompson (2008) and others who argue that even long samples have little power to distinguish expected returns from zero. Third, and most relevant for our analysis, the VIX and SVIX in Panel (d) exhibit no comparable upward trend. The SVIX-implied lower bound fluctuates around a mean just under 5%, spiking in crises but reverting quickly. The correlation between the VIX and the survey or valuation-based ERP measures is generally low: the highest pairwise correlation is 44.1% with the Livingston

Survey.[26] This disconnect between trending ERP proxies and measures of risk, vis-a-vis a non-trending VIX is a pattern our framework can rationalize — a rising correlation between valuations and returns, or a rising probability of disasters, can increase the ERP while leaving the SVIX largely unchanged.

6. CONCLUSION

This paper investigates volatility bounds through the lens of macro-finance models. We show that the equity risk premium (ERP) can fluctuate substantially without corresponding movements in the SVIX or VIX. Within a canonical asset-pricing framework, we characterize the gap between the ERP and the SVIX-implied lower bound (Martin, 2017) as an equilibrium object which depends on both preference parameters and moments of payoffs and valuations.

We demonstrate (analytically for the case of short-lived equity, quantitatively otherwise) that as the correlation between valuations and dividends (payouts) rises, while the ERP and the Hansen and Jagannathan (1991) upper bound increase, the lower bound is largely insensitive to it. We provide a simple macroeconomic mechanism for this in the data: when equity holders' consumption is imperfectly correlated with dividends, changes in the aggregate profit share alter the correlation between consumption growth and equity returns. Moreover, simulating our model to match the SVIX-implied lower bound quarter by quarter, alongside other asset pricing moments, implies a gap to the ERP that is positive outside of the GFC and highly time-varying. We further confirm our results in the RBC model of Farhi and Gourio (2018).

Our analysis has stark implications for both theory and policy. On the theory side, we show that quantitative macroeconomic models risk substantially understating the relevance of the risk-neutral variance of returns, even when they are calibrated to match the unconditional averages of equity premia. In particular, we show that relying on a high degree of risk aversion necessarily implies these models will struggle to match the level of the (S)VIX. From a policy perspective, while the (S)VIX is our best measure of equity premia, we argue that a stable (S)VIX should not be immediately interpreted as evidence of unchanged aggregate risk or stable expected returns. Structural changes

[26] The correlations of VIX with the SPF, CLMO, and DR are 12.2%, -32.4%, and -4.9%, respectively.

that reallocate risk—such as shifts in factor shares or wealth concentration, or changes in perceived disaster probabilities—can raise risk premia without increasing market-implied volatility.

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A. Data Description

Rates The equity return is the quarterly value-weighted return including dividends on the NYSE, AMEX, and NASDAQ from the Center for Research in Security Prices (CRSP). The nominal risk-free rate is measured as the quarterly average of monthly 3-month Treasury Bill secondary market rate from the Federal Reserve Bank of St. Louis (FRED). We construct real equity returns and risk-free rates by adjusting with an expected inflation rate, estimated by regressing a GDP deflator inflation on 1-year lag of the inflation, where the seasonally-adjusted GDP deflator is from the Bureau of Economic Analysis (BEA).

Net Value Added and Profit Share We follow Greenwald et al. (2025) in constructing the net value added (NVA) Y_t and the profit share S_t. We focus on domestic corporate business and take nominal NVA from the BEA NIPA Table 1.14. We construct the price deflator as nominal NVA divided by the chained dollar real net value added of nonfinancial corporate business and take it as a proxy of corporate business deflator. The profit share is total earnings over NVA where the total earnings consist of domestic after-tax profits and foreign earnings. We measure foreign earnings by subtracting the domestic corporate profits with IVA and CCAdj from the Table 1.14 from the corporate profits with IVA and CCAdj in National Income level from the BEA NIPA Table 1.12.

(S)VIXs and Valuation The VIXs at 30-days and 3-month horizons are available on Bloomberg terminal (ticker: `VIX Index`, `VIX3M Index`). For the series of SVIX^2 used to construct the lower bound, we use the spreadsheet `SVIX2.xls` from Ian Martin's website. The file provides daily SVIX^2 of 1, 2, 3, 6, and 12 months horizons from Jan. 4, 1996 to Jan. 31, 2012. We take 3-month horizon SVIX^2 and take averages for each quarter to get quarterly SVIX^2. The dividend per share is obtained from CRSP value-weighted returns both including and excluding dividends. Using the return excl. dividends, we calculate a series of price per share. To annualize the yield, we take a sum of quarterly dividends per share paid over four quarters.

Correlation between Consumption and Dividend Growth Table 3 illustrates the results. c^{agg} is log of real personal consumption expenditures (PCE) and c^{nd} is

Table 3

Variables:	Δc^{agg}	Δc^{nd}	Δc^{asset}	$\Delta \hat{c}^{CC}$	$\Delta \hat{c}^{BY}$	Δc_k
Δd	30.6%	19.9%	12.6%	20.0%	55.0%	58.1%

log of real PCE of nondurable goods and services from the BEA NIPA data over the period 1960:Q1-2023:Q4. c^{asset} is log of asset holders' consumption from Elkamhi and Jo (2023). Assetholders are defined as households who own any stocks, bonds, mutual funds, or other such securities by using the Consumer Expenditure Survey (CEX) by the Bureau of Labor Statistics (BLS). Correlations for each $\hat{c}^{CC}$ and $\hat{c}^{BY}$ are from Campbell and Cochrane (1999) and Bansal and Yaron (2004), respectively. c_k and d are our model-implied log of investor consumption and net earnings (payouts).

Measurement Issues Ideally, we would confirm the model-implied consumption dynamics with equity or asset holder consumption in the data from, e.g. CEX data. There are at least two issues with this. First, Brav, Constantinides and Geczy (2002) emphasize that restricting the sample to asset holders dramatically reduces sample size (in some quarters, as few as 13-71 households meet high-wealth thresholds), leading to questionable representation. Second, Kozliakov et al. (2026) show that the consumption dynamics of high income, high wealth individuals (which constitute 41% of stockholders in CEX data) actually predict a *negative* Sharpe ratio, and argue this is evidence that measures of consumption growth is a poor proxy for the valuations of these households.

With regards to the profit share, Koh, Santaeulàlia-Llopis and Zheng (2020) and Atkeson (2020) argue that secular trends in factor shares appear to depend heavily on the accounting treatment of intangible investment, as well as raising questions about the measurement of the profit share itself. GLL argue that the net profit share is much better measured and that most of these concerns do not apply to the post-1989 (which we focus on).[27] The GLL measure also adresses international profit-shifting by U.S. firms (Bertaut, Bressler and Curcuru, 2021; Guvenen, Mataloni Jr, Rassier and Ruhl, 2022) by including both domestic and foreign subsidiaries in the calculation of the

[27]The profit share over the period 1990Q1-2000Q4 is 12.2%, very comparable to our baseline of 11.4%.

profit share.

