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Risk Shocks and Housing Markets

Gabriel Lee
University of Regensburg, IHS, Wien
Victor Dorofeenko
IHS, Wien
Kevin Salyer
U.C. Davis

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This paper analyzes the role of uncertainty in a multi-sector housing model with financial frictions. We include time varying uncertainty (i.e. risk shocks) in the technology shocks that affect housing production. The analysis demonstrates that risk shocks to the housing production sector are a quantitatively important impulse mechanism for the business cycle. Also, we demonstrate that bankruptcy costs act as an endogenous markup factor in housing prices; as a consequence, the volatility of housing prices is greater than that of output, as observed in the data. The model can also account for the observed countercyclical behavior of risk premia on loans to the housing sector.

Department of Economics
One Shields Avenue
Davis, CA 95616
(530)752-0741

http://www.econ.ucdavis.edu/working_search.cfm

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Abstract

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- *JEL Classification: E4, E5, E2, R2, R3*
- Keywords: *agency costs, credit channel, time-varying uncertainty, residential investment, housing production, calibration*

Victor Dorofeenko
Department of Economics and Finance
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna, Austria

Gabriel S. Lee
IREBS
University of Regensburg
Universitaetstrasse 31
93053 Regensburg, Germany
And
Institute for Advanced Studies

Kevin D. Salyer (Corresponding Author)
Department of Economics
University of California
Davis, CA 95616
Contact Information:
Lee: ++49-941.943.5060; E-mail: gabriel.lee@wiwi.uni-regensburg.de
Salyer: (530) 554-1185; E-mail: kdsalyer@ucdavis.edu

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1 Introduction

The Great Recession of 2009 has dramatically underscored the importance that financial and housing markets have for the behavior of the macroeconomy. To better understand the role that these markets play in aggregate fluctuations, this paper presents a calibrated general equilibrium model that incorporates these factors but also introduces an impulse mechanism, time varying uncertainty, that, until recently, has not received much attention in the literature.[1] In particular, we analyze the role of time varying uncertainty (i.e. risk shocks) in a multi-sector real business cycle model that includes housing production (developed by Davis and Heathcote, (2005)) and a financial sector with lending under asymmetric information (e.g. Carlstrom and Fuerst, (1997), (1998); Dorofeenko, Lee, and Salyer, (2008)). We model risk shocks as a mean preserving spread in the distribution of the technology shocks affecting house production and explore how changes in uncertainty affect equilibrium characteristics.[2]

Our aim in examining this environment is twofold. First, we want to develop a framework that can capture one of the main components of the current financial crises, namely, changes in the risk associated with the housing sector. In our analysis, we focus entirely on the variations in risk associated with the production of housing and the consequences that this has for lending and economic activity. Hence our analysis is very much a fundamental-based approach so that we side step the delicate issue of modeling housing bubbles and departures from rational expectations. The results, as discussed below, suggest (to us) that this conservative approach is warranted.[3] Second, we want to cast the analysis of risk shocks in a model that is broadly consistent with some of

[1] Some recent papers that have examined the effects of uncertainty in a DSGE framework include Bloom et al. (2008), Fernandez-Villaverde et al. (2009), and Christiano et al. (2008). The last paper is most closely related to the analysis presented here in that it also uses a credit channel model.

[2] Some of the recent works which also examine housing and credit are: Iacoviello and Minetti (2008) and Iacoviello and Neri (2008) in which a new-Keynesian DGSE two sector model is used in their empirical analysis; Iacoviello (2005) analyzes the role that real estate collateral has for monetary policy; and Aoki, Proudman and Vliegh (2004) analyse house price amplification effects in consumption and housing investment over the business cycle. None of these analyses use risk shocks as an impulse mechanism.

[3] In a closely related analysis, Kahn (2008) also uses a variant of the Davis and Heathcote (2005) framework in order to analyze time variation in the growth rate of productivity in a key sector (consumption goods). He demonstrates that a change in regime growth, combined with a learning mechanism, can account for some of the observed movements in housing prices.

the important stylized facts of the housing sector such as: (i) residential investment is about twice as volatile as non-residential investment and (ii) residential investment and non-residential investment are highly procyclical.[4]

To that end, we employ the Davis and Heathcote (2005) housing model which, as demonstrated by the authors, can replicate the high volatility observed in residential investment despite the absence of any frictions in the economy. The recent analysis in Christiano et al. (2008), however, provides compelling evidence that financial frictions play an important role in business cycles and, given the recent financial events, it seems reasonable to investigate this role when combined with a housing sector.[5] Consequently, we modify the Davis and Heathcote (2005) analysis by adding a financial sector in the economy and require that housing producers must finance their inputs via loans from the banking sector. This modification appears to be important; for instance, we show that by incorporating an explicit financial market into this model, we can produce large movements in housing prices, a feature of the data that was missing in the Davis and Heathcote (2005) analysis. We also demonstrate that housing prices in our model are affected by expected bankruptcies and the associated agency costs; these serve as an endogenous, time-varying markup factor affecting the price of housing. The volatility in this markup translates into increased volatility in housing prices. Moreover, the model implies that this endogenous markup to housing as well as the risk premium associated with loans to the housing sector should be countercyclical; both of these features are seen in the data.[6]

Our analysis finds that plausible calibrations of the model with time varying uncertainty produce a quantitatively meaningful role for uncertainty over the housing and business cycles. For

[4] One other often mentioned stylized fact is that housing prices are persistent and mean reverting (e.g. Glaser and Gyourko (2006)). See Figure 1 and Table 4 for these cyclical and statistical features during the period of 1975 until the second quarter of 2007.

[5] Christiano et al. (2008) use a New Keynesian model to analyze the relative importance of shocks arising in the labor and goods markets, monetary policy, and financial sector. They find that time-varying second moments, i.e. risk shocks, are quantitatively important relative to the the other impulse mechanisms.

[6] In addition to these cyclical features, a marked feature of the housing sector has been the growth in residential and commercial real estate lending over the last decade. As shown in Figure 2, residential real estate loans (excluding revolving home equity loans) account for approximately 50% of total lending by domestically chartered commercial banks in the United States over the period October 1996 to July 2007. Figure 3 shows the strong co-movement between the amount of real estate loans and house prices.

instance, we compare the impulse response functions for aggregate variables (such as output, consumption expenditure, and investment) due to a 1% increase in technology shocks to the construction sector to a 1% increase in uncertainty to shocks affecting housing production. We find that, quantitatively, the impact of risk shocks is almost as great as that from technology shocks. This comparison carries over to housing market variables such as the price of housing, the risk premium on loans, and the bankruptcy rate of housing producers. The model is not wholly satisfactory in that it can not account for the lead-lag structure of residential and non-residential investment but this is not surprising given that the analysis focuses entirely on the supply of housing. Still, we think the approach presented here provides a useful start in studying the effects of time-varying uncertainty on housing, housing finance and business cycles.

2 Model Description

As stated above, our model builds on two separate strands of literature: Davis and Heathcote's (2005) multi-sector growth model with housing, and Dorofeenko, Lee and Salyer's (2008) credit channel model with time-varying uncertainty. For expositional clarity, we first briefly outline our variant of the Davis and Heathcote model and then introduce the credit channel model.

2.1 Production

2.1.1 Firms

The economy consists of two agents, a consumer and an entrepreneur, and four sectors: an intermediate goods sector, a final goods sector, a housing goods sector and a banking sector. The intermediate sector is comprised of three perfectly competitive industries: a building/construction sector, a manufacturing sector and a service sector. The output from these sectors are then combined to produce a residential investment good and a consumption good which can be consumed or used as capital investment; these sectors are also perfectly competitive. Entrepreneurs combine residential investment with a fixed factor (land) to produce housing; this sector is where the lending channel and financial intermediation play a role.

Turning first to the intermediate goods sector, the representative firm in each sector is characterized by the following Cobb-Douglas production function:

$$x_{it} = k_{it}^{\theta_i} \left(n_{it} \exp^{z_{it}} \right)^{1-\theta_i} \tag{1}$$

where $i = b, m, s$ (building/construction, manufacture, service), k_{it}, n_{it},and z_{it} are capital, household labor, and a labor augmenting productivity shock respectively for each sector, with θ_i being the share of capital for sector i.[7] In our calibration we set $\theta_b < \theta_m$ reflecting the fact that the manufacturing sector is more capital intensive (or less labor intensive) than the construction sector.

Productivity in each sector exhibits stochastic growth as given by:

$$z_i = t g_{z,i} + \tilde{z}_i \tag{2}$$

where $g_{z,i}$ is the trend growth rate in sector i.

The vector of technology shocks, $\tilde{\mathbf{z}} = (\tilde{z}_b, \tilde{z}_m, \tilde{z}_s)$, follows an $AR(1)$ process:

$$\tilde{\mathbf{z}}_{t+1} = B \cdot \tilde{\mathbf{z}}_t + \vec{\varepsilon}_{t+1} \tag{3}$$

The innovation vector $\vec{\varepsilon}$ is distributed normally with a given covariance matrix Σ_{ε}.[8]

These intermediate firms maximize a conventional static profit function every period. That is,

[7] Real estate developers, i.e. entrepreneurs, also provide labor to the intermediate goods sectors. This is a technical consideration so that the net worth of entrepreneurs, including those that go bankrupt, is positive. Labor's share for entrepreneurs is set to a trivial number and has no effect on output dynamics. Hence, for expositional purposes, we ignore this factor in the presentation.

[8] In their analysis, Davis and Heathcote (2005) introduced a government sector characterized by non-stochastic tax rates and government expenditures and a balanced budget in every period. We abstract from these features in order to focus on time varying uncertainty and the credit channel. Our original model included these elements but it was determined that they did not have much influence on the policy functions that characterize equilibrium (although they clearly influence steady-state values).

at time t, the objective function is:

$$\max_{\{k_{it},n_{it}\}} \left\{ \sum_i p_{it}x_{it} - r_t k_{it} - w_t n_{it} \right\} \tag{4}$$

which results in the usual first order conditions for factor demand:

$$r_t k_{it} = \theta_i p_{it} x_{it}, \qquad w_t n_{it} = (1-\theta_i) p_{it} x_{it} \tag{5}$$

where r_t, w_t, and p_{it} are the capital rental, wage, and output prices (with the consumption good as numeraire).

The intermediate goods are then used as inputs to produce two final goods, y_j, where $j = c, d$ (consumption/capital investment and residential investment respectively). This technology is also assumed to be Cobb-Douglas with constant returns to scale:

$$y_{jt} = \prod_{i=b,m,s} x1_{ijt}^{\rho_{ij}}, \quad j = c, d. \tag{6}$$

Note that there are no aggregate technology shocks in the model. The input matrix is defined by

$$\mathbf{x1} = \begin{pmatrix} b_c & b_d \\ m_c & m_d \\ s_c & s_d \end{pmatrix}, \tag{7}$$

where, for example, m_j denotes the quantity of manufacturing output used as an input into sector j. The shares of construction, manufactures and services for sector j are defined by the matrix

$$\rho = \begin{pmatrix} B_c & B_d \\ M_c & M_d \\ S_c & S_d \end{pmatrix}. \tag{8}$$

The relative shares of the three intermediate inputs differ in producing the two final goods. For example, in the calibration of the model, we set $B_c < B_d$ to represent the fact that residential investment is more construction input intensive relative to the consumption good sector. The first degree homogeneity of the production processes implies $\sum_i \rho_{ij} = 1; \quad j = c, d$ while market clearing in the intermediate goods markets requires $x_{it} = \sum_j x1_{ijt}; \quad i = b, m, s.$

With intermediate goods as inputs, the final goods' firms solve the following static profit maximization problem at t (as stated earlier, the price of consumption good, p_{ct}, is normalized to 1):

$$\max_{x_{ijt}} \left\{ y_{ct} + p_{dt} y_{dt} - \sum_j \sum_i p_{it} x1_{ijt} \right\} \tag{9}$$

subject to the production functions (eq.(6)) and non-negativity of inputs.

