Sowing the Seeds of Financial Crises: Endogenous Asset Creation and Adverse Selection

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Abstract

What sows the seeds of financial crises, and what policies can help avoid them? I model the interaction between the *ex-ante* production of assets and *ex-post* adverse selection in financial markets. Positive shocks that increase market prices exacerbate the production of low-quality assets and can increase the likelihood of a financial market collapse. The interest rate and the liquidity premium are endogenous and depend on the functioning of financial markets as well as the total supply of assets (private and public). Optimal policy balances the economy's liquidity needs *ex-post* with the production incentives *ex-ante*, and it can be implemented with three instruments: government bonds, asset purchase programs, and transaction taxes. Public liquidity improves incentives but implies a higher deadweight loss than private market interventions. Optimal policy does not rule out private market collapses but mitigates the fluctuations in the total liquidity.

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1 Introduction

It is widely believed that the recession that hit the US economy in 2008 originated in the financial sector. The years previous to the Great Recession were characterized by a rapid increase in the private production of assets that were considered safe, mostly through securitization. Many of the markets for these assets later collapsed, marking the starting point of the deepest recession in the post-war era. To mitigate the adverse consequences of the financial crisis, the Fed responded aggressively using a variety of instruments, such as the direct purchase of mortgage-back securities (MBS) as part of the Quantitative Easing (QE) 1 program in 2008, as well as the active management of public liquidity, with the purchase of long-term Treasuries as part of QE 2 in 2010.[1] However, relatively little theoretical work has analyzed the optimal policy *mix* in economies exposed to financial distress. This is particularly important in contexts in which policy can be a contributing factor to sowing the seeds of the next crisis. For example, policies aimed at improving the efficiency of private markets might induce excessive risk build-ups, while policies targeted at increasing public liquidity can be costly and even exacerbate the malfunctioning of private markets by crowding out the private sector. Therefore, a formal analysis requires understanding the interplay between the frictions in the economy and the dynamic effects of interventions.

In this paper, I build a model of an economy susceptible to risk build-ups that can lead to financial crises, and use it as a laboratory to study the role of policy in improving market outcomes. The model features an endogenous determination of *financial market fragility*, i.e., the probability of a *discontinuous* drop in the volume traded in private financial markets, which disrupts the flow of funds to the agents with the highest valuation. I analyze constrained efficiency by considering the problem of a planner who faces the same constraints as the private economy and study what type of tools can implement a Pareto improvement. I show that a government can implement the constrained efficient allocation by a combination of three instruments: government bonds, asset purchase programs, and transaction (or "Tobin") taxes. Notably, the optimal implementation depends on the government's ability to commit to a plan, and a government with imperfect commitment might choose to distort its policy choices *ex-ante* to influence its decisions *ex-post*.

I develop a model of asset quality determination in which the ex-ante production of assets interacts with ex-post adverse selection in financial markets. Agents in the economy face idiosyncratic risk, and financial markets are incomplete. Assets play a dual role and derive their value from the dividends they pay and the liquidity services they provide. Better-quality assets pay higher dividends, but because of adverse selection in markets, they sell at a pooling price with lower-quality assets. This cross-subsidization between high- and low-quality assets introduces a motive for agents to produce more lemons when they expect prices to be high, since they expect

[1] Since the Great Recession, the Fed has adopted these instruments as part of its standard toolkit to deal with large shocks. For example, in 2019, the New York Fed increased its repo operations in response to a sudden spike of the repo rate in September of that year. In March of 2020, the Fed implemented a new round of securities purchases to mitigate the financial turmoil caused by the COVID-19 crises.

to sell the assets rather than keep them until maturity. As a consequence, the theory predicts that the production of low-quality assets is more responsive to market conditions than that of high-quality assets. Shocks that improve the functioning of financial markets or increase their scarcity value exacerbate the production of lemons and may even increase the exposure of the economy to a financial market collapse –a process that disrupts liquidity.

An important distinctive feature of the model is that the supplies of privately produced tradable assets and government bonds (i.e., private and public liquidity) interact through an endogenously determined *liquidity premium*. Because of the market incompleteness, the allocation of resources in the competitive equilibrium is always imperfect, and the liquidity premium is a measure of the *scarcity* of assets that can facilitate the flow of funds. Private and public assets can fulfill this role, but only private assets suffer from adverse selection. My theory predicts that reductions in the supply of government bonds increase the production of private assets that can serve as substitutes.[2] But because low-quality assets are more sensitive to changes in the value of liquidity services, the private production is biased towards low-quality assets. Hence, a shortage of safe assets induces a deterioration of private asset quality. Indeed, my model predicts that the reductions in US government bonds in the late 1990s due to sustained fiscal surpluses, as well as the increased foreign demand for US-produced safe assets in the early 2000s (a consequence of the so-called "savings glut"), generated perverse effects on the quality composition of privately produced assets.[3]