B. DERIVATIONS FOR SECTION. 3

SDF Consider the utility function:

$$U_t = \mathbb{E}_t\Big[\sum_{j=0}^{\infty}\Big(\frac{1}{1+\delta}\Big)^j \frac{1}{1-\sigma}\Big(\frac{C_t}{(C_t^a)^{\gamma_0}(C_{t-1}^a)^{\gamma_1}(G^t)^{\gamma_2}}\Big)^{1-\sigma}\Big].$$

The SDF is defined as:

$$\begin{aligned} M_{t+1} &\equiv \frac{\partial U_t/\partial C_{t+1}}{\partial U_t/\partial C_t} \\ &= \Big(\frac{1}{1+\delta}\Big)\Big(\frac{C_{t+1}}{C_t}\Big)^{-\sigma}\Big(\Big(\frac{C_{t+1}^a}{C_t^a}\Big)^{\gamma_0}\Big(\frac{C_t^a}{C_{t-1}^a}\Big)^{\gamma_1} G^{\gamma_2}\Big)^{\sigma-1}. \end{aligned}$$

In equilibrium $\dfrac{C_{t+1}}{C_t} = \dfrac{C_{t+1}^a}{C_t^a} \equiv x_{t+1}$ yielding the result:

$$M_{t+1} = \frac{G^{\gamma_2(\sigma-1)}}{1+\delta}\, x_{t+1}^{-\sigma+\gamma_0(\sigma-1)}\, x_t^{\gamma_1(\sigma-1)}.$$

Equity Returns Consider n-period equity which pays D_{t+n-j}^{λ} in the period that is j periods before it matures. By letting the price at period t as $p_t(n,\lambda)$, we can write

$$R_{t+1}(n,\lambda) = \frac{p_{t+1}(n-1,\lambda) + D_{t+1}^{\lambda}}{p_t(n,\lambda)}.$$

Following Abel (1999), we can guess the price as $p_t(n,\lambda) = \omega(n,\lambda)\, x_t^{\theta}\, D_t^{\lambda}$. assuming processes are i.i.d. With the SDF given above, the Euler equation for asset price $\mathbb{E}_t\big(R_{t+1}(n,\lambda)\, M_{t+1}\big) = 1$ implies a difference equation for $\omega(n,\lambda)$:

$$\omega(n,\lambda) = \beta\,\mathbb{E}(x^{\theta-A}z^{\lambda})\,\omega(n-1,\lambda) + \beta\,\mathbb{E}(x^{-A}z^{\lambda}) \;\text{ for } n \geq 1.$$

For convenience, we assume the payoff happens at the beginning of each period which implies a boundary condition for the difference equation: $\omega(0,\lambda) = 0$. Then by solving by iteration, we get:

$$\omega(n,\lambda) = \beta\,\mathbb{E}(x^{-A}z^{\lambda})\sum_{i=0}^{n-1}\kappa(\lambda)^i,$$

where $\kappa(\lambda) \equiv \beta\,\mathbb{E}(x^{\theta-A}z^{\lambda}) < 1$ is assumed. By substituting $\omega(n,\lambda)$ into the return equation, we get

$$R_{t+1}(n,\lambda) = \left[\Big(\frac{1-\kappa(\lambda)^{n-1}}{1-\kappa(\lambda)^{n}}\Big)x_{t+1}^{\theta}z_{t+1}^{\lambda} + \Big(\frac{1-\kappa(\lambda)}{1-\kappa(\lambda)^{n}}\Big)\frac{1}{\beta\,\mathbb{E}(x^{-A}z^{\lambda})}z_{t+1}^{\lambda}\right]x_t^{-\theta} \quad \text{for } n \geq 2.$$

For the infinitely lived equity as we take in the paper, the equity return $R_{t+1} \equiv R_{t+1}(\infty,\lambda)$ is:

$$R_{t+1} = \left[x_{t+1}^{\theta}z_{t+1}^{\lambda} + \frac{1-\beta\,\mathbb{E}(x^{\theta-A}z^{\lambda})}{\beta\,\mathbb{E}(x^{-A}z^{\lambda})}z_{t+1}^{\lambda}\right]x_t^{-\theta}.$$

Deriving the Lower Bound Note that the risk-neutral variance of $R_{t+1}/R_{f,t}$ can be re-written as:

$$\begin{aligned} Var_t^*\Big(\frac{R_{t+1}}{R_{f,t}}\Big) &= \frac{1}{R_{f,t}^2}\Big(\mathbb{E}_t^*(R_{t+1}^2) - \mathbb{E}_t^*(R_{t+1})^2\Big) \\ &= \frac{1}{R_{f,t}^2}\Big(R_{f,t}\,\mathbb{E}_t(M_{t+1}R_{t+1}^2) - R_{f,t}^2\Big) \\ &= \frac{\mathbb{E}_t(M_{t+1}R_{t+1}^2)}{R_{f,t}} - 1. \end{aligned}$$

After substituting the SDF and equity return, the lower bound can be simplified as:

$$\begin{aligned} \mathcal{L}^b &= \ln\Big(Var_t^*\Big(\frac{R_{t+1}}{R_{f,t}}\Big)+1\Big) = \ln\Big(\frac{\mathbb{E}_t(M_{t+1}R_{t+1}^2)}{R_{f,t}}\Big) \\ &= \ln\left(\frac{\mathbb{E}(x^{-A})\times\Omega(\lambda) + Var(x^{-A}z^{\lambda}) - Cov(x^{-A}, x^{-A}z^{2\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})^2} + 1\right), \end{aligned}$$

where,

$$\Omega(\lambda) \equiv \begin{bmatrix} \beta^2\,\mathbb{E}(x^{-A}z^{\lambda})\Big\{Cov(x^{\theta-A}z^{\lambda}, x^{\theta-A}z^{2\lambda}) - Cov(x^{-A}z^{\lambda}, x^{2\theta-A}z^{2\lambda})\Big\} \\ +\big(\beta^2\,\mathbb{E}(x^{\theta-A}z^{\lambda}) - 2\beta\big)\Big\{Cov(x^{-A}z^{\lambda}, x^{\theta-A}z^{2\lambda}) - Cov(x^{-A}z^{2\lambda}, x^{\theta-A}z^{\lambda})\Big\} \end{bmatrix}.$$

Results with Epstein-Zin To extend to Epstein and Zin (1989, 1991) utility, we use SDF (17) and first derive a closed-form of the aggregate return on wealth R_{t+1}^a, and then we get the equity return by using R_{t+1}^a. Since R_{t+1}^a must satisfy the asset pricing equation, we have:

$$\mathbb{E}_t(M_{t+1}R_{t+1}^a) = \mathbb{E}_t\Big(\beta^{\frac{1-A}{1-\sigma}}\,x_{t+1}^{\frac{-\sigma(1-A)}{1-\sigma}}\,(R_{t+1}^a)^{\frac{1-A}{1-\sigma}}\Big) = 1.$$

Following Abel (1999), we guess a price of the wealth portfolio maturing in ∞-period as $p_t^a = \omega^a C_{k,t+1}^a$ so that $R_{t+1}^a = \frac{\omega^a+1}{\omega^a} x_{t+1}$. Substituting it into the asset pricing equation with the nature of i.i.d. yields:

$$R_{t+1}^a = \beta^{-1} \mathbb{E}(x^{1-A})^{-\frac{1-\sigma}{1-A}} x_{t+1}.$$

Then the SDF is $M_{t+1} = \beta\, \mathbb{E}(x^{1-A})^{\frac{A-\sigma}{1-A}} x_{t+1}^{-A}$ and the risk-free rate is as follows:

$$R_{f,t} = \frac{1}{\mathbb{E}_t(M_{t+1})} = \frac{1}{\beta\, \mathbb{E}(x^{1-A})^{\frac{A-\sigma}{1-A}} \mathbb{E}\big(x^{-A}\big)} = \frac{\mathbb{E}\big(x^{1-A}\big)}{\beta^* \,\mathbb{E}\big(x^{-A}\big)},$$

where $\beta^* \equiv \mathbb{E}_t(M_{t+1} x_{t+1}) = \beta\, \mathbb{E}\big(x^{1-A}\big)^{\frac{1-\sigma}{1-A}}$.