The first order conditions associated with profit maximization are given by the typical marginal conditions

$$p_{it} x1_{ijt} = \rho_{ij} p_{jt} y_{jt}; \quad i = b, m, s; \quad j = c, d \tag{10}$$

Constant returns to scale implies zero profits in both sectors so we have the following relationships:

$$\sum_j p_{jt} y_{jt} = \sum_i p_{it} x_{it} = r_t k_t + w_t n_t \tag{11}$$

Finally, new housing structures, y_{ht}, are produced by entrepreneurs (i.e. real estate developers) using the residential investment good, y_{dt}, and land, x_{lt}, as inputs. For entrepreneur a, the production function is denoted $F(x_{alt}, y_{adt})$ and is assumed to exhibit constant returns to scale. Specifically, we assume:

$$y_{aht} = \omega_{at} F(x_{alt}, y_{adt}) = \omega_{at} x_{alt}^{\zeta} y_{adt}^{1-\zeta} \tag{12}$$

where, ζ denotes the share of land. It is assumed that the aggregate quantity of land is fixed. The technology shock, ω_{at}, is an idiosyncratic shock affecting real estate developers. The technology shock is assumed to have a unitary mean and standard deviation of $\sigma_{\omega,t}$. The standard deviation,

$\sigma_{\omega,t}$, follows an $AR(1)$ process:

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^{\chi} \exp^{\varepsilon_{\sigma,t+1}} \tag{13}$$

with the steady-state value $\sigma_0, \chi \in (0,1)$ and $\varepsilon_{\sigma,t+1}$ is a white noise innovation.[9]

Each period, the production of new housing is added to the depreciated stock of existing housing units. Davis and Heathcote (2005) exploit the geometric depreciation structure of housing in order to define a stock of effective housing units, denoted h_t. Given the lack of aggregate uncertainty in new housing production, the law of motion for per-capita effective housing can be written as:

$$\eta h_{t+1} = x_{lt}^{\zeta} y_{dt}^{1-\zeta} (1 - ac_t) + (1 - \delta_h) h_t \tag{14}$$

where δ_h is the depreciation on effective housing units, η represents the population growth rate (the same for households and entrepreneurs), and ac_t represents the agency costs due to bankruptcy of a fraction of real estate developers.[10] The last factor is critical and is discussed in more detail in the discussion of the lending channel presented below (see eq. (24) below).

2.1.2 Households

The representative household derives utility each period from consumption, c_t, housing, h_t, and leisure, $1-n_t$; all of these are measured in per-capita terms. Instantaneous utility for each member of the household is defined by the Cobb-Douglas functional form of

$$U(c_t, h_t, 1 - n_t) = \frac{\left(c_t^{\mu_c} h_t^{\mu_h} (1 - n_t)^{1-\mu_c-\mu_h}\right)^{1-\sigma}}{1-\sigma} \tag{15}$$

[9] This autoregressive process is used so that, when the model is log- linearized, $\hat{\sigma}_{\omega,t}$ (defined as the percentage deviations from σ_0) follows a standard, mean-zero AR(1) process.

[10] Davis and Heathcote (2005) derive the law of motion for effective housing units (with no agency costs) and demonstrate that the depreciation rate δ_h is related to the depreciation rate of structures. As mentioned in the text, it is not necessary to keep track of the stock of housing structures as an additional state variable; the amount of effective housing units, h_t, is a sufficient statistic.

where μ_c and μ_h are the weights for consumption and housing in utility, and σ represents the coefficient of relative risk aversion. The household maximizes expected lifetime utility as given by:

$$E_0 \sum_{t=0}^{\infty} (\beta\eta)^t U(c_t, h_t, 1-n_t) \tag{16}$$

Each period agents combine labor income with income from assets (capital, housing, land and loans to the banking sector, denoted b_t) and use these to purchase consumption, new housing and investment. These choices are represented by the per-capita budget constraint:

$$c_t + \eta k_{t+1} + \eta p_{ht} h_{t+1} = w_t n_t + (r_t + 1 - \delta_\kappa) k_t + (1-\delta_h) p_{ht} h_t + p_{lt} x_{lt} + (R_t - 1)\, b_t \tag{17}$$

where δ_k and δ_h are the capital and house depreciation rates respectively and R_t is the return on bank deposits.[11] Note that loans to the banking sector are intra-period loans and, because financial intermediation eliminates all idiosyncratic risk as discussed below, the equilibrium interest on these loans will be unity, i.e. $R_t = 1$.

The optimization problem leads to the following necessary conditions which represent, respectively, the Euler conditions associated with capital and housing and the intra-temporal labor-leisure decision:

$$1 = \beta\eta E_t[(r_t + 1 - \delta_\kappa)\frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}], \tag{18}$$

$$p_{ht} = \beta\eta E_t[\frac{U_2(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)} + (1-\delta_h) p_{ht+1} \frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}], \tag{19}$$

$$w_t = \frac{U_3(c_t, h_t, 1-n_t)}{U_1(c_t, h_t, 1-n_t)}. \tag{20}$$

[11] Note that lower case variables for capital, labor and consumption represent per-capita quantities while upper case denote will denote aggregate quantities. Also, in addition to household's income from renting capital and providing labor, he also receives income from selling land to developers.

2.2 The Credit Channel

2.2.1 Housing Entrepreneurial Contract

The economy described above is identical to that studied in Davis and Heathcote (2005) except for the addition of productivity shocks affecting housing production.[12] We describe in more detail the nature of this sector and the role of the banking sector. It is assumed that a continuum of housing producing firms with unit mass are owned by risk-neutral entrepreneurs (developers). The costs of producing housing are financed via loans from risk-neutral intermediaries. Given the realization of the idiosyncratic shock to housing production, some real estate developers will not be able to satisfy their loan payments and will go bankrupt. The banks take over operations of these bankrupt firms but must pay an agency fee. These agency fees, therefore, affect the aggregate production of housing and, as shown below, imply an endogenous markup to housing prices. That is, since some housing output is lost to agency costs, the price of housing must be increased in order to cover factor costs.

The timing of events is critical:

1. The exogenous state vector of technology shocks and uncertainty shocks, denoted $(z_{i,t}, \sigma_{\omega,t})$, is realized.

2. Firms hire inputs of labor and capital from households and entrepreneurs and produce intermediate output via Cobb-Douglas production functions. These intermediate goods are then used to produce the two final outputs.

3. Households make their labor, consumption and savings/investment decisions.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of the loan (fp_{at}) and a cutoff level of productivity for the entrepreneurs' technology

[12] Also, as noted above, we abstract from taxes and government expenditures.

shock, $\bar{\omega}_t$.

5. Entrepreneurs use their net worth and loans from the banking sector in order to purchase the factors for housing production. The quantity of factors (residential investment and land) is determined and paid for before the idiosyncratic technology shock is known.

6. The idiosyncratic technology shock of each entrepreneur is realized. If $\omega_{at} \geq \bar{\omega}_t$ the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional to total factor payments.

7. Entrepreneurs that are solvent make consumption choices; these in part determine their net worth for the next period.

A schematic of the implied flows is presented in Figure 5.

Each period, entrepreneurs enter the period with net worth given by nw_{at}. Developers use this net worth and loans from the banking sector in order to purchase inputs. Letting fp_{at} denote the factor payments associated with developer a, we have:

$$fp_{at} = p_{dt} y_{adt} + p_{lt} x_{alt} \tag{21}$$

Hence, the size of the loan is $(fp_{at} - nw_{at})$. The realization of ω_{at} is privately observed by each entrepreneur; banks can observe the realization at a cost that is proportional to the total input bill. Letting μ denote the proportionality factor, the cost is therefore given by μfp_{at}.

With a positive net worth, the entrepreneur borrows $(fp_{at} - nw_{at})$ consumption goods and agrees to pay back $\left(1 + r^L\right)(fp_{at} - nw_{at})$ to the lender, where r^L is the interest rate on loans. As Carlstrom and Fuerst (1998) demonstrate, the cutoff value of productivity, $\bar{\omega}_t$, that determines solvency (i.e. $\omega_{at} \geq \bar{\omega}_t$) or bankruptcy (i.e. $\omega_{at} < \bar{\omega}_t$) can be used to define two functions (denoted $f(\bar{\omega}_t; \sigma_{\omega,t})$ and $g(\bar{\omega}_t; \sigma_{\omega,t})$) which determine the allocation of the value of housing production between producers and lenders, respectively. Denoting the *c.d.f.* and *p.d.f.* of ω_t as $\Phi(\omega_t;\sigma_{\omega,t})$ and $\phi(\omega_t;\sigma_{\omega,t})$, these functions are defined as:[13]

$$f(\bar{\omega}_t;\sigma_{\omega,t}) = \int_{\bar{\omega}_t}^{\infty} (\omega - \bar{\omega}_t)\,\phi(\omega;\sigma_{\omega,t})\,d\omega = \int_{\bar{\omega}_t}^{\infty} \omega\phi(\omega;\sigma_{\omega,t})\,d\omega - [1 - \Phi(\bar{\omega}_t;\sigma_{\omega,t})]\,\bar{\omega}_t \tag{22}$$

and

$$g(\bar{\omega}_t;\sigma_{\omega,t}) = \int_{0}^{\bar{\omega}_t} \omega\phi(\omega;\sigma_{\omega,t})\,d\omega + [1 - \Phi(\bar{\omega}_t;\sigma_{\omega,t})]\,\bar{\omega}_t - \Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu \tag{23}$$

Note that these two functions sum to:

$$f(\bar{\omega}_t;\sigma_{\omega,t}) + g(\bar{\omega}_t;\sigma_{\omega,t}) = 1 - \Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu \tag{24}$$

Hence, the term $\Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu$ captures the loss of housing due to the agency costs associated with bankruptcy. Note that that loss of output due to agency costs combined with the constant returns to scale production function implies that the value of housing output must exhibit a markup over factor costs. Denote this markup as $\bar{s}_t > 1$ which is taken as parametric for both lender and real estate developer so that, by definition: $p_{ht}y_{aht} = \bar{s}_t f p_{at}$. The optimal borrowing contract is defined by the pair $(fp_{at}, \bar{\omega}_t)$ that maximizes the entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur). That is, the optimal contract is determined by the solution to:

$$\max_{\bar{\omega}_t, fp_{at}} \bar{s}_t f p_{at} f(\bar{\omega}_t;\sigma_{\omega,t}) \text{ subject to } \bar{s}_t f p_{at} g(\bar{\omega}_t;\sigma_{\omega,t}) \geqslant f p_{at} - n w_{at} \tag{25}$$

A necessary condition for the optimal contract problem is given by:

$$\frac{\partial(.)}{\partial\bar{\omega}_t} : \bar{s}_t f p_{at} \frac{\partial f(\bar{\omega}_t;\sigma_{\omega,t})}{\partial\bar{\omega}_t} = -\lambda_t \bar{s}_t f p_{at} \frac{\partial g(\bar{\omega}_t;\sigma_{\omega,t})}{\partial\bar{\omega}_t} \tag{26}$$

[13] The notation $\Phi(\omega;\sigma_{\omega,t})$ is used to denote that the distribution function is time-varying as determined by the realization of the random variable, $\sigma_{\omega,t}$.

where λ_t is the shadow price of the lender's resources. Using the definitions of $f(\bar{\omega}_t;\sigma_{\omega,t})$ and $g(\bar{\omega}_t;\sigma_{\omega,t})$, this can be rewritten as:[14]

$$1-\frac{1}{\lambda_t}=\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}\mu \tag{27}$$

As shown by eq.(27), the shadow price of the resources used in lending is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of housing production, i.e. $\lambda_t = 1$.