The mechanics of the model hinge upon the agents' valuation of the different asset qualities. Agents with high-quality assets sell them only if their liquidity needs are high relative to the price discount they suffer in the market due to the adverse selection problem. In contrast, agents with low-quality assets always sell their holdings. Anticipating that this will be their strategy in the market, agents adjust their quality production decisions to the expected market conditions. If the market's expectations are high –in the sense that expected prices are high– agents anticipate that the probability they will sell their assets is relatively high, independent of their quality. In this case, more low-quality assets are produced. That is, low-quality assets are produced for *speculative motives*: not for their fundamental value, but for the profit the agent can make from selling in the market. This result is an extension of Akerlof (1970), who shows that the decision to sell non-lemons is more sensitive to prices than the decision to sell lemons. In my model, Akerlof's result still holds in the market for assets. But the lower exposure of the private valuation of high-quality assets to market conditions results in the opposite sensitivity in the production stage.

While the theory presented is silent about the specifics of the safe asset production process, the economic forces that it highlights are typical of the full process of transforming illiquid assets into liquid ones. Safety refers to a characteristic of assets that are perceived as high quality, have an

[2]This channel has been found empirically, for example, by Greenwood et al. (2015) and Krishnamurthy and Vissing-Jorgensen (2015).

[3]See Caballero (2006) and Caballero (2010) for a discussion of safe asset shortages. For a quantitative analysis, see Barclays Capital (2012) and Caballero et al. (2017).

active (*liquid*) market, and facilitate financial transactions.[4] While traditionally this characteristic was mostly limited to government bonds and bank deposits, in the last 30 years there has been a large increase in the use of other privately produced assets, such as asset- and mortgage-backed securities (see Gorton et al. (2012)). This process was particularly stark in the mortgage market, which saw an explosion of non-standard, low-documentation mortgages, and low credit score borrowers.[5] In fact, the Bank for International Settlements (2001) articulated an early warning about the deterioration of the quality of assets used as collateral. In my interpretation, the production of assets comprises both the origination of loans (e.g., mortgages) and their posterior securitization (e.g., AAA-rated private-label mortgage-backed securities).[6] In both cases, the "producers" know more than other market participants about the underlying quality of these products, either because they have collected information that cannot be credibly transmitted, or because they know how much effort they put into the process. Hence, the problem of quality production and adverse selection can be present in the whole intermediation chain.[7]

I then solve the problem of a planner who faces the same constraints as the private economy. Solving for the constrained optimal allocation in the presence of moral hazard, adverse selection, and aggregate risk is a complex task. First, the optimal policy involves *functions* of the aggregate state. Second, because the planner internalizes the effects that its actions have on the equilibrium determination of the economy, its calculations involve a two-way interaction between a direct effect on liquidity (and the functioning of private markets) and an indirect effect on the production incentives in the induced equilibrium of the economy across different dates and states. Third, the discontinuities in the private market generate kinks in the planner's problem. Nonetheless, I present a solution method that allows me to provide insights from analytical results.

I first solve the problem of a planner that can fully commit to a plan, even if it is time-inconsistent. I find that the optimal policy balances two forces: a liquidity effect and an incentives effect. The liquidity effect captures the impact of the planner's plan on the reallocation of resources for a given composition of asset quality. This is the force that justifies government intervention in models in which the private sector fails to fully reallocate resources to those agents with the highest valuations, as in Woodford (1990) and Holmström and Tirole (1998). The liquidity effect is static: since it takes the asset quality composition as given, it is optimized state by state and does not incorporate any intertemporal feedback effects. In contrast, the incentives effect captures

[4]This has been recently emphasized, for instance, by Calvo (2013), Gorton et al. (2012) and Gorton (2017).

[5]See Ashcraft and Schuermann (2008). While origination of non-agency mortgages (subprime, Alt-A and Jumbo) was \$680 billion in 2001, it increased to \$1, 480 billion in 2006, a growth of 118%. In contrast, origination of agency (prime) mortgages decreased by 27%, from \$1443 billion in 2001 to \$1040 billion in 2006. Moreover, while only 35% of non-agency mortgages were securitized in 2001, that figure grew to 77% in 2006.

[6]An important question is whether tranching can help avoid adverse selection. If the balance sheets of financial intermediaries are difficult to monitor, then intermediaries can always go back to the market to sell any remaining fraction of assets, limiting the role for "skin-in-the-game." Second, certification by third parties (e.g., rating agencies) can have limited success if agents learn to game the rating models, or if the incentives of the third party are compromised.

