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Peter Duersch
University of Heidelberg
Joerg Oechssler
University of Heidelberg
Burkhard Schipper
University of California, Davis

December 09, 2011

Paper # 11-12

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Department of Economics
One Shields Avenue
Davis, CA 95616
(530)752-0741

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Once Beaten, Never Again: Imitation in Two-Player Potential Games*

Peter Duersch† Jörg Oechssler ‡ Burkhard C. Schipper§

April 26, 2012

Abstract

We show that in symmetric two-player exact potential games, the simple decision rule "imitate-if-better" cannot be beaten by any strategy in a repeated game by more than the maximal payoff difference of the one-period game. Our results apply to many interesting games including examples like 2x2 games, Cournot duopoly, price competition, public goods games, common pool resource games, and minimum effort coordination games.

Keywords: Imitate-the-best, learning, exact potential games, symmetric games, relative payoffs, zero-sum games.

JEL-Classifications: C72, C73, D43.

*Some of the material was previously circulated in a companion paper "Unbeatable Imitation". We thank Carlos Alós-Ferrer, Chen Bo, Drew Fudenberg, Alexander Matros, Klaus Ritzberger, Karl Schlag, and John Stachurski for interesting discussions. Seminar audiences at Australian National University, Melbourne University, Monash University, UC Davis, UC San Diego, the Universities of Heidelberg, Konstanz, Wien, and Zürich, the University of Queensland, the University of Oregon, Calpoly, at the International Conference on Game Theory in Stony Brook, 2009, the Midwestern Economic Theory Conference in Evanston 2010, and at the Econometric Society World Congress 2010 in Shanghai contributed helpful comments.

†Department of Economics, University of Heidelberg, Email: peter.duersch@awi.uni-heidelberg.de

‡Department of Economics, University of Heidelberg, Email: oechssler@uni-hd.de

§Department of Economics, University of California, Davis, Email: bcschipper@ucdavis.edu

1 Introduction

Suppose you play a repeated symmetric game against one opponent. Suppose further that you know that this opponent uses a very simple learning algorithm, namely the rule "imitate-if-better", the rule that simply prescribes to mimic the action of the other player if and only if the other player received a higher payoff in the previous period. Since the rule is deterministic, you therefore know exactly what your opponent will do in all future periods and how he will react to your actions. The question we pose in this paper is whether you can use this knowledge to exploit the imitator in the sense hat you achieve a higher payoff than the imitator.

We show that surprisingly there are meaningful classes of games like all symmetric 2x2 games, Cournot duopoly, Bertrand duopoly, public goods games, common pool resource games, minimum effort coordination games, and Diamond's search, where this is essentially not possible. To be precise, we call imitation "essentially unbeatable" if in the repeated game there exists no strategy of the opponent with which the opponent can obtain, in total, over an infinite number of periods, a payoff difference that is more than the maximal payoff difference for the one–period game.

To gain some intuition for this, consider the game of "chicken" presented in the following payoff matrix.

$$\begin{array}{ll} & \begin{array}{cc} \text{swerve} & \text{straight} \end{array} \\ \begin{array}{c} \text{swerve} \\ \text{straight} \end{array} & \begin{pmatrix} 3,3 & 1,4 \\ 4,1 & 0,0 \end{pmatrix} \end{array}$$

Suppose that initially the imitator starts out with playing "swerve". What should a forward looking opponent do? If she decides to play "straight", she will earn more than the imitator today but will be copied by the imitator tomorrow. From then on, the imitator will stay with "straight" forever. If she decides to play "swerve" today, then she will earn the same as the imitator and the imitator will stay with "swerve" as long as the opponent stays with "swerve". Suppose the opponent is a dynamic relative payoff maximizer. In that case, the dynamic relative payoff maximizer can beat the imitator at most by the maximal one-period payoff differential of 3. Now suppose the opponent maximizes the sum of her *absolute* payoffs. The best an absolute payoff maximizer can do is to play swerve forever. In this case the imitator cannot be beaten at all as he receives the same payoff as his opponent. In either case, imitation comes very close to the top–performing heuristics and there is no evolutionary pressure against such an heuristic.