Next, the return $R_{t+1}(n,\lambda) = \dfrac{p_{t+1}(n-1,\lambda) + D_{t+1}^{\lambda}}{p_t(n,\lambda)}$ on the equity which pays a payout D_t^{λ} each period satisfies:

$$\mathbb{E}_t\Big(\beta\, \mathbb{E}(x^{1-A})^{\frac{A-\sigma}{1-A}} x_{t+1}^{-A} \times \frac{\omega(n-1,\lambda)+1}{\omega(n,\lambda)} z_{t+1}^{\lambda}\Big) = 1,$$

where we use $p_t(n,\lambda) = \omega(n,\lambda) D_t^{\lambda}$. Following the same procedure in solving the difference equation of $\omega(n,\lambda)$ with assumption of $\beta\, \mathbb{E}(x^{1-A})^{\frac{A-\sigma}{1-A}}\, \mathbb{E}(x^{-A} z^{\lambda}) < 1$, we can derive:

$$\begin{aligned} R_{t+1}(n,\lambda) &= \beta^{-1}\, \mathbb{E}(x^{1-A})^{-\frac{A-\sigma}{1-A}}\, \mathbb{E}(x^{-A} z^{\lambda})^{-1}\, z_{t+1}^{\lambda} \\ &= \frac{\mathbb{E}(x^{1-A})}{\beta^*\, \mathbb{E}(x^{-A} z^{\lambda})} z_{t+1}^{\lambda} \quad \text{for all } n. \end{aligned}$$

The ERP and lower bound are given by:

$$\begin{aligned} ERP &= \ln\Big(\frac{\mathbb{E}_t(R_{t+1}(n,\lambda))}{R_{f,t}}\Big) = \ln\Big(\frac{\mathbb{E}(x^{-A})\, \mathbb{E}(z^{\lambda})}{\mathbb{E}(x^{-A} z^{\lambda})}\Big), \\ \mathcal{L}^b &= \ln\Big(\frac{\mathbb{E}(M_{t+1} R_{t+1}(n,\lambda)^2)}{R_{f,t}}\Big) = \ln\Big(\frac{\mathbb{E}(x^{-A})\, \mathbb{E}(x^{-A} z^{2\lambda})}{\mathbb{E}(x^{-A} z^{\lambda})^2}\Big), \end{aligned}$$

respectively. Under the log-normality, the distance between the ERP and lower bound corresponds to Corollary 1: $ERP - \mathcal{L}^b = \lambda\sigma_z \times (A\,\sigma_x \rho_{x,z} - \lambda\sigma_z)$.

Disaster Risk Considering the consumption and payout in the macro model are proportion of output, we assume the disaster risk as an additional mean-preserving shock on the output so that x_t and z_t are equally affected. Specifically, we take $Y_{t+1} = Y_t S_{t+1}$ with $S_{t+1} = S_t e^{\chi_{t+1}}$ so that $\tilde{x}_{t+1} \equiv \frac{\tilde{C}_{t+1}}{\tilde{C}_t} = \frac{C_{t+1}}{C_t} e^{\chi_{t+1}} = x_{t+1} e^{\chi_{t+1}}$ and $\tilde{z}_{t+1} \equiv \frac{\tilde{D}_{t+1}}{\tilde{D}_t} = \frac{D_{t+1}}{D_t} e^{\chi_{t+1}} = z_{t+1} e^{\chi_{t+1}}$ where χ_{t+1} is the disaster shock which is independent with x_{t+1} and z_{t+1} as well as is i.i.d. and follows three-point distribution:

$$\chi_{t+1} = \begin{cases} 0 & \text{with prob. } 1-2p \\ \ln(1+b_H) & \text{with prob. } p \\ \ln(1-b) & \text{with prob. } p, \end{cases}$$

where b captures how much the growth rates decline in disaster episode, p captures the probability of disaster, and b_H is chosen so that $\mathbb{E}(e^{\chi_{t+1}}) = 1$.

Then the stochastic discount factor is given by:

$$\tilde{M}_{t+1} = \beta \, x_{t+1}^{-A} x_t^{\theta} \, e^{-A\chi_{t+1} + \theta\chi_t}$$

and a disaster-adjusted risk-free rate is

$$\tilde{R}_{f,t} = \frac{1}{\beta \, \mathbb{E}(x^{-A} e^{-A\chi})} x_t^{-\theta} \, e^{-\theta\chi_t}.$$

Additionally, by following the same procedure as above with the modified moments, the equity return can be derived as

$$\tilde{R}_{t+1} = \Big(x_{t+1}^{\theta} z_{t+1}^{\lambda} \, e^{(\theta+\lambda)\chi_{t+1}} + \frac{1 - \beta \, \mathbb{E}(x^{\theta-A} z^{\lambda}) \, \mathbb{E}(e^{(\theta+\lambda-A)\chi})}{\beta \, \mathbb{E}(x^{-A} z^{\lambda}) \, \mathbb{E}(e^{(\lambda-A)\chi})} z_{t+1}^{\lambda} e^{\lambda\chi_{t+1}} \Big) x_t^{-\theta} e^{-\theta\chi_t}.$$

Then we can derive the ERP and its bounds in the exactly same form as the benchmark while updating the moments of x and z with χ_t:

$$E\tilde{R}P_t = \ln\left(\frac{\beta\big(Cov(\tilde{x}^{\theta-A}\tilde{z}^{\lambda}, \tilde{z}^{\lambda}) - Cov(\tilde{x}^{-A}\tilde{z}^{\lambda}, \tilde{x}^{\theta}\tilde{z}^{\lambda})\big)}{\mathbb{E}(\tilde{z}^{\lambda})} + 1 \right) + \ln\left(\frac{-Cov(\tilde{x}^{-A}, \tilde{z}^{\lambda})}{\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{\lambda})} + 1 \right), \tag{27}$$

$$\tilde{\mathcal{U}}_t^b = \ln\left(\beta \, \sigma(\tilde{x}^{-A}) \, \sigma\Big(\tilde{x}^{\theta} \tilde{z}^{\lambda} + \tfrac{1-\beta \, \mathbb{E}(\tilde{x}^{\theta-A}\tilde{z}^{\lambda})}{\beta \, \mathbb{E}(\tilde{x}^{-A}\tilde{z}^{\lambda})} \tilde{z}^{\lambda} \Big) + 1 \right),$$

$$\tilde{\mathcal{L}}_t^b = \ln\Big(\frac{\tilde{\Omega}(\lambda)}{\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{2\lambda})} + 1 \Big) + \ln\Big(\frac{Var(\tilde{x}^{-A}\tilde{z}^{\lambda}) - Cov(\tilde{x}^{-A}, \tilde{x}^{-A}\tilde{z}^{2\lambda})}{\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{\lambda})^2} + 1 \Big), \tag{28}$$

where $\tilde{\Omega}(\lambda) \equiv \begin{bmatrix} \beta^2\,\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{\lambda})\Big\{Cov(\tilde{x}^{\theta-A}\tilde{z}^{\lambda},\tilde{x}^{\theta-A}\tilde{z}^{2\lambda}) - Cov(\tilde{x}^{-A}\tilde{z}^{\lambda},\tilde{x}^{2\theta-A}\tilde{z}^{2\lambda})\Big\} \\ +\big(\beta^2\,\mathbb{E}(\tilde{x}^{\theta-A}\tilde{z}^{\lambda}) - 2\beta\big)\Big\{Cov(\tilde{x}^{-A}\tilde{z}^{\lambda},\tilde{x}^{\theta-A}\tilde{z}^{2\lambda}) - Cov(\tilde{x}^{-A}\tilde{z}^{2\lambda},\tilde{x}^{\theta-A}\tilde{z}^{\lambda})\Big\}. \end{bmatrix}$

B.1. Proof to Lemma 1

Recall the n-period equity return:

$$R_{t+1}(n,\lambda) = \begin{cases} \dfrac{1}{\beta\,\mathbb{E}(x^{-A}z^{\lambda})} z_{t+1}^{\lambda} x_t^{-\theta} & \text{for } n = 1, \\ \Big[\Big(\dfrac{1-\kappa(\lambda)^{n-1}}{1-\kappa(\lambda)^n}\Big) x_{t+1}^{\theta} z_{t+1}^{\lambda} + \Big(\dfrac{1-\kappa(\lambda)}{1-\kappa(\lambda)^n}\Big) \dfrac{1}{\beta\,\mathbb{E}(x^{-A}z^{\lambda})} z_{t+1}^{\lambda}\Big] x_t^{-\theta} & \text{elsewhere.} \end{cases}$$

Then the ERP in case of n can be decomposed into a term premium and a short-term risk premium, which can be further simplified as follows:

$$\begin{aligned} ERP_t^n &= \ln\Big(\frac{\mathbb{E}_t\big(R_{t+1}(n,\lambda)\big)}{\mathbb{E}_t\big(R_{t+1}(1,\lambda)\big)}\Big) + \ln\Big(\frac{\mathbb{E}_t\big(R_{t+1}(1,\lambda)\big)}{R_{f,t}}\Big) \\ &= \ln\left(\frac{\kappa(\lambda)-\kappa(\lambda)^n}{1-\kappa(\lambda)^n} \times \frac{\mathbb{E}(x^{-A}z^{\lambda})\,\mathbb{E}(x^{\theta}z^{\lambda}) - \mathbb{E}(z^{\lambda})\,\mathbb{E}(x^{\theta-A}z^{\lambda})}{\mathbb{E}(z^{\lambda})\mathbb{E}(x^{\theta-A}z^{\lambda})} + 1\right) + \ln\left(\frac{\mathbb{E}(x^{-A})\,\mathbb{E}(z^{\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})}\right). \end{aligned}$$

Taking $n \to \infty$ and re-writing the moments with covariances, we get ERP in Lemma 1.