[7]There is an empirical literature that measures the extent of adverse selection in financial markets (see, e.g., Keys et al. (2010), Demiroglu and James (2012), Downing et al. (2009), Krainer and Laderman (2014), and Piskorski et al. (2015)).

how (expected) changes in market conditions *ex-post* affect the incentives to produce asset quality *ex-ante*. This effect is specific to the problem with endogenous asset quality production and moral hazard. Moreover, the incentives effect is dynamic: changes in market conditions and the total liquidity available affect the private decisions to produce asset quality, affecting the equilibrium in the economy in *all* possible states in the future, which feeds back into the optimal policy decisions.

Notably, the different policy instruments interact. On the one hand, public provision of liquidity and private market interventions affect the economy through similar channels. Both can increase the total available liquidity, and they both shape the incentives to produce asset quality. However, the two instruments differ in important ways. Interventions aimed at restoring private market functioning require an increase in the price received by sellers, which tends to increase the incentives to produce low-quality assets. In contrast, direct provisions of liquidity reduce the liquidity premium and therefore reduce the incentives to produce low-quality assets but they tend to be more costly than private market interventions. I find that the optimal policy prescribes an aggressive increase in the supply of public liquidity in times of crisis, i.e., when private markets collapse, while it aims to stabilize market prices, interest rates, and trading volume in normal times (in a *leaning against the wind* type of policy) and provide only a small amount of public liquidity. In fact, reducing the probability of a (private) financial market collapse is not an objective of the planner *per se*, as (private) market fragility can be higher under the optimal policy than in the *laissez-faire* equilibrium. Instead, the planner tries to mitigate variation in the *total liquidity* in the economy (private and public) across states, trading off between the cost of interventions and the incentives the policies generate.

These results change when the planner has imperfect commitment. I assume that the planner can commit to the provision of liquidity *ex-ante*, but the intervention in the private market is chosen *ex-post*. In this case, the optimal private market intervention maximizes only the liquidity effect, which has pervasive effects on the incentives to produce asset quality. However, the planner can manipulate its future-self incentives by distorting its public provision of liquidity *ex-ante*. In particular, I find that the planner *increases* its provision of public liquidity in order to reduce the benefits of a future intervention. By increasing the supply of public liquidity beyond the cost-benefit analysis of the full commitment case, the planner reduces its future-self incentives to intervene through two channels. First, it reduces the marginal benefit of an extra unit of liquidity, reducing the benefits of stimulating the private market. Second, it increases the cost of interventions, as a lower liquidity premium increases the interest rate of the economy, and, therefore, the planner's funding cost. Thus, these results highlight the importance of understanding how the private and public supply of liquidity interact for the optimal policy design.

Finally, I show that the optimal policy can be implemented with three sets of instruments: state-contingent government bonds, asset purchase programs, and transaction taxes. Two features of the implementation are worth noting. First, mapping the provision of public liquidity to government bonds provides a justification for assuming that the planner could commit to the transfers *ex-ante*.[8] Second, while the planner's problem features market wedges that map into transaction subsidies, I present a novel partial equivalence result for asset purchase programs. This is important from a policy point of view, as transaction subsidies can generate spurious trades aimed exclusively at collecting the subsidy, defeating the purpose of the instrument. Asset purchase programs do not suffer from this problem. Moreover, asset purchase programs are part of the toolkit recently used by the Fed to improve the liquidity in private markets after the Great Recession, so their study can be of independent interest.

Literature Review. This paper contributes to several strands of the literature. First, the paper is related to the literature that incorporates adverse selection in financial markets into macroeconomic models. An early paper that studies this problem is Eisfeldt (2004). Kurlat (2013) and Bigio (2015) build a model in which adverse selection in financial markets is used to explain the sudden collapse of the market for mortgage-related securities during the Great Recession. However, these papers take the distribution of asset quality as exogenously given, and abstract from the role of government bonds as a source of liquidity. My paper builds on these insights but, taking a step back, focuses on how the endogenous determination of asset quality distribution interacts with the state of the economy and the policy stance. This extension is key to understanding the build-ups of risks emphasized in these papers, as well as for the design of the optimal policy plan.