These results extend results from our recent paper Duersch, Oechssler, and Schipper (2011a), in which we show that imitation is subject to a money pump (i.e., can be exploited without bounds) if and only if the relative payoff game is of the rock-paper-scissors variety. The current results are stronger because they show that imitation can only be exploited with a bound that is equal to the payoff difference in the one–period game. However, this comes at the cost of restricting ourselves to the class of symmetric two-player exact potential games. But as mentioned above, many economically relevant games satisfy this property. Exact potential games have been introduced by Monderer and Shapley (1996) and are studied widely in learning in games (see Sandholm, 2010).

The behavior of learning heuristics has previously been studied mostly for the case when all players use the same heuristic. For the case of imitate-the-best,[1] Vega-Redondo (1997) showed that in a symmetric Cournot oligopoly with imitators, the long run outcome converges to the competitive output if small mistakes are allowed. This result has been generalized to aggregative quasisubmodular games by Schipper (2003) and Alós-Ferrer and Ania (2005). Huck, Normann, and Oechssler (1999), Offerman, Potters, and Sonnemans (2002), and Apesteguia et al. (2007, 2010) provide some experimental evidence in favor of imitative behavior. In contrast to the above cited literature, the current paper deals with the interaction of an imitator and a possibly forward looking, very rational and patient player. Apart from experimental evidence in Duersch, Kolb, Oechssler, and Schipper (2010) and our own paper Duersch at al. (2011a) we are not aware of any work that deals with this issue. For a Cournot oligopoly with imitators and myopic best reply players, Schipper (2009) showed that the imitators' long run average payoffs are strictly higher than the best reply players' average payoffs.[2]

A recent paper by Feldman, Kalai, and Tennenholtz (2010) has a similar but complementary objective to ours. They study whether a strategy which they call "copycat" can be beaten in a symmetric two-player game by an arbitrary opponent who may have full knowledge of the game and may play any history dependent strategy. Remarkably, the copycat strategy can nearly match the average payoff of the opponent. Yet, their copycat rule is far more sophisticated than our imitation rule as it entails finding a (possibly

[1]For the two-player case, imitate-the-best and imitate-if-better are almost equivalent, the difference being that the latter specifically prescribes a tie-breaking rule (for the case of both players having equal payoffs in the previous round). Since we use imitate-if-better only in the two-player case, we do not need to specify what happens if more than one other player is observed.

[2]In a related paper, Hehenkamp and Kaarbøe (2008) study a Cournot duopoly with one imitator and one myopic optimizer in a rapidly changing environment with strategic substitutes. If the optimizer can adjust decisions as fast as some parameters of the game change (like the demand), then it is possible that the optimizer is better off than the imitator if strategic substitutes are sufficiently weak.

mixed) minmax strategy in the auxiliary zero–sum game in each round. On the other hand, the possible opponents are less omniscient than the opponents in our setting. In particular, the opponents in their setting cannot perfectly predict the imitator's action in the next round.

Somewhat farther related is the literature on playing games by a population with heterogeneous learning rules. For instance, Droste, Hommes, and Tuinstra (2002) consider a large population of firms matched randomly into a Cournot duopoly. A fraction of firms play a best response to the average output in the previous period whereas others perfectly forecast the output of either decision rule, play a best response to the forecast but bear an extra information cost. They show that in an evolutionary dynamics with noise both decision rules survive in the long run with fluctuating fractions. Josephson (2008) introduces an evolutionary stability criterion for learning rules and studies with simulations versions of experience weighted attraction in three types of 2x2 games. Josephson (2009) analyzes stochastic adaptation in finite games played by heterogeneous populations containing best repliers, better repliers, and imitators. He provides sufficient conditions for the convergence to minimal sets closed under better replies.

The article is organized as follows. In the next section, we present the model and provide a formal definition for being unbeatable. Sufficient conditions for imitation to be essentially unbeatable are given in Section 3. We finish with Section 4, where we summarize and discuss the results.

2 Model

We consider a symmetric two–player game (X, π), in which both players are endowed with the same (finite or infinite) set of pure actions X. For each player, the bounded payoff function is denoted by $\pi : X \times X \longrightarrow \mathbb{R}$, where $\pi(x, y)$ denotes the payoff to the player choosing the first argument when his opponent chooses the second argument. We will frequently make use of the following definition.