The second necessary condition is:

$$\frac{\partial(.)}{\partial f p_{at}}:\bar{s}_t f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)=-\lambda_t\left[1-\bar{s}_t g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\right] \tag{28}$$

These first-order conditions imply that, in general equilibrium, the markup factor, $\bar{s}_t$, will be endogenously determined and related to the probability of bankruptcy. Specifically, using the first order conditions, we have that the markup, $\bar{s}_t$, must satisfy:

$$\begin{aligned}\bar{s}_t^{-1} &= \left[\left(f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)+g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\right)+\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\mu f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\frac{\partial f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}}\right] \\ &= \left[1-\underbrace{\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\mu}_{A}-\underbrace{\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}\mu f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}_{B}\right]\end{aligned} \tag{29}$$

First note that the markup factor depends only on economy-wide variables so that the aggregate markup factor is well defined. Also, the two terms, A and B, demonstrate that the markup factor is affected by both the total agency costs (term A) and the marginal effect that bankruptcy has on the entrepreneur's expected return. That is, term B reflects the loss of housing output, μ, weighted by

[14] Note that we have used the fact that $\frac{\partial f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}=\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)-1<0$

the expected share that would go to entrepreneur's, $f(\bar{\omega}_t; \sigma_{\omega,t})$, and the conditional probability of bankruptcy (the Inverse Mill's ratio). Finally, note that, in the absence of credit market frictions, there is no markup so that $\bar{s}_t = 1$. In the partial equilibrium setting, it is straightforward to show that equation (29) defines an implicit function $\bar{\omega}(\bar{s}_t, \sigma_{\omega,t})$ that is increasing in $\bar{s}_t$.

The incentive compatibility constraint implies

$$fp_{at} = \frac{1}{(1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}))} nw_{at} \tag{30}$$

Equation (30) implies that the size of the loan is linear in entrepreneur's net worth so that aggregate lending is well-defined and a function of aggregate net worth.

The effect of an increase in uncertainty on lending can be understood in a partial equilibrium setting where $\bar{s}_t$ and nw_{at} are treated as parameters. As shown by eq. (29), the assumption that the markup factor is unchanged implies that the costs of default, represented by the terms A and B, must be constant. With a mean-preserving spread in the distribution for ω_t, this means that $\bar{\omega}_t$ will fall (this is driven primarily by the term A). Through an approximation analysis, it can be shown that $\bar{\omega}_t \approx g(\bar{\omega}_t; \sigma_{\omega,t})$ (see the Appendix in Dorofeenko, Lee, and Salyer (2008)). That is, the increase in uncertainty will reduce lenders' expected return $(g(\bar{\omega}_t; \sigma_{\omega,t}))$. Rewriting the binding incentive compatibility constraint (eq. (30)) yields:

$$\bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}) = 1 - \frac{nw_{at}}{fp_{at}} \tag{31}$$

the fall in the left-hand side induces a fall in fp_{at}. Hence, greater uncertainty results in a fall in housing production. This partial equilibrium result carries over to the general equilibrium setting.

The existence of the markup factor implies that inputs will be paid less than their marginal products. In particular, profit maximization in the housing development sector implies the following necessary conditions:

$$\frac{p_{lt}}{p_{ht}} = \frac{F_{xl}\left(x_{lt}, y_{dt}\right)}{\bar{s}_t} \tag{32}$$

$$\frac{p_{dt}}{p_{ht}} = \frac{F_{yd}\left(x_{lt}, y_{dt}\right)}{\bar{s}_t} \tag{33}$$

These expressions demonstrate that, in equilibrium, the endogenous markup (determined by the agency costs) will be a determinant of housing prices. The production of new housing net of agency costs is denoted $y_{ht} = x_{lt}^{\zeta} y_{dt}^{1-\zeta}\left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\mu\right].$

2.2.2 Housing Entrepreneurial Consumption and House Prices

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the household. This is represented by following expected utility function:

$$E_0 \sum_{t=0}^{\infty} \left(\beta\eta\gamma\right)^t c_t^e \tag{34}$$

where c_t^e denotes entrepreneur's per-capita consumption at date t, and $\gamma \in (0,1)$. This new parameter, γ, will be chosen so that it offsets the steady-state internal rate of return due to housing production.

Each period, entrepreneur's net worth, nw_t is determined by the value of capital income and the remaining capital stock.[15] That is, entrepreneurs use capital to transfer wealth over time (recall that the housing stock is owned by households). Denoting entrepreneur's capital as k_t^e, this implies:[16]

$$nw_t = k_t^e\left[r_t + 1 - \delta_\kappa\right] \tag{35}$$

The law of motion for entrepreneurial capital stock is determined in two steps. First, new

[15] As stated in footnote 6, net worth is also a function of current labor income so that net worth is bounded above zero in the case of bankruptcy. However, since entrepreneur's labor share is set to a very small number, we ignore this component of net worth in the exposition of the model.

[16] For expositional purposes, in this section we drop the subscript a denoting the individual entrepreneur.

capital is financed by the entrepreneurs' value of housing output after subtracting consumption:

$$\eta k_{t+1}^{e}=p_{ht}h_{t}f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)=\bar{s}_{t}fp_{t}f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)-c_{t}^{e} \tag{36}$$

Then, using the incentive compatibility constraint, eq. (30), and the definition of net worth, this can be written as:

$$\eta k_{t+1}^{e}=k_{t}^{e}\left(r_{t}+1-\delta_{\kappa}\right)\frac{\bar{s}_{t}f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)}{1-\bar{s}_{t}g\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)}-c_{t}^{e} \tag{37}$$

The term $\bar{s}_{t}f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)/\left(1-\bar{s}_{t}g\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)\right)$ represents the entrepreneur's internal rate of return due to housing production. Or, alternatively, it reflects the leverage enjoyed by the entrepreneur. Multiplying numerator and denominator by nw_t and again using the incentive compatibility constraint we have:

$$\frac{\bar{s}_{t}f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)}{1-\bar{s}_{t}g\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)}=\frac{\bar{s}_{t}fp_{t}f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)}{nw_{t}} \tag{38}$$

That is, entrepreneurs use their net worth to finance factor inputs of value fp_t, this produces housing which sells at the markup $\bar{s}_t$ with entrepreneur's retaining fraction $f\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)$ of the value of housing output. Given this setting, the optimal path of consumption implies the following Euler equation:

$$1=\beta\eta\gamma E_{t}\left[\left(r_{t+1}+1-\delta_{\kappa}\right)\frac{\bar{s}_{t+1}\ f\left(\bar{\omega}_{t+1};\sigma_{\omega,t+1}\right)}{1-\bar{s}_{t+1}g\left(\bar{\omega}_{t+1};\sigma_{\omega,t+1}\right)}\right] \tag{39}$$

Finally, we can derive an explicit relationship between entrepreneur's capital and the value of the housing stock using the incentive compatibility constraint and the fact that housing sells at a markup over the value of factor inputs. That is, since $p_{ht}F\left(x_{alt},y_{adt}\right)=\bar{s}_{t}fp_{t}$, the incentive compatibility constraint implies:

$$p_{ht}\left(x_{lt}^{\zeta}y_{dt}^{1-\zeta}\right)=k_{t}^{e}\frac{\left(r_{t}+1-\delta_{\kappa}\right)}{1-\bar{s}_{t}g\left(\bar{\omega}_{t};\sigma_{\omega,t}\right)}\bar{s}_{t} \tag{40}$$

Again, it is important to note that the markup parameter plays a key role in determining housing prices and output.

2.2.3 Financial Intermediaries

The banks in the model act as risk-neutral financial intermediaries that, in equilibrium, earn zero profits. There is a clear role for banks in this economy since, through pooling, all aggregate uncertainty of housing production can be eliminated. The banking sector receives deposits from households and these are repaid by funds from two sources: loan repayment from solvent housing producers and the value of housing output, net of monitoring costs, of insolvent housing firms.

3 Equilibrium

Prior to solving for equilibrium, it is necessary to express the growing economy in stationary form. Given that preferences and technologies are Cobb-Douglas, the economy will have a balanced growth path. Hence, it is possible to transform all variables by the appropriate growth factor. As discussed in Davis and Heathcote (2005), the output value of all markets (e.g. $p_d y_d, y_c, p_i x_i$ for $i = (b, m, s)$) are growing at the same rate as capital and consumption, g_k. This growth rate, in turn, is a geometric average of the growth rates in the intermediate sectors: $g_k = g_{zb}^{B_c} g_{zm}^{M_c} g_{zs}^{S_c}$. It is also the case that factor prices display the normal behavior along a balanced growth path: interest rates are stationary while the wage in all sectors is growing at the same rate. The growth rates for the various factors are presented in Table 1 (again see Davis and Heathcote (2005) for details). These growth factors were used to construct a stationary economy; all subsequent discussion is in terms of this transformed economy.

Equilibrium in the economy is described by the vector of factor prices (w_t, r_t), the vector of intermediate goods prices, (p_{bt}, p_{mt}, p_{st}), the price of residential investment (p_{dt}), the price of land (p_{lt}), the price of housing (p_{ht}), and the markup factor $(\bar{s}_t)$. In total, therefore, there are nine

Table 1: Growth Rates on the Balanced Growth Path

n_b, n_m, n_s, n, r	1
$k_b, k_m, k_s, k, c, y_c, w$	$g_k = \left[g_{zb}^{B_c(1-\theta_b)} g_{zm}^{M_c(1-\theta_m)} g_{zs}^{S_c(1-\theta_s)}\right]^{(1/(1-B_c\theta_b-M_c\theta_m-S_c\theta_s))}$
b_c, b_d, x_b	$g_b = g_k^{\theta_b} g_{zb}^{1-\theta_b}$
m_c, m_d, x_m	$g_m = g_k^{\theta_m} g_{zm}^{1-\theta_m}$
s_c, s_d, x_s	$g_s = g_k^{\theta_s} g_{zs}^{1-\theta_s}$
y_d	$g_d = g_b^{B_h} g_m^{M_h} g_s^{S_h}$
x_l	$g_l = \eta^{-1}$
y_h, h	$g_h = g_l^{\zeta} g_d^{1-\zeta}$
$p_h y_h, p_d x_d, p_l x_l, p_b x_b, p_m x_m, p_s x_s$	g_k

equilibrium prices. In addition, the following quantities are determined in equilibrium: the vector of intermediate goods (x_{mt}, x_{bt}, x_{st}) , the vector of labor inputs (n_{mt}, n_{bt}, n_{st}) , the total amount of labor supplied, (n_t) ,the vector of inputs into the final goods sectors $(b_{ct}, b_{dt}, m_{ct}, m_{dt}, s_{ct}, s_{dt})$, the vector of capital inputs (k_{mt}, k_{bt}, k_{st}) , entrepreneurial capital (k_t^e) , household investment (k_{t+1}) , the vector of final goods output (y_{ct}, y_{dt}) ,the technology cutoff level $(\bar{\omega}_t)$, the effective housing stock (h_{t+1}) , and the consumption of households and entrepreneurs (c_t, c_t^e) . In total, there are 24 quantities to be determined; adding the nine prices, the system is defined by 33 unknowns.