While the upper bound is immediately obtained by substituting Equations (10) and (13) into $\mathcal{U}_t^b = \ln\big(\sigma_t(M_{t+1})\sigma_t(R_{t+1}) + 1\big)$, we can further decompose the lower bound in case of n as follows:

$$\begin{aligned} \mathcal{L}_t^{b,n} &= \ln\Big(\frac{E_t\big(M_{t+1}\,R_{t+1}(\infty,\lambda)^2\big)}{E_t\big(M_{t+1}\,R_{t+1}(1,\lambda)^2\big)}\Big) + \ln\Big(\frac{E_t\big(M_{t+1}\,R_{t+1}(1,\lambda)^2\big)}{R_{f,t}}\Big) \\ &= \ln\left(\frac{(1-\kappa(\lambda)^{n-1})\big(1-\kappa(\lambda)^n\cdot H(\lambda)\big)}{(1-\kappa(\lambda)^n)^2} \times \frac{\Omega(\lambda)}{\mathbb{E}(x^{-A}z^{2\lambda})} + 1\right) + \ln\left(\frac{\mathbb{E}(x^{-A})\,\mathbb{E}(x^{-A}z^{2\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})^2}\right), \end{aligned}$$

where $H(\lambda) \equiv \dfrac{E(x^{-A}z^{\lambda})^2\,E(x^{2\theta-A}z^{2\lambda}) - E(x^{-A}z^{2\lambda})\,E(x^{\theta-A}z^{\lambda})^2}{\Omega(\lambda)\,E(x^{\theta-A}z^{\lambda})}$. Taking $n \to \infty$ and re-writing the short-term premium term yields the lower bound in Lemma 1. □

B.2. Proof to Proposition 1

The ERP and its bounds for $n = 1$ are

$$
\begin{aligned}
ERP_t^{n=1} &= \ln\left(\frac{\mathbb{E}(x^{-A})\,\mathbb{E}(z^{\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})}\right),\\
\mathcal{U}_t^{b,n=1} &= \ln\left(\frac{\sigma(x^{-A})\,\sigma(z^{\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})}+1\right),\\
\mathcal{L}_t^{b,n=1} &= \ln\left(\frac{\mathbb{E}(x^{-A})\,\mathbb{E}(x^{-A}z^{2\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})^2}\right).
\end{aligned}
$$

Since x_t and z_t are jointly log-normal, we can write

$$
\begin{aligned}
ERP_t^{n=1} &= \ln\left(\frac{e^{-A\mu_x+A^2\sigma_x^2/2}\times e^{\lambda\mu_z+\lambda^2\sigma_z^2/2}}{e^{-A\mu_x+\lambda\mu_z+(A^2\sigma_x^2+\lambda^2\sigma_z^2-2A\lambda\sigma_{x,z})/2}}\right),\\
\mathcal{U}_t^{b,n=1} &= \ln\left(\frac{e^{-A\mu_x+\lambda\mu_z+(A^2\sigma_x^2+\lambda^2\sigma_z^2)/2}\sqrt{e^{A^2\sigma_x^2}-1}\sqrt{e^{\lambda^2\sigma_z^2}-1}}{e^{-A\mu_x+\lambda\mu_z+(A^2\sigma_x^2+\lambda^2\sigma_z^2-2A\lambda\sigma_{x,z})/2}}+1\right),\\
\mathcal{L}_t^{b,n=1} &= \ln\left(\frac{e^{-A\mu_x+A^2\sigma_x^2/2}\times e^{-A\mu_x+2\lambda\mu_z+(A^2\sigma_x^2+4\lambda^2\sigma_z^2-4A\lambda\sigma_{x,z})/2}}{e^{-2A\mu_x+2\lambda\mu_z+A^2\sigma_x^2+\lambda^2\sigma_z^2-2A\lambda\sigma_{x,z}}}\right).
\end{aligned}
$$

Simplifying yields Proposition 1. $\square$

B.3. Proof to Proposition 2

From Equations (27) and (28), the ERP and lower bound of short-term equity are given as follows:

$$
\begin{aligned}
\tilde{ERP}^{n=1} &= \ln\left(\frac{\mathbb{E}(\tilde{x}^{-A})\,\mathbb{E}(\tilde{z}^{\lambda})}{\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{\lambda})}\right)\\
&= \ln\left(\frac{\mathbb{E}(x^{-A})\,\mathbb{E}(z^{\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})}\right)+\ln\left(\frac{\mathbb{E}(e^{-A\chi})\,\mathbb{E}(e^{\lambda\chi})}{\mathbb{E}(e^{(-A+\lambda)\chi})}\right)\\
\tilde{\mathcal{L}}^{b,n=1} &= \ln\left(\frac{\mathbb{E}(\tilde{x}^{-A})\,\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{2\lambda})}{\mathbb{E}(\tilde{x}^{-A}\tilde{z}^{\lambda})^2}\right)\\
&= \ln\left(\frac{\mathbb{E}(x^{-A})\,\mathbb{E}(x^{-A}z^{2\lambda})}{\mathbb{E}(x^{-A}z^{\lambda})^2}\right)+\ln\left(\frac{\mathbb{E}(e^{-A\chi})\,\mathbb{E}(e^{(-A+2\lambda)\chi})}{\mathbb{E}(e^{(-A+\lambda)\chi})^2}\right),
\end{aligned}
$$

where the second equality is from the definition of $\tilde{x}$ and $\tilde{z}$, and the i.i.d. of χ. Given values of b and b_H, define $D_k(p) \equiv 1-2p+p(1+b_H)^k+p(1-b)^k$. Then by using Proposition 1 and substituting the disaster terms, we get the result in Proposition 2.

Now we show that when $b = b_H \in (0, b^{max})$ and $A > \lambda$, the gap increases in $p \in [0,1]$. For any k, let $c(k) \equiv \partial_p D_k(p) = -2 + (1+b)^k + (1-b)^k$ where ∂_p denotes the partial derivative with respect to p so that $D_k(p) = 1 + c(k)p$. Since we have:

$$\partial_p E\tilde{R}P^{n=1} = \frac{c(-A)}{D_{-A}(p)} + \frac{c(\lambda)}{D_\lambda(p)} - \frac{c(\lambda-A)}{D_{\lambda-A}(p)},$$
$$\partial_p \tilde{\mathcal{L}}^{b,n=1} = \frac{c(-A)}{D_{-A}(p)} + \frac{c(2\lambda-A)}{D_{2\lambda-A}(p)} - 2\frac{c(\lambda-A)}{D_{\lambda-A}(p)},$$

a derivative of the gap between ERP and lower bound with respect to p is

$$\partial_p\big(E\tilde{R}P^{n=1} - \tilde{\mathcal{L}}^{b,n=1}\big)$$
$$= \frac{c(\lambda)c(\lambda-A)c(2\lambda-A)p^2 + 2c(\lambda)c(\lambda-A)p + c(\lambda) + c(\lambda-A) - c(2\lambda-A)}{D_\lambda(p)\, D_{\lambda-A}(p)\, D_{2\lambda-A}(p)} \tag{29}$$

We next show (29) is positive. We do so in three steps: the first step ensures the denominator is positive, the next three, jointly ensure that the numerator is positive.

Step 1: $D_k(p) > 0$ for all k.