Definition 1 (Relative payoff game) *Given a symmetric two-player game (X, π), the relative payoff game is (X, Δ), where the relative payoff function $\Delta : X \times X \longrightarrow \mathbb{R}$ is defined by*

$$\Delta(x, y) = \pi(x, y) - \pi(y, x).$$

Note that, by construction, every relative payoff game is a symmetric zero-sum game since $\Delta(x, y) = -\Delta(y, x)$.

The *imitator* follows the simple rule "imitate-if-better". To be precise, the imitator adopts the opponent's action if and only if in the previous round the opponent's payoff was strictly higher than that of the imitator. Formally, the action of the imitator y_t in period t given the action of the other player from the previous period x_{t-1} is

$$y_t = \begin{cases} x_{t-1} & \text{if } \Delta(x_{t-1}, y_{t-1}) > 0 \\ y_{t-1} & \text{else} \end{cases} \tag{1}$$

for some initial action $y_0 \in X$.

Our aim is to determine whether there exists a strategy of the imitator's opponent that obtains substantially higher payoffs than the imitator. We allow for any strategy of the opponent, including very sophisticated ones. In particular, the opponent may be infinitely patient and forward looking, and may never make mistakes. More importantly, she may know exactly what her opponent, the imitator, will do at all times, including the imitator's starting value. She may also commit to any closed loop strategy.

Consider now a situation in which an imitator starts out with a very unfavorable initial action. A clever opponent who knows this initial action can take advantage of it. Yet, from then on the opponent has no strategy that makes her better off than the imitator. Arguably, the disadvantage in the initial period should not play a role in the long run. This motivates the following definition:

Definition 2 (Essentially unbeatable) *We say that* imitation is essentially unbeatable *if for any initial action of the imitator and any strategy of the opponent, the imitator can be beaten in total by at most the maximal one-period payoff differential, i.e., if for any y_0 and any sequence $\{x_t\}$,*

$$\sum_{t=0}^{T} \Delta(x_t, y_t) \leq \max_{x,y} \Delta(x, y), \ \textit{for all } T \geq 0, \tag{2}$$

where y_t is given by equation (1).

3 Results

In this section we present two classes of games for which imitation is essentially unbeatable. The first class is the class of 2x2 games. The second class is the class of games with an exact potential.

3.1 Symmetric 2x2 games

In the chicken game discussed in the Introduction, imitation was essentially unbeatable since the maximal payoff difference was 3. In this section, we extend the "chicken" example of the introduction to all symmetric 2x2 games.

Proposition 1 *In any symmetric 2x2 game, imitation is essentially unbeatable.*

PROOF. Let $X = \{x, x'\}$. Consider a period t in which the opponent achieves a strictly positive relative payoff, $\Delta(x, x') > 0$. (If no such period t in which the opponent achieves a strictly positive relative payoff exists, then trivially imitation is essentially unbeatable.) Obviously, $\Delta(x, x') \leq \max_{x,y} \Delta(x, y)$. Since $\Delta(x, x') > 0$, the imitator imitates x in period $t + 1$. For there to be another period in which the opponent achieves a strictly positive relative payoff, it must hold that $\Delta(x', x) > 0$. This yields a contradiction since the relative payoff game is symmetric zero-sum and hence $\Delta(x', x) = -\Delta(x, x')$. Thus there can be at most one period in which the opponent achieves a strictly positive relative payoff. $\square$

Note that "Matching pennies" is not a counter-example since it is not symmetric.

3.2 Exact Potential Games

Next, we consider games that possess an exact potential function. As it turns out, a symmetric two-player game is an exact potential game if and only if it is also an exact potential games with respect to its relative payoff game. Furthermore, in our context, exact potential games have a relative payoff function that is additively separable, and has increasing and decreasing differences. All these definitions may appear to be restrictive. However, we will show below that there is a fairly large number of important examples that fall into this class.

The following notion is due to Monderer and Shapley (1996).