These are determined by the following conditions:

Factor demand optimality in the intermediate goods markets

$$r_t = \theta_i \frac{p_{it} x_{it}}{k_{it}} \quad (3 \textit{ equations}) \tag{41}$$

$$w_t = (1-\theta_i) \frac{p_{it} x_{it}}{n_{it}} \quad (3 \textit{ equations}) \tag{42}$$

Factor demand optimality in the final goods sector:

$$p_{ct} y_{ct} = \frac{p_{bt} b_{ct}}{B_c} = \frac{p_{mt} m_{ct}}{M_c} = \frac{p_{st} s_{ct}}{S_c} \quad (3 \textit{ equations}) \tag{43}$$

$$p_{dt} y_{dt} = \frac{p_{bt} b_{dt}}{B_d} = \frac{p_{mt} m_{dt}}{M_d} = \frac{p_{st} s_{dt}}{S_d} \quad (3 \textit{ equations}) \tag{44}$$

Factor demand in the housing sector (using the fact that, in equilibrium $x_{lt} = 1$) produces two more equations:

$$\frac{p_{lt}}{p_{ht}} = \frac{\zeta y_{dt}^{1-\zeta}}{\bar{s}_t} \tag{45}$$

$$\frac{p_{dt}}{p_{ht}} = \frac{(1-\zeta)\, y_{dt}^{-\zeta}}{\bar{s}_t} \tag{46}$$

The household's necessary conditions provide 3 more equations:

$$1 = \beta\eta E_t[(r_t + 1 - \delta_\kappa)\frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}], \tag{47}$$

$$p_{ht} = \beta\eta E_t[\frac{U_2(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)} + (1-\delta_h)p_{ht+1}\frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}], \tag{48}$$

$$w_t = \frac{U_3(c_t, h_t, 1-n_t)}{U_1(c_t, h_t, 1-n_t)}. \tag{49}$$

The financial contract provides the condition for the markup and the incentive compatibility constraint:

$$\bar{s}_t^{-1} = \left[\left(f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) + g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right) + \frac{\phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{\frac{\partial f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{\partial \bar{\omega}_t}} \right] \tag{50}$$

$$p_{ht} y_{dt}^{1-\zeta} = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \bar{s}_t \tag{51}$$

The entrepreneur's maximization problem provides the following Euler equation:

$$1 = \beta\eta\gamma E_t \left[(r_{t+1} + 1 - \delta_\kappa) \frac{\bar{s}_{t+1}\ f\left(\bar{\omega}_{t+1}; \sigma_{\omega,t+1}\right)}{1 - \bar{s}_{t+1} g\left(\bar{\omega}_{t+1}; \sigma_{\omega,t+1}\right)} \right] \tag{52}$$

To these optimality conditions, we have the following market clearing conditions:

Labor market clearing:

$$n_t = \sum_i n_{it}, \quad i = b, m, s \tag{53}$$

Market clearing for capital:

$$k_t = \sum_i k_{it}, \qquad i = b, m, s \tag{54}$$

Market clearing for intermediate goods:

$$x_{bt} = b_{ct} + b_{dt}, \quad x_{mt} = m_{ct} + m_{dt}, \quad x_{st} = s_{ct} + s_{dt}. \tag{55}$$

The aggregate resource constraint for the consumption final goods sector (i.e. the law of motion for capital)

$$\eta k_{t+1} = (1 - \delta_\kappa) k_t + y_{ct} - c_t - c_t^e \tag{56}$$

The law of motion for the effective housing units:

$$\eta h_{t+1} = (1 - \delta_h) h_t + y_{dt}^{1-\zeta} \left(1 - \Phi\left(\bar{\omega}_t\right) \mu\right) \tag{57}$$

The law of motion for entrepreneur's capital stock:

$$\eta k_{t+1}^e = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) - c_t^e \tag{58}$$

Finally, we have the production functions. Specifically, for the intermediate goods markets:

$$x_{it} = k_{it}^{\theta_i} (n_{it} \exp^{z_{it}})^{1-\theta_i}; \quad i = b, m, s \tag{59}$$

For the final goods sectors, we have:

$$y_{ct} = b_{ct}^{B_c} m_{ct}^{M_c} s_{ct}^{S_c} \tag{60}$$

Table 2: Key Preference and Production Parameters

Depreciation rate for capital: δ_κ	0.056
Depreciation rate for effective housing (h): δ_h	0.014
Land's share in new housing: ζ	0.106
Population growth rate: η	1.017
Discount factor: β	0.951
Risk aversion: σ	2.00
Consumption's share in utility: μ_c	0.314
Housing's share in utility: μ_h	0.044
Leisure's share in utility: 1-$\mu_c - \mu_h$	0.642

$$y_{dt} = b_{dt}^{B_d} m_{dt}^{M_d} s_{dt}^{S_d} \tag{61}$$

These provide the required 33 equations to solve for equilibrium. In addition there are the laws of motion for the technology shocks and the uncertainty shocks.

$$\tilde{\mathbf{z}}_{t+1} = B \cdot \tilde{\mathbf{z}}_t + \vec{\varepsilon}_{t+1} \tag{62}$$

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^{\chi} \exp^{\varepsilon_{\sigma,t+1}} \tag{63}$$

To solve the model, we log linearize around the steady-state. The solution is defined by 33 equations in which the endogenous variables are expressed as linear functions of the vector of state variables $(z_{bt}, z_{mt}, z_{st}, \sigma_{\omega t}, k_t, k_t^e, h_t)$.

4 Calibration and Data

A strong motivation for using the Davis and Heathcote (2005) model is that the theoretical constructs have empirical counterparts. Hence, the model parameters can be calibrated to the data. We use directly the parameter values chosen by the previous authors; readers are directed to their paper for an explanation of their calibration methodology. Parameter values for preferences, depreciation rates, population growth and land's share are presented in Table 2. In addition, the

Table 3: Intermediate Production Technology Parameters

	B	M	S
Input shares for consumption/investment good (B_c, M_c, S_c)	0.031	0.270	0.700
Input shares for residential investment (B_d, M_d, S_d)	0.470	0.238	0.292
Capital's share in each sector $(\theta_b, \theta_m, \theta_s)$	0.132	0.309	0.237
Sectoral trend productivity growth (%) (g_{zb}, g_{zm}, g_{zs})	-0.27	2.85	1.65

parameters for the intermediate production technologies are presented in Table 3.[17]

As in Davis and Heathcote (2005), the exogenous shocks to productivity in the three sectors are assumed to follow an autoregressive process as given in eq. (3). The parameters for the vector autoregression are the same as used in Davis and Heathcote (2005) (see their Table 4, p. 766 for details). In particular, we use the following values (recall that the rows of the B matrix correspond to the building, manufacturing, and services sectors, respectively):

$$B = \begin{pmatrix} 0.707 & 0.010 & -0.093 \\ -0.006 & 0.871 & -0.150 \\ 0003 & 0.028 & 0.919 \end{pmatrix}$$

Note this implies that productivity shocks have modest dynamic effects across sectors. The contemporaneous correlations of the innovations to the shock are given by the correlation matrix:

$$\Sigma = \begin{pmatrix} Corr\left(\varepsilon_b, \varepsilon_b\right) & Corr\left(\varepsilon_b, \varepsilon_m\right) & Corr\left(\varepsilon_b, \varepsilon_s\right) \\ & Corr\left(\varepsilon_m, \varepsilon_m\right) & Corr\left(\varepsilon_m, \varepsilon_s\right) \\ & & Corr\left(\varepsilon_s, \varepsilon_s\right) \end{pmatrix} = \begin{pmatrix} 1 & 0.089 & 0.306 \\ & 1 & 0.578 \\ & & 1 \end{pmatrix}$$

The standard deviations for the innovations were assumed to be: $(\sigma_{bb}, \sigma_{mm}, \sigma_{ss}) =$ (0.041, 0.036, 0.018).

For the financial sector, we use the same loan and bankruptcy rates as in Carlstrom and Fuerst (1997) in order to calibrate the steady-state value of $\bar{\omega}$, denoted ϖ, and the steady-state standard

[17] Davis and Heathcote (2005) determine the input shares into the consumption and residential investment good by analyzing the two sub-tables contained in the "Use" table of the 1992 Benchmark NIPA Input-Output tables. Again, the interested reader is directed to their paper for further clarification.

deviation of the entrepreneur's technology shock, σ_0. The average spread between the prime and commercial paper rates is used to define the average risk premium (rp) associated with loans to entrepreneurs as defined in Carlstrom and Fuerst (1997); this average spread is 1.87% (expressed as an annual yield). The steady-state bankruptcy rate (br) is given by $\Phi(\varpi, \sigma_0)$ and Carlstrom and Fuerst (1997) used the value of 3.9% (again, expressed as an annual rate). This yields two equations which determine (ϖ, σ_0):

$$\begin{aligned} \Phi(\varpi, \sigma_0) &= 3.90 \\ \frac{\varpi}{g(\varpi, \sigma_0)} - 1 &= 1.87 \end{aligned} \tag{64}$$

yielding $\varpi \approx 0.65$, $\sigma_0 \approx 0.23$.[18]

Finally, the entrepreneurial discount factor γ can be recovered by the condition that the steady-state internal rate of return to the entrepreneur is offset by their additional discount factor:

$$\gamma \left[\frac{\bar{s} f(\varpi, \sigma_0)}{1 - \bar{s} g(\varpi, \sigma_0)} \right] = 1$$

and using the mark-up equation for $\bar{s}$ in eq. (29), the parameter γ then satisfies the relation

$$\gamma = \frac{g_U}{g_K} \left[1 + \frac{\phi(\varpi, \sigma_0)}{f'(\varpi, \sigma_0)} \right] \approx 0.832$$

where, g_U is the growth rate of marginal utility and g_K is the growth rate of consumption (identical to the growth rate of capital on a balanced growth path). The autoregressive parameter for the risk shocks, χ, is set to 0.90 so that the persistence is roughly the same as that of the productivity shocks.