There is also a related literature that explores the interaction between the incentives to produce asset quality and ex-post adverse selection in financial markets (see, for example, Parlour and Plantin (2008), Chemla and Hennessy (2014), Vanasco (2017)). I contribute to this literature by studying the interaction between the private and public provision of liquidity in a setting with aggregate risk. In my model, the incentives to produce private assets interact with the supply of public assets through an endogenously determined liquidity premium, and optimal policy leverages this relationship. Moreover, the presence of aggregate risk introduces an endogenously determined probability of a financial crisis, and allows me to explore state-contingent policies that distinguish between "normal" and "crisis" states. In particular, I identify the conditions under which it is optimal to aggressively increase the supply of public liquidity (as with QE 2), when it is optimal to support the private markets (as with QE 1), or whether the private markets should be taxed. Contemporaneous work by Fukui (2018) and Neuhann (2017) also study how moral hazard and adverse selection in private markets can lead to boom-bust dynamics. Fukui (2018) focuses on the reallocation of physical capital while Neuhann (2017) studies changes in the demand for financial assets triggered by variations in the distribution of wealth. In contrast, I study the endogenous determination of the liquidity premium and the interaction between the private and public provision of liquidity. This distinction is crucial for the policy analysis.

[8] Bailouts could replace government bonds as long as they are "systemic," as emphasized by Bianchi (2016). However, bailouts present two drawbacks compared to government bonds. First, they are an *ex-post* policy, so they are subject to a time-inconsistency problem. Second, government bonds imply a cost of participation for the agents (as agents do not internalize that the government rebates the proceeds lump-sum). Thus, government bonds provide a self-selection mechanism that could lead to a lower cost of intervention.

My focus on the public provision of liquidity is shared by a large body of literature that emphasizes the role of government bonds in facilitating the flow of resources among agents in economies with financial frictions. Woodford (1990) shows that when agents face binding borrowing constraints, a higher supply of government bonds can increase welfare. Holmström and Tirole (1998) also highlight the role of tradable instruments when agents cannot fully pledge their future income. Geromichalos et al. (2007) is one of the first papers to study the effect of monetary policy on asset prices in a monetary-search environment. They show that when the supply of private "tradable" assets is not sufficient to satiate the agents' liquidity needs, money can increase welfare. Gorton and Ordoñez (2013) also study the interaction between public and private liquidity, but their focus is on the production of information, whereas my model highlights the liquidity premium and the production of asset quality.[9]

Finally, this paper is related to the literature that studies the scarcity of safe assets more generally. Kiyotaki and Moore (2012) and Del Negro et al. (2017) consider the adverse effects of an exogenous reduction in the collateral value of private assets.[10] Del Negro et al. (2017) argue that the Fed's aggressive response in 2008, which substantially increased the supply of public liquidity, helped avoid a deeper recession. My paper complements their analysis by microfounding the source of the private market deterioration, and by studying the optimal policy mix when both public provision of liquidity and private market interventions are available. Consistent with their findings, I find that, in the event of a crisis, an aggressive policy of providing public liquidity is optimal. However, there are important differences. First, the optimal policy mix also includes interventions in the private markets. Second, while Del Negro et al. (2017) focus on *ex-post* policies, my analysis includes the *ex-ante* incentives that contribute to risk build-ups. In this sense, the trade-offs in the economy are fundamentally different. Relatedly, Tirole (2012) and Philippon and Skreta (2012) study the optimal policy in a setting in which private markets have collapsed due to an adverse selection problem. However, their focus is on ex-post interventions on the private markets, while I study the ex-ante problem. Additionally, I consider the public provision of liquidity as an additional instrument. Angeletos et al. (2016) study the role of liquidity but abstract from asymmetric information and the possibility of financial crises.

Outline The rest of the paper is organized as follows. Section 2 presents the model, and Section 3 studies the equilibrium determination and its positive implications, including the response of the economy to changes in the supply of public liquidity. The normative analysis is developed in Section 4, and Section 5 concludes. All the proofs are presented in the appendix.

[9] A significant number of papers have documented that private production of safe assets increases when the supply of government bonds is low (and vice versa). See, e.g., Gorton et al. (2012), Krishnamurthy and Vissing-Jorgensen (2015), Greenwood et al. (2015) and Sunderam (2015). Krishnamurthy and Vissing-Jorgensen (2012) show that an increase in the supply of government bonds reduces the liquidity premium.

[10] Caballero and Farhi (2018) study the effects of an exogenous reduction in the supply of safe assets in an economy with sticky prices.

2 The Model

The economy lasts for three periods and is populated by a measure one of ex-ante identical agents. Agents choose the quality of the assets they produce, anticipating that in the future they will face a "liquidity shock" that affects their intertemporal preference for consumption and a market for private assets that suffers from adverse selection. Agents can also trade government bonds.