Definition 3 (Exact potential games) *The symmetric game (X, π) is an exact potential game if there exists an exact potential function $P : X \times X \longrightarrow \mathbb{R}$ such that for all*

$y \in X$ and all $x, x' \in X$,[3]

$$\begin{aligned} \pi(x,y) - \pi(x',y) &= P(x,y) - P(x',y), \\ \pi(x,y) - \pi(x',y) &= P(y,x) - P(y,x'). \end{aligned}$$

Definition 4 (Additively Separable) *A relative payoff function Δ is additively separable if $\Delta(x,y) = f(x) + g(y)$ for some functions $f, g : X \longrightarrow \mathbb{R}$.*

Definition 5 (Increasing/decreasing Differences) *Let X be a totally ordered set. A (relative) payoff function Δ has decreasing (resp. increasing) differences on $X \times X$ if for all $x'', x', y'', y' \in X$ with $x'' > x'$ and $y'' > y'$,*

$$\Delta(x'', y'') - \Delta(x', y'') \leq (\geq) \Delta(x'', y') - \Delta(x', y'). \tag{3}$$

Δ is a valuation if it has both decreasing and increasing differences.

Proposition 2 *Let (X, π) be a symmetric two-player game. Suppose that X is a compact and totally ordered set and π is continuous. Then imitation is essentially unbeatable if any of the following conditions holds:*

(i) (X, π) is an exact potential game

(ii) (X, Δ) is an exact potential game

(iii) Δ has increasing differences

(iv) Δ has decreasing differences

(v) Δ is additively separable.

Proof. We first note that all five conditions are equivalent in our context. Duersch, Oechssler, and Schipper (2011b, Theorem 20) show that (i) and (ii) are equivalent. There, we also show that (iii) and (iv) are equivalent for all symmetric two-player zero-sum games (Proposition 13). Hence, (iii) or (iv) imply that Δ is a valuation. Brânzei, Mallozzi, and Tijs (2003, Theorem 1) show that (ii) is equivalent to Δ being a valuation for zero-sum games. Finally, Topkis (1998, Theorem 2.6.4.) shows equivalence of (v) and Δ being a valuation for zero-sum games. Thus, it suffices to prove the claim for condition (v).

[3] Given the symmetry of (X, π), the second equation plays the role usually played by the quantifier "for all players" in the definition of potential games.

Let Δ be additively separable, i.e. $\Delta(x,y) = f(x) + g(y)$ for some functions $f, g : X \longrightarrow \mathbb{R}$. Thus we have for all $x'', x', x \in X$,

$$\Delta(x'', x) - \Delta(x', x) = \Delta(x'', x') - \Delta(x', x'),$$

which is equivalent to

$$\Delta(x'', x) = \Delta(x'', x') + \Delta(x', x) \tag{4}$$

because $\Delta(x', x') = 0$ since the relative payoff game is a symmetric zero–sum game.

Let $(x_0, x_1, ...)$ be a sequence of actions generated by the opponent's strategy, and let $\{\Delta(x_t, y_t)\}_{t=0,1,...}$ be her associated sequence of relative payoffs when the imitator follows his imitation rule in equation (1) with an initial action y_0. Now consider the subsequence of strictly positive relative payoffs of the opponent, $\{\Delta(x_t, y_t) | t = 0, 1, ...; \Delta(x_t, y_t) > 0\}$. Assume the case that $\{\Delta(x_t, y_t) | t = 0, 1, ...; \Delta(x_t, y_t) > 0\}$ is not a singleton. (Otherwise the Proposition follows trivially.) Observe that for any adjacent elements of the subsequence, say $\Delta(x_k, y_k)$ and $\Delta(x_{k+\ell}, y_{k+\ell})$ (for some $\ell > 0$), we must have $\Delta(x_{k+\ell}, y_{k+\ell}) = \Delta(x_{k+\ell}, x_k)$. This is because an imitator mimics the opponent if the opponent obtained a strictly positive relative payoff and stays with his own action if the opponent's relative payoff was less than or equal to zero. Note that $\sum_{t=k}^{k+\ell} \Delta(x_t, y_t) \leq \Delta(x_{k+\ell}, x_k) + \Delta(x_k, y_k) = \Delta(x_{k+\ell}, y_k)$, where the inequality follows from the fact that all elements of the sequence strictly between k and $k + \ell$ are non-positive and the equality follows from equation (4) above. Applying this argument inductively yields that for any y_0 and $T > 0$ for which $\Delta(x_T, y_0) > 0$, we have that

$$\sum_{t=0}^{T} \Delta(x_t, y_t) \leq \Delta(x_T, y_0) \leq \max_{x,y} \Delta(x, y),$$

where $\max_{x,y} \Delta(x, y)$ exists because π is continuous and X is compact. □

A sufficient condition for the additive separability of relative payoffs is provided in the next result.