Figure 1 and Table 4 show the cyclical and statistical features for the period from 1975 through

[18] Note that the risk premium can be derived from the markup share of the realized output and the amount of payment on borrowing: $\bar{s}_t \bar{\omega}_t fp_t = (1 + rp)(fp_t - nw_t)$. And using the optimal factor payment (project investment), fp_t, in equation (30), we arrive at the risk premium in equation (64).

Table 4: Business Cycle Properties (1975:1 - 2007:2)

Data: all series are Hodrick-Prescott filtered with the smoothing parameter set to 1600			
% S.D.			
GDP	1.2		
Consumption	0.69		
House Price Index (HPI)	1.9		
Non - Residential Fixed Investment (Non-Res)	4.5		
Residential Fixed Investment (Res)	8.7		
Correlations			
GDP, Consumption	0.83		
GDP, HPI	0.31		
GDP, HPI (for pre 1990)	0.21		
GDP, HPI (for post 1990)	0.51		
Non-Res, Res	0.29		
GDP, Non-Res	0.81		
GDP, Res	0.30		
GDP, Real Estate Loans (from 1985:1)	0.15		
Real Estate Loans, HPI	0.47		
Lead - Lag correlations	$i = -3$	$i = 0$	$i = 3$
$GDP_t, Non-res_{t-i}$	0.47	0.78	0.31
GDP_t, res_{t-i}	-0.27	0.20	0.32
$Non-res_{t-i}, res_t$	0.63	0.26	-0.27

the second quarter of 2007 using quarterly data. The U.S. business cycle properties for various aggregate and housing variables are listed in Table (4). As mentioned in the Introduction, the stylized facts for housing are readily seen i): Housing prices are much more volatile than output; ii) Residential investment is almost twice as volatile as non-residential investment; iii) GDP, consumption, the price of housing, non-residential - and residential investment all co-move positively; iv) and lastly, residential investment leads output by three quarters.

Table 5: Steady - State Values: Ratios to GDP

Variables	Our model	Davis and Heathcote (D & H)	Data (1948 - 2001)	Data (1948 - 2007)
Capital Stock (K)	1.96	1.52	1.52	1.43
Residential structures stock($P_d \times S$)	3.20	1.00	1.00	1.00
Private consumption (PCE)	0.77	0.64	0.64	0.65
Nonresidential investment (i_c)	0.18	0.14	0.14	0.13
Residential investment (i_d)	0.05	0.04	0.05	0.05
Construction ($b = p_b x_b$)	0.05	0.05	0.05	0.05
Manufacturing ($m = p_m x_m$)	0.24	0.25	0.33	0.31
Services ($p_s x_s + qh$)	0.71	0.71	0.62	0.63

5 Results

5.1 Steady State Values, Second Moments and Lead - Lag Patterns

Table 5 shows some of the selected steady-state values from our model that includes the financial friction. These steady state values differ somewhat from those in Davis and Heathcote (2005) but the calibrated parameter values produce steady-state values that are broadly in line with the data.

Our main interest is in the business cycle, i.e. second moment, properties of the model. To examine this, we solve the model for three different levels of stochastic volatility: a low level in which the innovation to stochastic volatility has a standard deviation of 0.20, a medium level in which the innovation to stochastic volatility has a standard deviation of 0.50, and a high level of volatility in which the innovation to stochastic volatility has a standard deviation of 1.10.[19] That is we assume that for the low volatility economy, $\varepsilon_\sigma \sim N(0, 0.20)$, the medium volatility economy has $\varepsilon_\sigma \sim N(0, 0.50)$ and the high volatility economy has $\varepsilon_\sigma \sim N(0, 1.10)$. (For all economies the autoregressive parameter for the shock process was held constant at $\chi = 0.90$.) The results from this exercise are shown in Table 6. We compare the second moments from our economy to that in the Davis and Heathcote (2005) model and to the data. For the last comparison, we present a two-standard deviation range as a crude measure of a 95% confidence interval.

[19] Recent research by Gilchrist et al. (2009) suggests that a relative standard deviation of 110% is not unreasonable.

Table 6: Standard Deviations in ratio to GDP with low volatility of variance

Variables (in relation to GDP)	Our model			(D & H)	Data ($\pm$2s.d.) (1948 - 2007)
Volatility of σ_ω	low: 20 %	mid: 50%	high:110%		
Markup ($\overline{s}$)	**0.82**	1.90	3.97	-	0.96 ($\pm$0.22)
Private consumption (PCE)	0.52	0.52	0.55	0.48	0.78 ($\pm$0.15)
Labor (N)	0.39	0.42	0.53	0.41	1.01 ($\pm$0.20)
Nonresidential investment (i_c)	3.17	3.74	5.52	3.21	2.51 ($\pm$0.45)
Residential investment (i_d)	**5.91**	12.2	25.0	6.12	5.04 ($\pm$0.98)
House price (p_h)	0.46	0.76	**1.44**	0.4	1.36 ($\pm$0.31)
Construction output (x_b)	3.85	6.15	11.5	4.02	2.74 ($\pm$0.53)
Manufacturing output (x_m)	**1.51**	**1.50**	1.46	1.58	1.85 ($\pm$0.36)
Service output (x_s)	**0.98**	**1.00**	1.06	0.99	0.85 ($\pm$0.16)
Construction labor (n_b)	**2.62**	5.46	11.2	2.15	2.37 ($\pm$0.45)
Manufacturing (n_m)	0.38	0.38	0.39	0.39	1.53 ($\pm$0.30)
Service (n_s)	0.39	0.45	**0.64**	0.37	0.66 ($\pm$0.13)
Construction Investment (i_b)	2.87	5.60	**11.3**	25.9	9.69 ($\pm$1.88)
Manufacturing Investment (i_m)	1.05	1.04	1.02	3.23	3.53 ($\pm$0.69)
Service Investment (i_s)	1.04	1.05	1.11	3.43	2.91 ($\pm$0.46)

Note that when the stochastic volatility in the housing sector is low, the model basically replicates the frictionless economy which, in turn, is consistent with many features seen in the data. However, a glaring inconsistency from the low volatility economy is the behavior of housing prices: the model severely underpredicts the volatility of housing prices. The model improves in this dimension as stochastic volatility in the housing sector increases and, in the high volatility economy, the model comes close to the matching the observed volatility. But success in this dimension comes at a cost in that the volatility of residential investment is over five times greater than that seen in the data. The Davis and Heathcote model makes the strong assumption of no investment adjustment costs and relaxing this assumption would moderate the volatility in residential investment.

The contemporaneous correlations of several key variables are presented in Table 7 (for expositional purposes, we only report that values for the low volatility economy ($\varepsilon_\sigma \sim N(0, 0.20)$) as the correlations were consistent across the three model economies). All variables co-move positively with the exception of house price and residential investment. This inconsistency with the

Table 7: Correlations

Variables $\varepsilon_\sigma \sim N(0, 20)$	Our Model	D & H	Data ($\substack{+\\-}$2s.d.) (1948 - 2007)
Correlations			
Monitoring Costs	$\mu = 0.25$		
(GDP, PCE)	0.96	0.95	0.79 (+0.08,-0.12)
(GDP, p_h)	**0.56**	0.65	0.73 (+0.13,-0.22)
(i_c, PCE)	0.86	0.91	0.61 (+0.15,-0.20)
(i_d, PCE)	0.21	0.26	0.66 (+0.13,-0.18)
(i_c, i_d)	-0.07	0.15	0.25 (+0.23,-0.27)
(i_d, p_h)	-0.34	-0.2	0.34 (+0.22,-0.26)
$(\bar{s}, p_h)$	**0.40**	-	0.55 (+0.21,-0.30)
$(\bar{s}, GDP)$	**0.04**	-	0.11 (+0.33,-0.36)
$(Risk\ Premium, GDP)$	-0.06	-	-0.65 (+0.29,-0.18)

data is not surprising in that the shocks to our model economy all emanate from the supply side whereas demand shocks (say in the form of easier access to credit for home buyers) no doubt play an important role in the behavior of the actual data. Our model critically has predictions for two variables that are not present in the Davis and Heathcote (2005) model: namely, the housing markup parameter, $\bar{s}_t$, and the risk premium associated with loans to housing producers. The time series for these two variables are presented in Figure 4. It is clear that the housing markup is procyclical while the risk premium is countercyclical. As reported in Table 7, the model with financial frictions can replicate these features, although the risk premium is only mildly countercyclical. The model's ability to replicate these two features provides a compelling argument for the inclusion of a credit market.

The last set of housing stylized facts that is in question is the lead - lag patterns of residential and non-residential investments. Table 8 shows the results. As in Davis and Heathcote (2005), we also fail to reproduce this feature of the data. Consequently, the propagation mechanism of agency costs model does amplify prices and other real variables, but does not contribute in explaining the lead-lag features.

Table 8: Lead - Lag Patterns: Annual Frequency

Variables $\varepsilon_\sigma \sim N(0,20)$	Our Model	D & H	Data ($\pm$2 s.d.) (1948 - 2007)
Monitoring Costs	$\mu = 0.25$		
$(i_c[-1], GDP[0])$	0.52	0.45	0.25(+0.24,-0.27)
$(i_c[0], GDP[0])$	0.90	0.94	0.75 (+0.10,-0.15)
$(i_c[1], GDP[0])$	**0.29**	0.33	0.47 (+0.18,-0.24)
$(i_d[-1], GDP[0])$	0.11	0.19	0.52 (+0.17,-0.23)
$(i_d[0], GDP[0])$	**0.34**	0.44	0.47 (+0.19,-0.24)
$(i_d[1], GDP[0])$	0.31	0.14	-0.22 (+0.27,-0.24)
$(i_c[-1], i_d[0])$	0.25	0.07	-0.37 (+0.26,-0.21)
$(i_c[0], i_d[0])$	-0.07	0.15	0.25 (+0.24,-0.27)
$(i_c[1], i_d[0])$	0.02	0.08	0.53 (+0.17,-0.23)

5.2 Dynamics: Impulse Response Functions

While the results discussed above provide some support for the housing cum credit channel model, the role of the lending channel is not easily seen because of the presence of the other impulse shocks (i.e., the sectoral productivity shocks). To analyze how the lending channel influences the effects of a risk shock, we analyze the model's impulse response functions to risk shocks under two different scenarios. In the first scenario, we set the monitoring cost parameter to zero ($\mu = 0$) while in the second scenario we use the value employed in the stochastic simulations ($\mu = 0.25$). With no monitoring costs, risk shocks should not influence the behavior of housing prices and residential investment (see eqs. $(29), (45)$, and (46). We also examine the economy's response to an innovation to productivity in the construction sector (this being the most important input into the residential investment good). The impulse response functions (to a 1% innovation in both shocks) for a selected set of key variables are presented in Figures 7-9.