Since $D_k(0) = 1$, $D_k(1) = 1 + c(k)$, and $D_k'(p) = c(k)$, it is obvious that $D_k(p)$ is always positive if and only if $c(k) > -1$, or equivalently, $f(b) \equiv (1+b)^k + (1-b)^k > 1$ for any $b \in (0,1)$. For $k \geq 1$: since $(1+b)^k$ and $(1-b)^k$ are convex, we have $\big(\frac{(1+b)^k+(1-b)^k}{2}\big) \geq \big(\frac{(1+b)+(1-b)}{2}\big)^k = 1$ by the Jensen's inequality, and thus, $f(b) \geq 2 > 1$. For $k \in (0,1)$: note that $f'(b) = k(1+b)^{k-1} - k(1-b)^{k-1} \leq 0$ since $k-1 < 0$. Hence, we have $f(b) \geq \min f(b) = f(1) = 2^k > 1$. For $k = 0$: $f(b) = 2 > 1$. For $k < 0$: $(1+b)^k > 0$ is well defined. Since $1 - b \in (0,1)$, $(1-b)^k > 1$ so that $f(b) > 1$.

What we show next is: $G(p) \equiv c(\lambda)c(\lambda-A)c(2\lambda-A)p^2 + 2c(\lambda)c(\lambda-A)p + c(\lambda) + c(\lambda - A) - c(2\lambda - A)$ is positive in $p \in [0,1]$.

Step 2: $G(0) > 0$.

$$\begin{aligned} G(0) &= c(\lambda) + c(\lambda-A) - c(2\lambda-A) \\ &= \underbrace{\big((1+b)^\lambda - 1\big)}_{>0\,\because\,\lambda>0}\underbrace{\big(1-(1+b)^{\lambda-A}\big)}_{>0\,\because\,\lambda-A<0} + \underbrace{\big((1-b)^\lambda - 1\big)}_{<0\,\because\,\lambda>0}\underbrace{\big(1-(1-b)^{\lambda-A}\big)}_{<0\,\because\,\lambda-A<0} > 0. \end{aligned}$$

Step 3: $G'(p) \geq 0$ if $\lambda \geq 1$ and $G'(p) < 0$ if $0 < \lambda < 1$.

Consider $G'(p) = 2c(\lambda)c(\lambda-A)\big(c(2\lambda-A)p+1\big)$, and we have $c(2\lambda-A)p+1 > 0$ since we have shown that $c(k) > -1$ for any k. Now, note that $c(0) = c(1) = 0$;

and from the f.o.c. of $c(k)$, we get $k^* = \ln\Big(-\dfrac{\ln(1-b)}{\ln(1+b)}\Big)/\ln\Big(\dfrac{1+b}{1-b}\Big) \in (0,1)$; and from the s.o.c., we get $c''(k) = \big(\ln(1+b)\big)^2(1+b)^k + \big(\ln(1-b)\big)^2(1-b)^k > 0$. Hence, it is trivial that $c(k) \geq 0$ for $k \geq 1$ and $k \leq 0$ where equality holds at $k = 0$ and $k = 1$, and $c(k) < 0$ for $k \in (0,1)$. Therefore, we have $c(\lambda - A) > 0$ and $c(\lambda) \geq 0$ when $\lambda \geq 1$ and $c(\lambda) < 0$ when $0 < \lambda < 1$.

Step 4: $G(1) > 0$ for small enough b.

Note that $G(1) = G(0) + c(\lambda)c(\lambda - A)\big(c(2\lambda - A) + 2\big)$. If $\lambda \geq 1$, then $c(\lambda) \geq 0$, and so $G(1) > 0$. If $\lambda < 1$, we have $G(1) < G(0)$. For small enough b, we can prevent the case where the absolute value of the second term to be greater than $G(0)$ so that $G(1)$ is still positive.

Since $G(p)$ is positive at $p = 0$ and $p = 1$ without changing the sign of the slope, $G(p) > 0$ for any $p \in [0,1]$. □

C. Further Results for Section 4

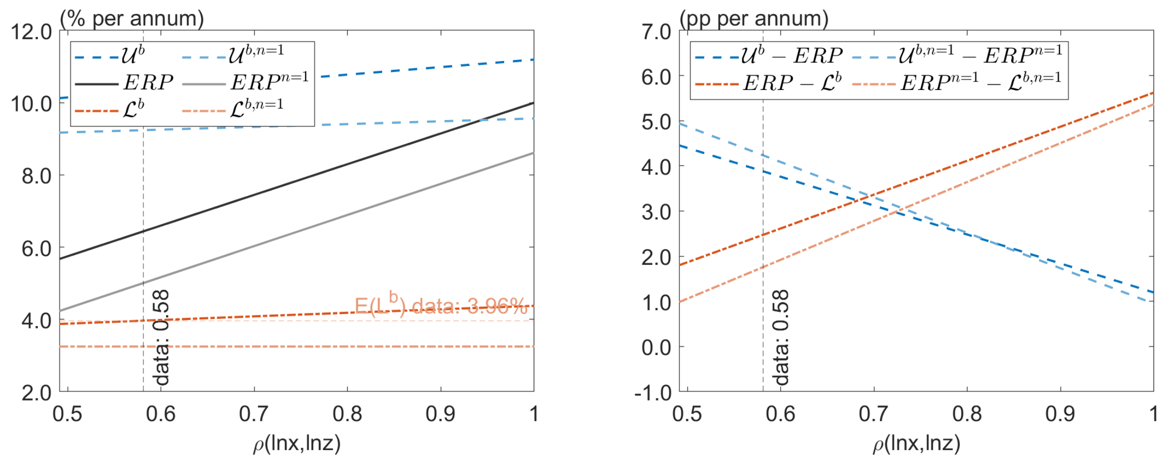
C.1. Term Premia vs. Risk Premia

In our quantitative exercise (Section 4.1), term premia are modest, implying that risk premia are the key contributor to our results. As such, the gap between the ERP and the lower bound is driven by the forces outlined in the Corollary. Figure 9 contrasts the objects in Lemma 1 to their short-term (n=1) counterparts. In our calibration, the short-term ERP explains about 78% of the total ERP. This is because, in our estimates, $\theta = 0.87$, which is modest. Finally, while the lower bound on short-term equity is flat (see Proposition 1), the lower bound is itself rising with the correlation, albeit at a much slower pace than ERP.

Table 4 below illustrates how our calibration changes when $\theta = 0$, keeping all other parameters constant. A key finding is that the lower bound drops by 0.7 p.p. With i.i.d. processes, the risk-free rate is not time-varying without habit formation. Additionally, in the absence of habit, the risk-free rate tends to be higher, reflecting a weaker precautionary saving motive. As such, the model-implied ERP and its bounds also tend to be lower without habit.

Critically, without habits, the $\mathcal{L}^b$ is orthogonal to the correlation while the ERP

Figure 9



Note: The figure plots the ERP and its bounds across the correlation same as Figure 4 with the terms of short-term premia in lighter colors.