Corollary 1 *Consider a game (X, π) with a compact action set X and a payoff function that can be written as $\pi(x, y) = f(x) + g(y) + a(x, y)$ for some continuous functions $f, g : X \longrightarrow \mathbb{R}$ and a symmetric function $a : X \times X \longrightarrow \mathbb{R}$ (i.e., $a(x, y) = a(y, x)$ for all $x, y \in X$). Then imitation is essentially unbeatable.*

The following examples demonstrate that the assumption of additively separable relative payoffs is not as restrictive as may be thought at first glance. All of those games are also exact potential games. However, often the conditions on the relative payoffs are easier to verify than finding an exact potential function.

Example 1 (Cournot Duopoly with Linear Demand) Consider a (quasi) Cournot duopoly given by the symmetric payoff function $\pi(x, y) = x(b - x - y) - c(x)$ with $b > 0$. Since $\pi(x, y)$ can be written as $\pi(x, y) = bx - x^2 - c(x) - xy$, Corollary 1 applies, and imitation is essentially unbeatable.

Example 2 (Bertrand Duopoly with Product Differentiation) Consider a differentiated duopoly with constant marginal costs, in which firms 1 and 2 set prices x and y, respectively. Firm 1's profit function is given by $\pi(x, y) = (x - c)(a + by - \frac{1}{2}x)$, for $a > 0$, $b \in [0, 1/2)$. Since $\pi(x, y)$ can be written as $\pi(x, y) = ax - ac + \frac{1}{2}cx - \frac{1}{2}x^2 - bcy + bxy$, Corollary 1 applies, and imitation is essentially unbeatable. This example with strategic complementarities also shows that the result is not restricted to strategic substitutes.

Example 3 (Public Goods) Consider the class of symmetric public good games defined by $\pi(x, y) = g(x, y) - c(x)$ where $g(x, y)$ is some symmetric monotone increasing benefit function and $c(x)$ is an increasing cost function. Usually, it is assumed that g is an increasing function of the sum of provisions, $x + y$. Various assumptions on g have been studied in the literature such as increasing or decreasing returns. In any case, Corollary 1 applies, and imitation is essentially unbeatable.

Example 4 (Common Pool Resources) Consider a common pool resource game with two appropriators. Each appropriator has an endowment $e > 0$ that can be invested in an outside activity with marginal payoff $c > 0$ or into the common pool resource. Let $x \in X \subseteq [0, e]$ denote the opponent's investment into the common pool resource (likewise y denotes the imitator's investment). The return from investment into the common pool resource is $\frac{x}{x+y}(a(x + y) - b(x + y)^2)$, with $a, b > 0$. So the symmetric payoff function is given by $\pi(x, y) = c(e - x) + \frac{x}{x+y}(a(x + y) - b(x + y)^2)$ if $x, y > 0$ and ce otherwise (see Walker, Gardner, and Ostrom, 1990). Since $\Delta(x, y) = (c(e - x) + ax - bx^2) - (c(e - y) + ay - by^2)$, Proposition 2 implies that imitation is essentially unbeatable.

Example 5 (Minimum Effort Coordination) Consider the class of minimum effort games given by the symmetric payoff function $\pi(x, y) = \min\{x, y\} - c(x)$ for some cost function $c(\cdot)$ (see Bryant, 1983, and Van Huyck, Battalio, and Beil, 1990). Corollary 1 implies that imitation is essentially unbeatable.

Example 6 (Synergistic Relationship) Consider a synergistic relationship among two individuals. If both devote more effort to the relationship, then they are both better off, but for any given effort of the opponent, the return of the player's effort first increases and then decreases. The symmetric payoff function is given by $\pi(x,y) = x(c+y-x)$ with $c > 0$ and $x, y \in X \subset \mathbb{R}_+$ with X compact (see Osborne, 2004, p.39). Corollary 1 implies that imitation is essentially unbeatable.