We first turn to the behavior of three key macroeconomic variables, namely GDP, household consumption (denoted PCE), and residential investment (denoted RESI) seen in Figure 7. The response to a technology shock to the construction sector has the predicted effect that GDP increases. On the other hand, consumption falls slightly while residential investment increases; this reflects agents response to the expected high productivity (due to the persistence of the shock) in the construction sector. Note that the response in residential investment is much larger than in the other two variables since this sector is most affected by the greater productivity in this critical input. Also note that monitoring costs (i.e. whether $\mu = 0$ or $\mu = 0.25$) play a rather insignificant role in the dynamic effects of a technology shock. And, as the model implies, when $\mu = 0$, risk shocks have no effect on the economy. (For this reason, in Figures 8 and 9, we report only the responses for $\mu = 0.25$.) When $\mu = 0.25$, a risk shock which affects housing production results in a modest fall in GDP but a relatively dramatic fall in residential investment. Recall, as discussed in the partial equilibrium analysis of the credit channel model, an increase in productivity risk results in a leftward shift in the supply of housing; since residential investment is the primary input into housing, it too falls in response to the increased risk. Consumption responds positively which is consistent with models that have an investment specific technology shock (e.g. Greenwood, Hercowitz, and Krusell (2000)).

Figure 8 reports the impulse response functions of the housing markup, the risk premium on loans to the housing producers and the bankruptcy rate. A positive technology shock to the construction sector increases the demand for housing and, ceteris paribus, will result in an increase in the price of housing. This will result in greater lending to the housing producers which will result in a greater bankruptcy rate and risk premium; both of these effects imply that the housing markup will increase. Note the counterfactual implication that both the bankruptcy rate and the risk premium on loans will be procyclical; this was also the case in the original Carlstrom and Fuerst (1997) model and for exactly the same reason. In contrast, a risk shock produces countercyclical behavior in these three variables. Hence, this argues for inclusion of risk shocks as an important impulse mechanism in the economy.

Finally, we report in Figure 9, the impulse response functions of the prices of land and housing to the two shocks. A technology shock to the construction sector results in lower cost of housing inputs due to the increased output in residential investment so that the price of housing falls. However, the price of land, i.e. the fixed factor, increases. For an uncertainty shock, the resulting fall in the supply of housing causes the demand for the fixed factor (land) to fall and the price of the final good (housing) to increase.

In ending this section, a word of caution is needed in interpreting the quantitative magnitudes seen in the impulse response functions. In particular, note that the response of housing prices to a productivity increase in the construction sector is greater than the response due to a risk shock. One might deduce that the housing sector and risks shocks play a minor role in the movement of housing prices. As the results from the full model (i.e. when the all technology and risk shocks are present) imply, such a conclusion would be incorrect (see Table 6).

5.3 Some Final Remarks

Our primary findings fall into two broad categories. First, risk shocks to the housing producing sector imply a quantitatively large role for uncertainty over the housing and business cycles. Second, our model can account for most of the salient features of housing stylized facts, in particular, housing prices are more volatile than output. The lead - lag pattern of residential and non-residential, however, is still not reconciled within our framework.[20]

For future research, modelling uncertainty due to time variation in the types of entrepreneurs would be fruitful. One possibility would be an economy with a low risk agent whose productivity shocks exhibit low variance and a high risk agent with a high variance of productivity shocks. Because of restrictions on the types of financial contracts that can be offered, the equilibrium is a pooling equilibrium so that the same type of financial contract is offered to both types of agents. Hence the aggregate distribution for technology shocks hitting the entrepreneurial sector is a mixture of the underlying distributions for each type of agent. Our conjecture is that this form of uncertainty has important quantitative predictions and, hence, could be an important impulse mechanism in the credit channel literature that, heretofore, has been overlooked. It also anecdotally corresponds with explanations for the cause of the current credit crisis: a substantial

[20] Recently, Jonas Fisher (2007) presents a model with household production which does produce the lead-lag pattern pattern observed in residential and non-residential investment.

fraction of mortgage borrowers had higher risk characteristics than originally thought.

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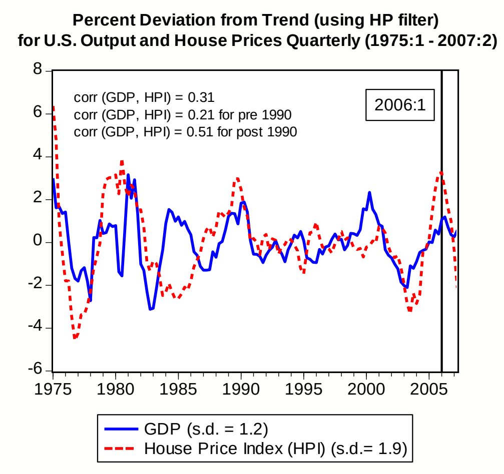
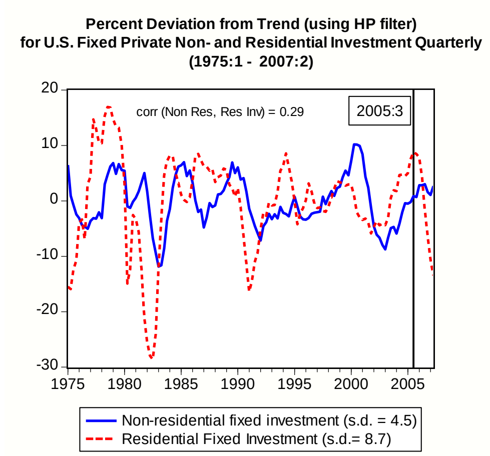
6 Data Appendix

- Loans: Federal Reserve Board, Statistics: Releases and Historical Data Assets and Liabilities of Commercial Banks in the U.S. - h.8. Seasonally adjusted, adjusted for mergers, billions of dollars. http://www.federalreserve.gov/releases/h8/data.htm
  - Total Loans: Total loans and leases at commercial banks.
  - Residential Real Estate Loans: Loans to residential sector excluding revolving home equity loans.
  - Commercial Real Estate Loans: Loans to commercial sector excluding revolving home equity loans.
  - Commercial and Industrial Loans (Business Loans): Commercial and industrial loans at all Commercial Banks.
  - Consumer Loans: Consumer (Individual) loans at all commercial banks.
- Gross Domestic Product (GDP), Personal Consumption Expenditures (PCE), Aggregate of gross private domestic investment (Non-RESI), Residential gross private domestic investment (RESI), and the Price Indexes for private residential Investment (PRESI) are all from the National Income and Product Accounts Tables (NIPA) at the Bureau of Economic Analysis.
  - To calculate the implied markup, $\overline{s}$, we used the house price index (HPI), residential investment (RESI) and the price for the RESI (PRESI).
  - We use the equation $p_{ht} = (1 - \zeta)\, y_{dt}^{-\zeta} p_{dt} \overline{s}_t$.
- House Price Index. (HPI): Constructed based on conventional conforming mortgage transactions obtained from the Federal Home Loan Mortgage Corporation(Freddie Mac) and the Federal National Mortgage Association (Fannie Mae). Source: The Office of Federal Housing Enterprise Oversight (OFHEO).

- Risk Premium: To calculate our risk premium, we used the spread between the prime rate and the three month commercial paper. These data can be obtained from Federal Reserve Bank of St. Louis, FRED Dataset under the category of Interest Rates.
  - http://research.stlouisfed.org/fred2/categories/22. These two series are monthly.
  - Commercial rate: CP3M, 3-Month Commercial Paper Rate :1971-04 till 1997-08.
  - Prime rate: MPRIME, Bank Prime Loan Rate: 1949-01 till 2009-08.

Figure 1: U.S. GDP, House Price, Non – and Residential Fixed Investments (1975:1 – 2007:2)





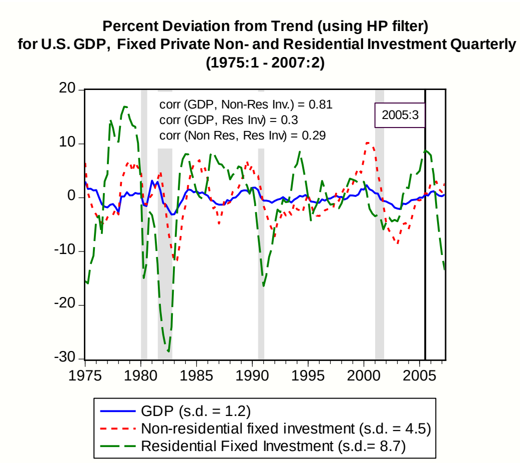


Figure 2: Different Loans at All U.S. Commercial Banks (1990:1 to 2007:7)

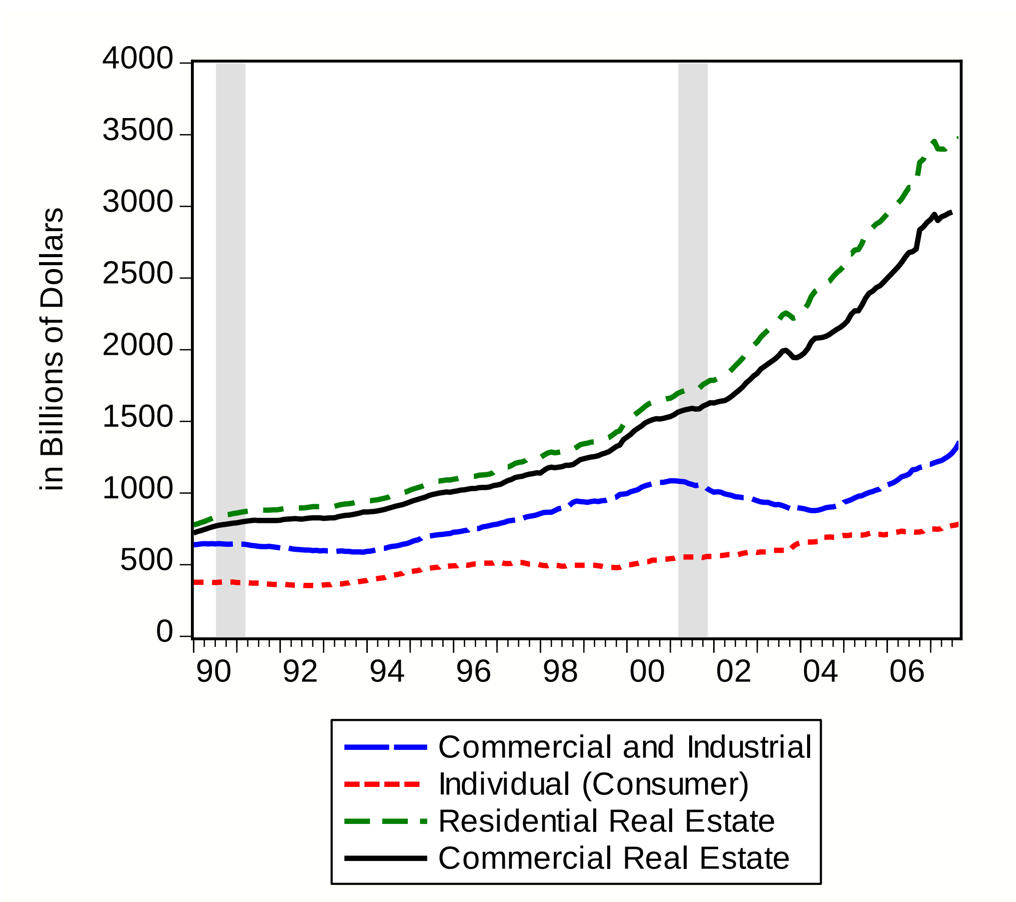
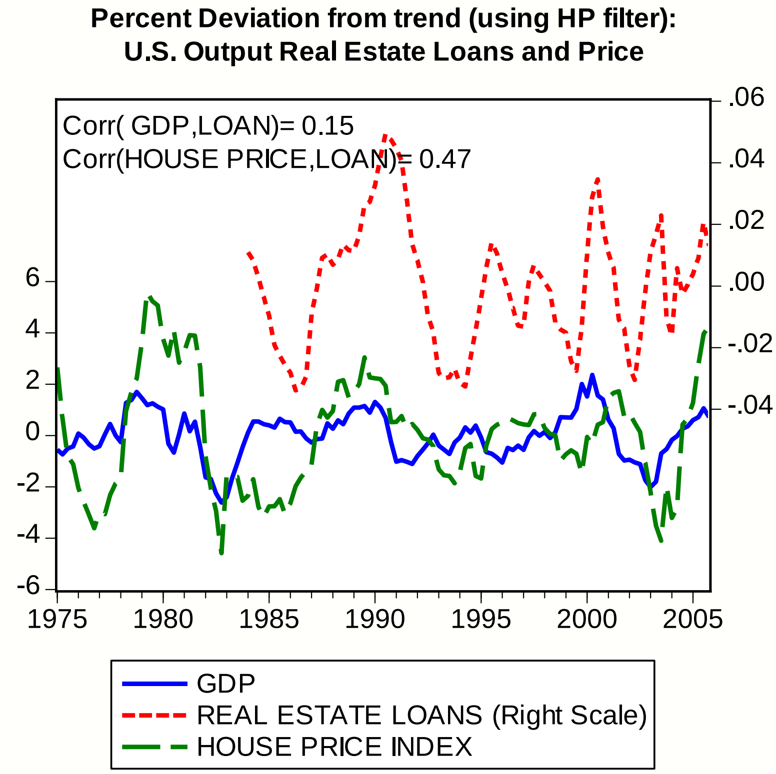