Table 4: *Model-Implied Moments without Habit Formation*

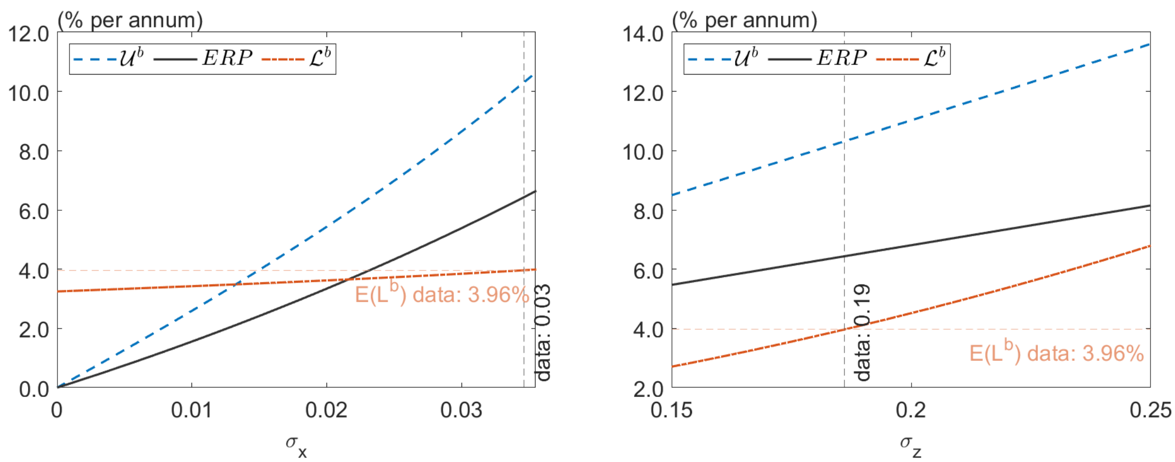
Unconditional Moments (p.a.):				
	Data	Baseline	$\theta = 0$	$\theta = 0$
Correlation	58%	58%	58%	100%
$E(R)$	7.865%	7.865%	9.595%	13.625%
$E(R_f)$	1.142%	1.142%	4.246%	4.246%
$E(\mathcal{L}^b)$	3.964%	3.964%	3.248%	3.248%
$E(D^a/P)$	2.777%	2.777%	10.521%	24.937%
$\sigma(R)$	17.023%	21.944%	19.912%	20.645%
$\sigma(R_f)$	1.214%	3.051%	0.000%	0.000%
ERP		6.437%	5.005%	8.615%
$\mathcal{U}^b$		10.316%	9.239%	9.563%

and $\mathcal{U}^b$ still increase in $\rho_{x,z}$. This is because the effect of correlation on lower bound is solely captured by the term premium (Proposition 1), but the term premium is zero without habit.

C.2. Comparative statics with respect to Volatilities

Figure 10 illustrates how the ERP and its bounds change with respect to the volatility of valuations σ_x and payouts σ_z. While both the ERP and its bounds all rise with higher volatilities, the gradients of the increase vary. In particular, the gap between the ERP and its lower bound is increasing in σ_x (assuming the NCC covariance is satisfied, so for $\sigma_x > 0.021$) but decreasing in σ_z.

Figure 10



Note: Left (Right) panel shows the ERP and bounds across the volatility of log consumption (payout) growth.

Table 5 below summarizes the effect of a change in the profit share on the moments $\{\sigma_x, \sigma_z, \rho_{x,z}\}$ and the resulting changes to the objects of interest. While the increase in the steady-state profit share $\overline{S}$, ceteris-paribus, leads to a significant fall in the volatility of dividends σ_z drawing the ERP and lower bound further apart, the change in the volatility of valuations σ_x is negligible.

C.3. Robustness

Correlation of Δc_k and Δd Since the correlation between Δc_k and Δd is central to our analysis of the ERP and its bounds, we test how the correlation depends on the assumed parameters of the model $\{\omega, \zeta\}$. First, we consider lower values of ζ, corresponding to a lower portion of labour income in investors' consumption (or a smaller share of investors in the labour pool), and recompute the implied correlation for each case. Figure 11 (LHS) illustrates how the correlation changes across different ζ. For lower values of ζ, C_k depends more heavily on payouts, so $\rho_{x,z}$ increases. Moreover, the change in $\overline{S}$ implies a 11% increase in the correlation (0.88 to 0.95) with $\zeta = 0.246$, vis-a-vis the 18% increase (0.56 to 0.66) in our baseline of $\zeta = 0.71$.

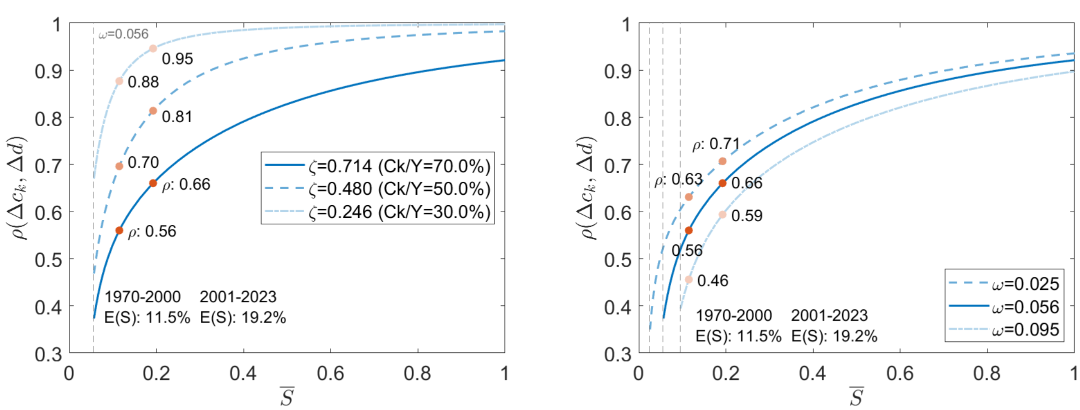
Table 5: *Summary of the ERP and Bounds: 1970-2000 to 2001-23*

Comparative Statics: $\overline{S}$ 11.5% → 19.2%								
	σ_x		σ_z		$\rho_{x,z}$		All	
	3.4% → 3.5%		20.0% → 14.9%		56% → 66%		three	
$\mathcal{U}^b$	10.15	10.59	10.93	8.70	10.27	10.48	10.87	9.03
ERP	6.39	6.51	6.44	6.43	6.26	7.11	6.76	6.45
$\mathcal{L}^b$	3.96	3.97	4.47	2.79	3.94	4.04	4.82	3.05
$\mathcal{U}^b - ERP$	3.76	4.08	4.49	2.28	4.04	3.37	4.11	2.58
$ERP - \mathcal{L}^b$	2.43	2.54	1.97	3.64	2.32	3.07	1.94	3.40

Note: The table shows implied-changes in the ERP and its bounds by shifts in the volatilities and correlation of $\ln x$ and $\ln z$ from increase in $\overline{S}$ from 11.5% to 19.2%. The shifts in volatilities and correlation are approximated by using Equation (24) and (25).

Figure 11 (RHS) instead shows how the correlation varies with values of ω. In our framework, we model the investment share as a constant (Jermann, 1998), but this is not necessarily true in the data. Higher values for ω imply a lower correlation and larger increment in the correlation given a change in $\overline{S}$. Encouragingly, however, we show that even very large changes in the net investment share from 2.5% to 9.5%, have only relatively modest effects on $\rho_{x,z}$ (46%–63%).

Figure 11: *Testing the correlation of Δc_k and Δd.*



(a) *Varying ζ* **(b)** *Varying ω*

Note: Panel (a) varies ζ with fixed $\omega = 0.0558$, corresponding to $C_k/Y = 0.7, 0.5$, and 0.3. Panel (b) varies $\omega = 0.025, 0.0558$, and 0.095 with fixed $\zeta = 0.714$.