Example 7 (Diamond's Search) Consider two players who exert effort searching for a trading partner. Any trader's probability of finding another particular trader is proportional to his own effort and the effort by the other. The payoff function is given by $\pi(x,y) = \alpha xy - c(x)$ for $\alpha > 0$ and c increasing (see Milgrom and Roberts, 1990, p. 1270). The relative payoff game of this two-player game is additively separable. By Proposition 2 imitation is essentially unbeatable.

Note that any symmetric 2x2 game is also an exact potential game.

Finally, a natural question is whether additive separability of relative payoffs (or equivalently the existence of an exact potential function for the underlying game) are also necessary conditions for imitation to be essentially unbeatable. The following counter-example shows that this is not the case.

Example 8 (Coordination game with outside option) Consider the following coordination game with an outside option (C) for both players of not participating (left matrix).

$$\pi = \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 4 & -1 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \qquad \Delta = \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0 & -3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Note that the relative payoff game Δ (right matrix) does not have constant differences. E.g., $\Delta(A,B) - \Delta(B,B) = -3 \neq \Delta(A,C) - \Delta(B,C) = 0$. Thus, by Topkis (1998, Theorem 2.6.4.) it is not additively separable, and by Duersch, Oechssler, and Schipper (2011, Theorem 3) (X, π) is not an exact potential game. Yet, imitation is essentially unbeatable. If the imitator's initial action is A, the opponent can earn at most a relative payoff differential of 3 after which the imitator adjusts and both earn zero from there on. For other initial actions of the imitator, the maximal payoff difference is at most 0.

4 Discussion

We have shown in this paper that imitation is a behavioral rule that is surprisingly robust to exploitation by any strategy in symmetric two-player exact potential games. This includes strategies by truly sophisticated opponents. The property that imitate-if-better is unbeatable in these games seems to be unique among commonly used learning rules. We are not aware of any rule that shares this property with imitate-if-better.[4] For example, there are important differences between imitate-if-better and *unconditional* imitation, when behavior is imitated regardless of its success. A well known example of the latter is tit–for–tat. To see the difference, consider the following game.

$$\pi = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0,0 & 0,-1 & -1,0 \\ -1,0 & 0,0 & 0,10 \\ 0,-1 & 10,0 & 0,0 \end{pmatrix} \end{array} \qquad \Delta = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & -10 \\ 1 & 10 & 0 \end{pmatrix} \end{array}$$

It is easy to see that imitate-if-better is essentially unbeatable for this game. However, tit–for–tat could be exploited without any bound by following a cycle ($A \to B \to C \to A \ldots$). The reason for this difference is that an imitate-if-better player would never leave action C whereas a tit–for–tat player can be induced to follow the opponent from C to A.

There are other modifications that may cause the imitate-if-better rule to lose the property of being unbeatable. For instance, we assumed that an imitator sticks to his action in case of a tie in payoffs. To see what goes wrong with an alternative tie-braking rule consider a homogenous Bertrand duopoly with constant marginal costs. Suppose the imitator starts with a price equal to marginal cost. If the opponent chooses a price strictly above marginal cost, her profit is also zero. If nevertheless, the opponent were imitated, she could start a money pump by undercutting the imitator until they reach again price equal to marginal cost and then start the cycle again.

Similarly, many commonly used belief learning rules, for example, best response learning or fictitious play, can easily be exploited in all games in which a Stackelberg leader achieves a higher payoff than the follower (as e.g. in Cournot games). Against such rules, the opponent can simply stubbornly choose the Stackelberg leader action knowing that the belief learning player will eventually converge to the Stackelberg follower action. Thus, belief learning rules can be beaten without bounds in such games. Yet,

[4] Apart from close variants of imitate-if-better like rules that imitate only with a certain probability, see e.g. Schlag's (1998) proportional imitation rule.

it remains an open question for future research whether there are other behavioral rules that perform equally well as imitate-if-better.

Finally, in Duersch, Oechssler, and Schipper (2011a) we show some extensions to more general classes of two-player games including relative payoff games with a generalized ordinal potential that come at cost of weakening the criterion of essentially unbeatable. Extensions to n-player games must be left for further research.

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