Figure 3: U.S. GDP, House Price and Residential Real Estate Loans



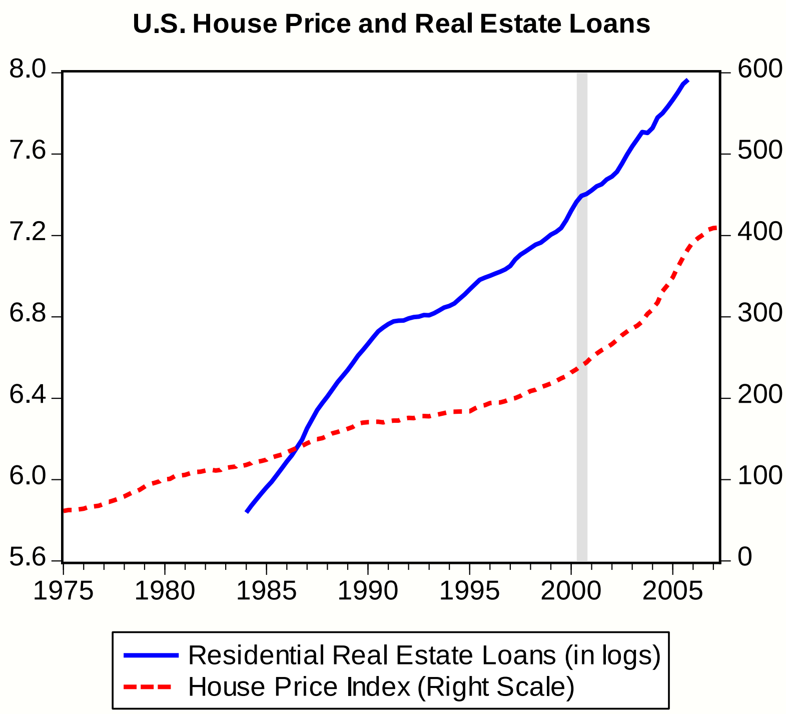
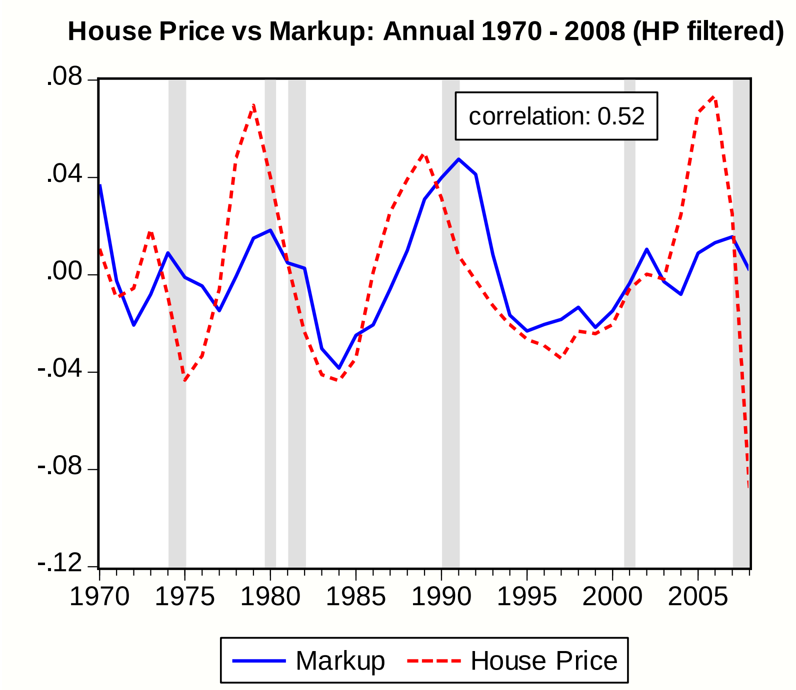


Figure 4: House Price, Housing Markups, and Risk Premium



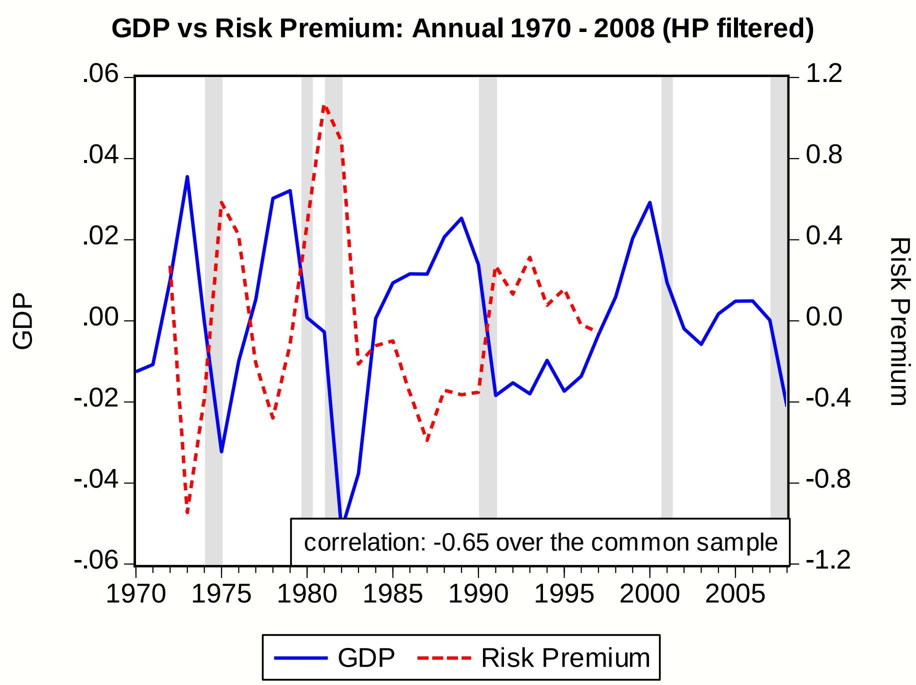


Figure 5: Flow of Funds in Credit Channel with Housing Model
(Household land income and Entrepreneur labor input and income are not shown)

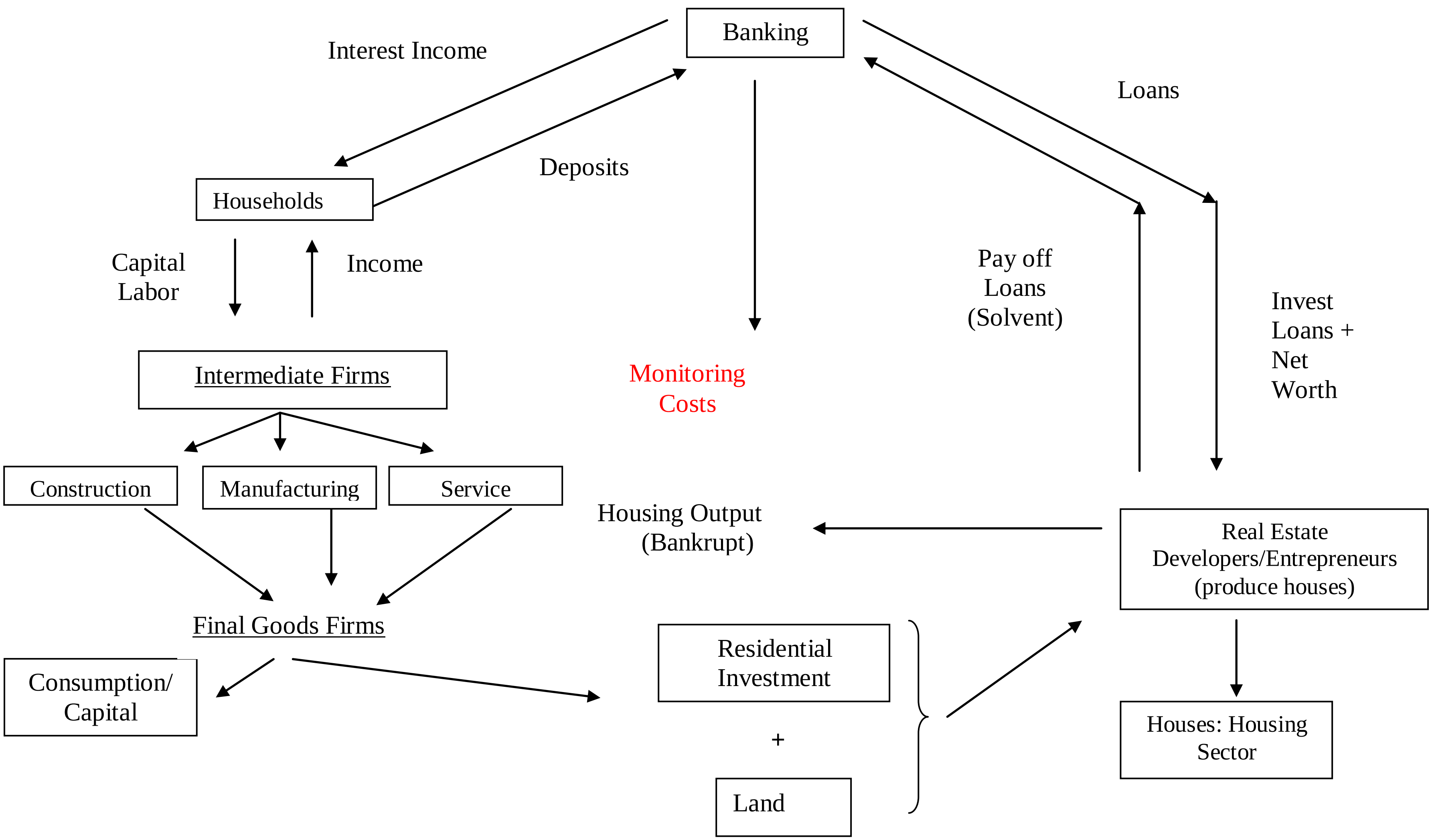


Figure 6: Technology and Uncertainty Shocks:
Effects on Housing Demand and Supply