C.4. Further Details for Section 4.2

C.4.1 Moments, Asset Prices and Parameter Estimates

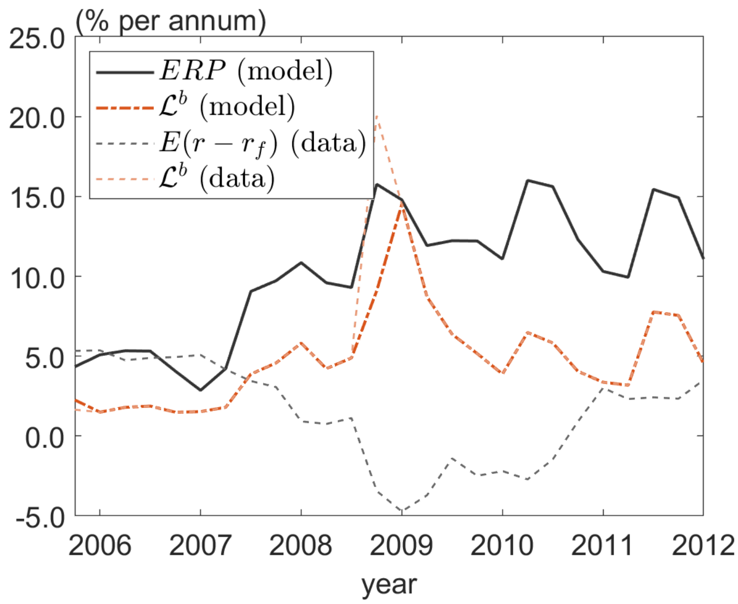
Figure 13 describes the underlying moments, the resulting parameters and the model fit. Specifically, we report the moments $\{\mu_{x,t}, \mu_{z,t}, \sigma_{x,t}, \sigma_{z,t}\}$, constructed using a 10 year backward rolling average of (22) and (23) in the top row. The GFC appears to have introduced a structural break in these series (pre vs. post 2008). Namely, the volatilities of valuations σ_x and payoffs σ_z both rise about 68% and 46% respectively. The correlation also rises significantly, almost doubling. The second row reports parameter estimates $\{\theta, A, \beta, \lambda, p\}$. Finally, the bottom row reports the corresponding mean and variance of risk free rates, as well as the term premium on a 5-year TIPS and the dividend yield. The mean risk-free rate decreases over the sample, while its variance is roughly constant, as are the term premia. Dividend yields rise slowly over the sample. The model does a good job matching these moments over time, although we slightly overshoot the volatility of interest rates.

C.4.2 Estimation with fixed A

As a robustness check, we repeat the estimation holding risk aversion A fixed. We report our main results in Figure 12. The model struggles to match the peak SVIX-implied lower bound during the GFC, but does well outside of this. The estimated ERP series is comparable to the baseline. Figure 14 reports the parameter estimates and the goodness of fit. In this specification, we target only mean risk-free rate, the volatility of the risk-free rate and the dividend yield. We leave the term premium untargetted.

For each period, the lower bound $\mathcal{L}^b$ is targeted which is observed from SVIX^2. The log excess return of equity from data is 10-year rolling average of realized excess return. The ERP is conditional expectation of log excess return implied by our model.

Figure 12: *Rolling Estimation with Fixed A*



Note: This is the analogue of Figure 5 , but with risk aversion A fixed to 13.79.

Figure 13: *Rolling Moments, Parameter Estimates, and Target Moments*

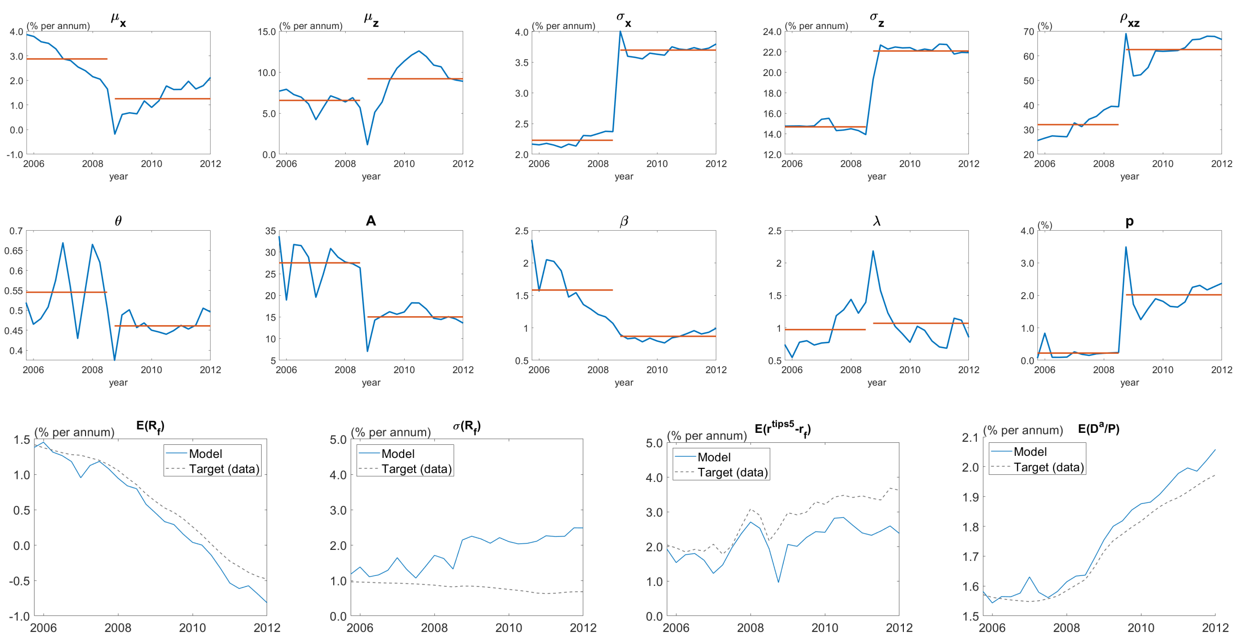
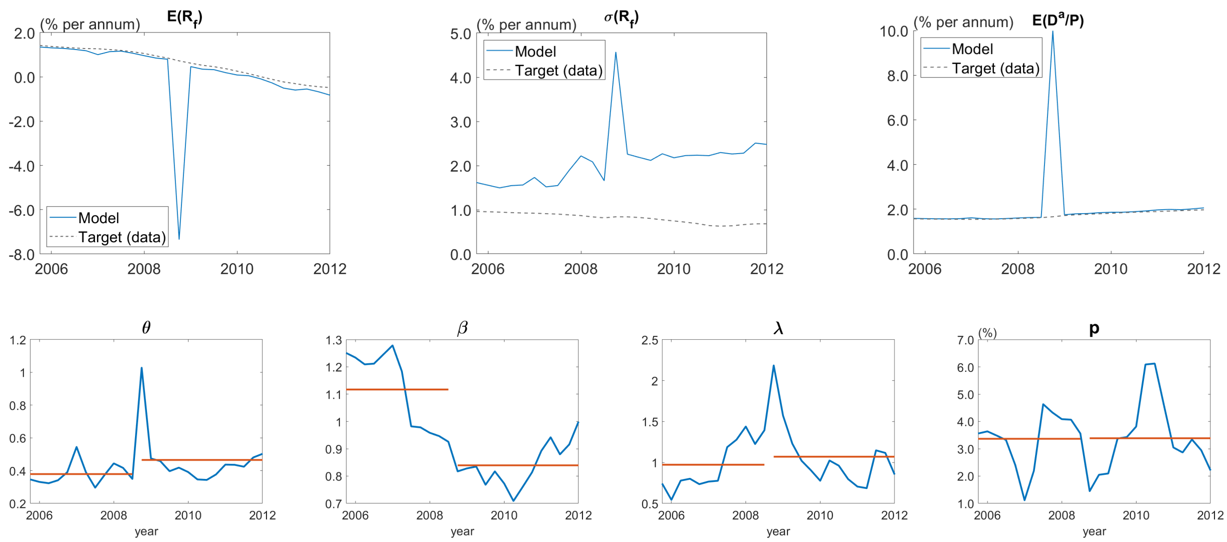


Figure 14: *Rolling Target Moments and Parameter Estimates with Fixed A*



D. Further Details for Section 5

Here we describe the details of the model in Farhi and Gourio (2018). Preferences are described by recursive utility as in Epstein and Zin (1989):

$$V_{k,t} = \Big((1-\beta)L_t c_{k,t}^{1-\sigma} + \beta\, \mathbb{E}_t(V_{k,t+1}^{1-A})^{\frac{1-\sigma}{1-A}}\Big)^{\frac{1}{1-\sigma}}$$

where $V_{k,t}$ is utility, L_t is population size of consumers, $c_{k,t}$ is per capita equity holder's consumption, σ is the inverse of EIS, and A captures risk aversion. A labor is fixed portion of the population $N_t = \overline{N}L_t$.

We introduce profits by assuming the Dixit-Stiglitz monopolistic competition framework. Final good is produced using the constant elasticity of substitution aggregate of differentiated intermediate goods:

$$Y_t = \Big(\int_0^1 y_{i,t}^{\frac{\epsilon-1}{\epsilon}}\, di\Big)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution. The intermediate goods follow the Cobb-Douglas technology:

$$y_{i,t} = Z_t k_{i,t}^{\alpha}(\tilde{S}_t n_{i,t})^{1-\alpha}$$

where $k_{i,t}$ and $n_{i,t}$ are capital and labor in firm i, Z_t is a deterministic productivity trend, and $\tilde{S}_t$ is a stochastic productivity process assumed to be a martingale: $\tilde{S}_{t+1} = \tilde{S}_t e^{\chi_{t+1}}$ where χ_{t+1} is i.i.d.

Capital accumulation is standard, albeit subject to investment-specific technological progress and a capital quality shock:

$$k_{i,t+1} = \big((1-\delta)\, k_{i,t} + Q_t\, \iota_{i,t}\big)\, e^{\chi_{t+1}},$$

where $\iota_{i,t}$ is investment of firm i and Q_t is the investment-specific progress which implies a relative price of capital to consumption good as $1/Q_t$. The capital quality shock is intended to capture a feature of disaster, and for the purpose of deriving BGP equilibrium, we assume it is equivalent to the productivity shock.

When the production factors are reallocated frictionlessly after the shock is realized, the economy can be aggregated in a representative-producer framework:

$$Y_t = Z_t K_t^{\alpha} (\tilde{S}_t N_t)^{1-\alpha}$$

where the first-order conditions of the firms imply $W_t N_t = \dfrac{1-\alpha}{\mu} Y_t$ and $R_t K_t = \dfrac{\alpha}{\mu} Y_t$ where W_t and R_t are real wage and rental rate of capital and $\mu \equiv \frac{\epsilon}{\epsilon - 1}$ is a gross markup. In addition, the law of motion of aggregate capital is

$$K_{t+1} = \big((1-\delta)\, K_t + Q_t\, I_t\big)\, e^{\chi_{t+1}}.$$

Given the preference above, the stochastic discount factor is

$$M_{t+1} = \beta \left(\frac{c_{k,t+1}}{c_{k,t}}\right)^{-\sigma} \left(\frac{v_{k,t+1}}{\mathbb{E}_t(v_{k,t+1}^{1-A})^{\frac{1}{1-A}}}\right)^{\sigma - A}$$

where $v_{k,t}$ is population-normalized utility, $v_{k,t} \equiv V_{k,t}\, L_t^{-\frac{1}{1-\sigma}}$. The resource constraint in the economy satisfies $C_{k,t} + I_t = Y_t$ where $C_{k,t} = L_t c_{k,t}$.

The equilibrium we are focusing on is the risky balanced growth path. The deterministic trends are given by population, TFP, and investment-specific progress: $L_{t+1} = (1+g_L)\, L_t$, $Z_{t+1} = (1+g_Z)\, Z_t$, $Q_{t+1} = (1+g_Q)\, Q_t$. In such equilibrium, the variables have following structure: $Y_t = T_t \tilde{S}_t y^*$, $I_t = T_t \tilde{S}_t \iota^*$, $C_{k,t} = T_t \tilde{S}_t c_k^*$ where $T_t = L_t Z_t^{\frac{1}{1-\alpha}} Q_t^{\frac{\alpha}{1-\alpha}}$ and small letters with asterisk determine constant ratios on the BGP.

Then the equity holder's SDF on the BGP is

$$M_{t+1} = \beta\,(1+g_{pc})^{-\sigma} e^{-A\chi_{t+1}}\, \mathbb{E}(e^{(1-A)\chi_{t+1}})^{\frac{A-\sigma}{1-A}} \tag{30}$$

where $1+g_{pc} = \frac{1+g_T}{1+g_L}$ and $1+g_T = (1+g_L)(1+g_Z)^{\frac{1}{1-\alpha}}(1+g_Q)^{\frac{\alpha}{1-\alpha}}$. And the risk-free rate is

$$R_{f,t} = \frac{1}{\mathbb{E}_t(M_{t+1})} = \frac{\mathbb{E}(e^{(1-A)\chi_{t+1}})}{\beta^*\,\mathbb{E}_t(e^{-A\chi_{t+1}})} \tag{31}$$

where $\beta^* \equiv \mathbb{E}_t(M_{t+1}\,e^{\chi_{t+1}}) = \beta\,(1+g_{pc})^{-\sigma}\,\mathbb{E}(e^{(1-A)\chi_{t+1}})^{\frac{1-\sigma}{1-A}}$.

The equity is defined as a claim to the dividend stream which is the profits less investment: $D_t = \Pi_t - I_t$. The equity return on the BGP can be obtained from the asset pricing equation $\mathbb{E}_t(M_{t+1}R_{t+1}) = 1$ and constant price-dividend ratio on the path:

$$\frac{p^*}{d^*} = \frac{\mathbb{E}_t\big(M_{t+1}\frac{D_{t+1}}{D_t}\big)}{1-\mathbb{E}_t\big(M_{t+1}\frac{D_{t+1}}{D_t}\big)} = \frac{\beta^*\,(1+g_T)}{1-\beta^*\,(1+g_T)}$$

where the first equality is from $R_{t+1} = \frac{D_{t+1}}{D_t}\frac{D_t}{P_t}\big(\frac{P_{t+1}}{D_{t+1}}+1\big)$ using a constant ratio, and the second equality is obtained by using $\frac{D_{t+1}}{D_t} = (1+g_T)\,e^{\chi_{t+1}}$ on the BGP. Substituting the price-dividend ratio yields the equity return on the BGP as follows:

$$R_{t+1} = \frac{1}{\beta^*}e^{\chi_{t+1}}. \tag{32}$$

D.1. Proof to Proposition 3

Using equations (31) and (32), the ERP is obtained as

$$ERP = \ln\Big(\frac{\mathbb{E}_t(R_{t+1})}{R_{f,t}}\Big) = \ln\frac{\mathbb{E}(e^{-A\chi})\,\mathbb{E}(e^{\chi})}{\mathbb{E}(e^{(1-A)\chi})}$$

and additionally with Equation 30, the lower bound can be derived as

$$\mathcal{L}^b = \ln\Big(\frac{\mathbb{E}_t(M_{t+1}R_{t+1}^2)}{R_{f,t}}\Big) = \ln\frac{\mathbb{E}(e^{-A\chi})\,\mathbb{E}(e^{(2-A)\chi})}{\mathbb{E}(e^{(1-A)\chi})^2}.$$

With the three-point distribution of χ_{t+1} where the size of the crash (bonanza) is given by b (b_H) and the probability is given by p, the moment for any coefficient k is

$$\mathbb{E}(e^{k\chi_{t+1}}) = 1 - 2p + pe^{k\ln(1+b_H)} + pe^{k\ln(1-b)}.$$

Substituting the moments yields the Proposition 3. □

D.2. Further Results for Section 5

The model exactly identifies the nine parameters $\{\beta, p, \mu, \delta, \alpha, g_Z, g_Q, g_L, \bar{N}\}$ by matching nine moments: means of profitability Π/K, capital share Π/Y, investment-capital ratio I/K, risk-free rate R_f, price-dividend ratio P/D, growth rates of population, TFP, and investment prices, and employment-population ratio N/L, for each two sub-periods 1984-2000 and 2001-2016.

Table 6: *Parameters and Targeting Moments in Farhi and Gourio (2018)*

Parameter Values (est.):			Unconditional Moments (target):		
	1984-2000	2001-2016		1984-2000	2001-2016
β	0.961	0.972	Π/K	14.012	14.890
p	0.034	0.065	Π/Y	29.887	33.992
μ	1.079	1.146	I/K	8.103	7.227
δ	0.028	0.032	R_f	2.787	-0.350
α	0.244	0.243	P/D	42.336	50.115
g_L	1.171	1.101	g_{pop}	1.171	1.101
g_Z	1.298	1.012	g_{tfp}	1.100	0.755
g_Q	1.769	1.127	g_{inv}	-1.769	-1.127
$\bar{N}$	62.344	60.838	N/L	62.344	60.838
Given Parameters:					
A=12; σ=0.5; b=0.15					

Note: All ratios except P/D and growth rates are in percentages.