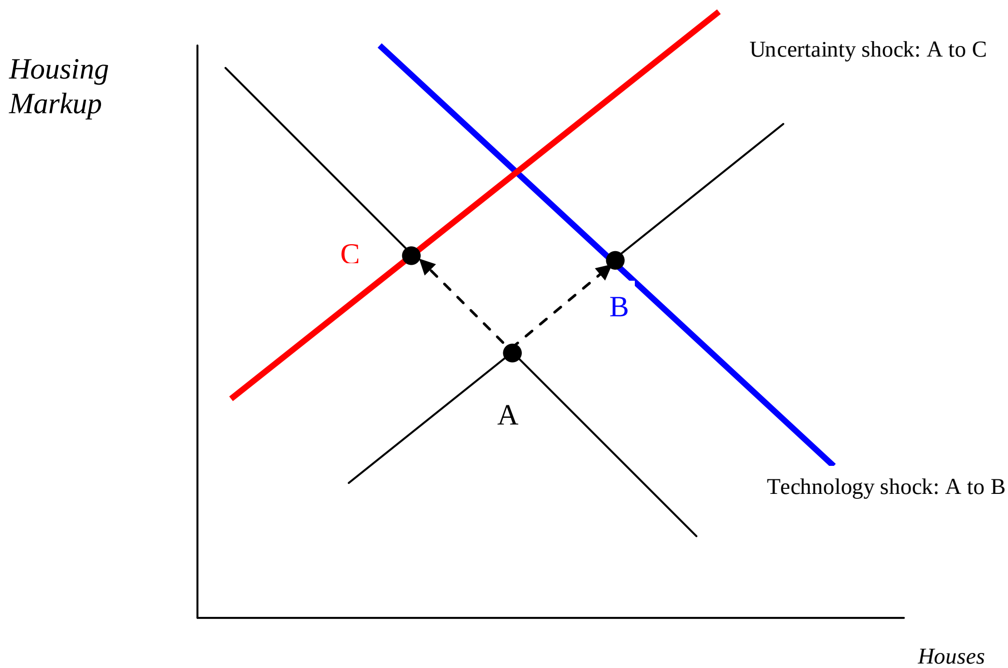


Figure 7: Response of Output, Private Consumption Expenditure, and Investment to 1% increase in Sector (Construction) Technology shocks and Uncertainty Shocks
(percentage deviations from steady-state values)

Technology Shock to Construction sector	Uncertainty Shock to Housing Developer
μ= 0.25 (monitoring cost: effects of agency cost) μ= 0 (no monitoring cost: multi-sector model with housing and no agency cost)	

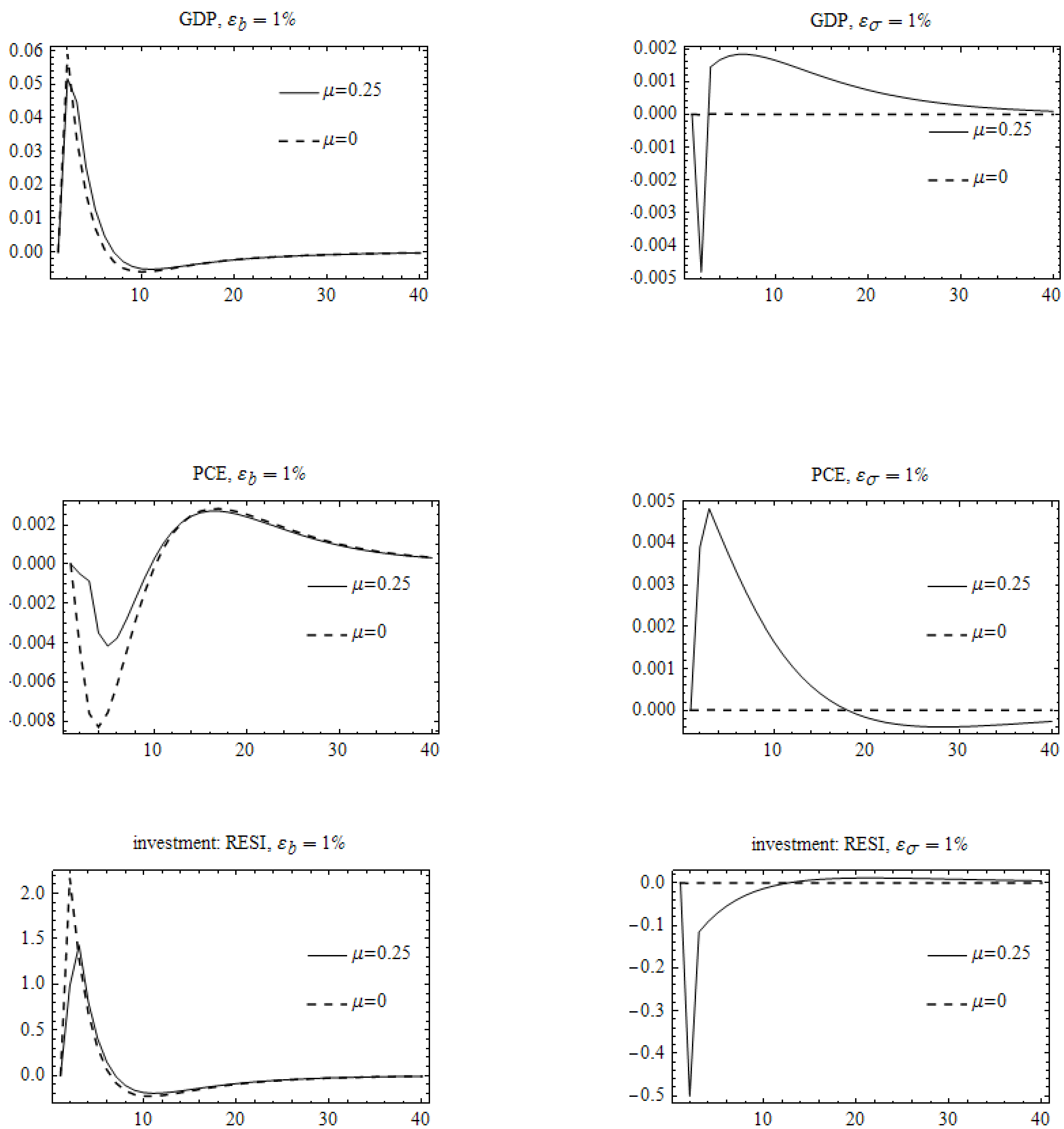


Figure 8: Response of Markup House Price, Risk Premia, and Bankruptcy Rate to a 1% increase in Sector (Construction) Technology shocks and Uncertainty Shocks

(percentage deviations from steady-state values)

Technology Shock to Construction sector	Uncertainty Shock to Housing Developer

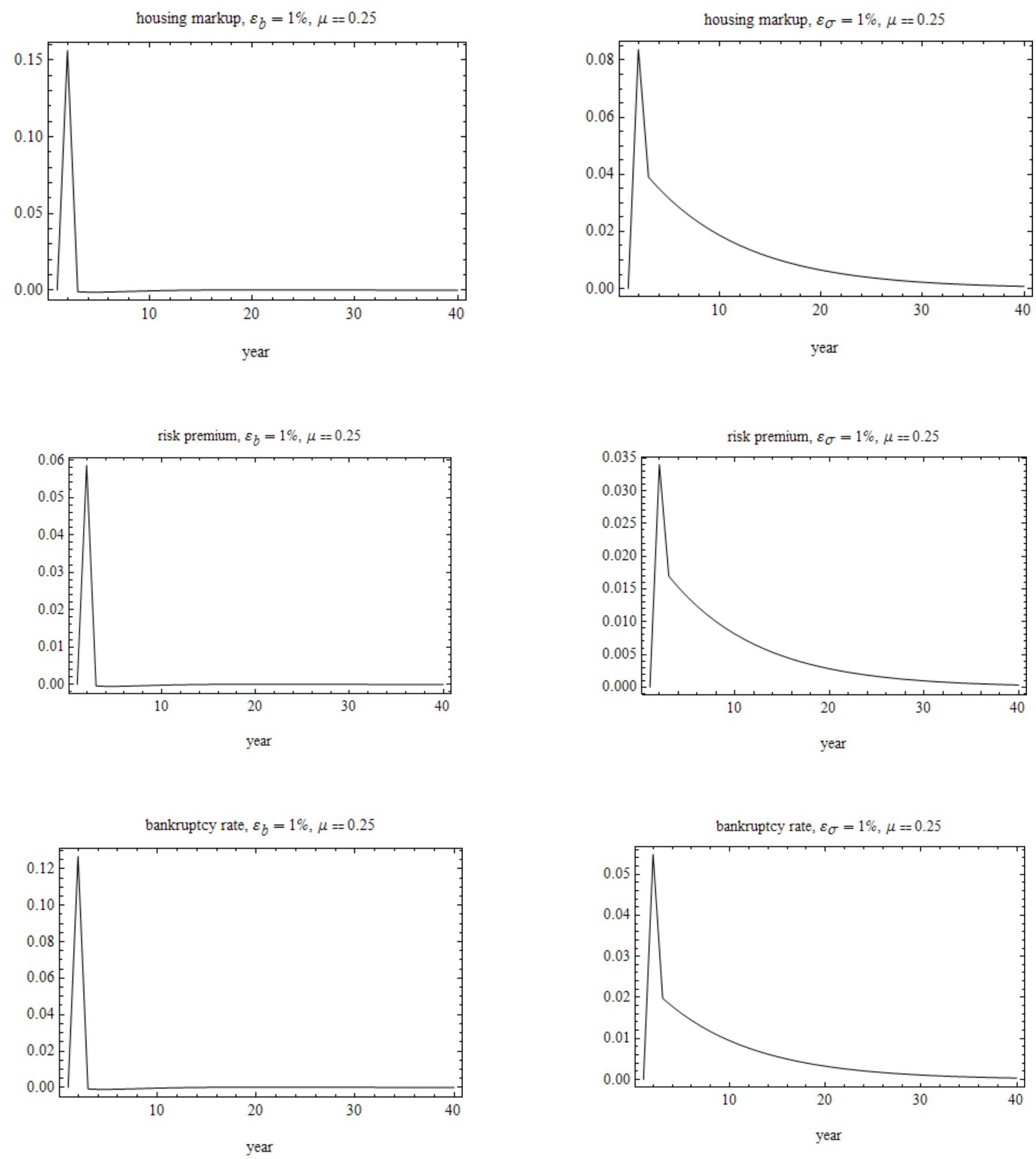


Figure 9: Response of Land and House Prices to a 1% increase in Sector (Construction) Technology Shocks and Uncertainty Shocks
(percentage deviations from steady-state values)

Technology Shock to Construction sector	Uncertainty Shock to Housing Developer