2.1 The Environment

Agents. There are three dates, 0, 1, and 2, and two types of goods: a final consumption good and Lucas trees. The economy is populated by a measure one of agents. Agents receive an endowment of final consumption good of $W_t > 0$ in period $t = 0, 1, 2$. In period 0, they operate a technology that transforms final consumption goods into trees, which pay a dividend in period 2.[11]

Agents' preferences are given by

$$U = E\left[\mu c_1 + c_2\right],$$

where c_t denotes consumption in $t = 1, 2$, μ is a random idiosyncratic "liquidity shock" (uncorrelated across agents), which is their private information, and the expectation is taken with respect to μ and an aggregate state, described below. The liquidity shock affects the agents' marginal utility of consumption in period 1. From period 0 point of view, μ is distributed according to the cumulative distribution function $G(\mu)$ in $[1, \mu^{\max}]$ with associated continuous density $g(\cdot)$.

Technology. Agents have access to a technology to produce trees in period 0. There are two types of trees. An agent can transform 1 unit of the consumption good into 1 unit of low-quality, or "bad," tree (denoted by H_B), and $C(H_G)$ units of the consumption good into 1 unit of high-quality, or "good," tree (denoted by H_G), where $C(0) = 0$, $C'(W_0) > 1$ and $C''(\cdot) > 0$. Let λ_E denote the fraction of good trees in the economy in period 1, that is $\lambda_E \equiv \frac{H_G}{H_G + H_B}$.

Trees deliver fruit in the form of final consumption good in period 2. A unit of good tree pays Z with certainty at maturity. In contrast, only a fraction α of bad trees deliver fruit in period 2, so that the expected payoff of a unit of bad tree is αZ. The fraction of bad trees that deliver fruit is known one period in advance. Thus, in period 1, the fraction α is common knowledge. However, in period 0 agents believe that α is a random variable distributed according to the cumulative distribution function F in the interval $[\underline{\alpha}, \overline{\alpha}] \subseteq [0, 1]$. One can interpret α as an aggregate shock to the productivity of bad trees, so that a higher α implies a higher quality of bad trees, or $1 - \alpha$ as a default rate of bad trees in period 2. I assume that F is continuous and non-degenerate, with associated continuous density $f(\cdot)$. Moreover, only the owner of the tree can determine its quality. This will be important when I describe the financial markets below.

[11] A Lucas tree in this economy is a technology that delivers an exogenous dividend in period 2. The trees stand for a privately produced asset, in contrast to publicly supplied assets, i.e., government bonds.

Financial Markets. Due to the idiosyncratic liquidity risk in period 1, there are gains from trade in this economy. I assume that financial markets are incomplete. In particular, I assume that agents can trade in only two markets: *i*) a market for the trees produced in period 0, and *ii*) a market for government bonds. These markets can be interpreted as a metaphor for collateralized debt markets, like "repos" or short-term commercial paper.[12]

I follow Kurlat (2013) and Bigio (2015) and assume that there is a unique market in which all tree qualities are traded, that buyers cannot distinguish the quality of a specific unit of tree but can predict what fraction of each type there is in the market, and that the market is anonymous, non-exclusive and competitive. These assumptions imply that the market features a pooling price, P_M. Buyers get a diversified pool of trees from the market, where λ_M is the fraction of good trees in the pool.

To make the distinction between good and bad trees stark, I make the following assumption.

Assumption 1. *The expected payoff of the trees satisfies:*

$$\frac{Z}{C'(C^{-1}(W_0))} > E[\alpha Z].$$

Assumption 1 implies that if the quality of trees were observable, the return of bad trees would be lower than the return of good trees for all relevant production scales. Thus, in an economy with perfect information, bad trees would not be produced.

Government. In period 0, the government supplies bonds which mature in period 2. The government's budget constraint in period 0 is

$$T_0 = Q_0^B B_0,$$

where Q_0^B denotes the price of government bonds, B_0 is the bond supply, and T_0 is a lump-sum transfer. The budget constraint in period 2 is

$$T_2 + B_0 = 0,$$

where T_2 is a lump-sum transfer in period 2.

Aggregate State and Timing. The exogenous state of the economy is given by the distribution of liquidity shocks in the population and the realized quality of bad trees, α. The endogenous state is given by the cross-section distribution of assets and shocks across agents. As a consequence of the linearity of the agents' preferences and constraints (see programs (P0), (P1) and (P2) below), prices and aggregate quantities do not depend on the distribution of portfolios in the population.

12 Bigio (2015) presents an equivalence result between a market for trading assets and a repo contract when there is no cost of defaulting besides delivering the collateral to the creditor. This is a standard assumption in papers on collateralized debt. See, for example, Geanakoplos (2010) and Simsek (2013).

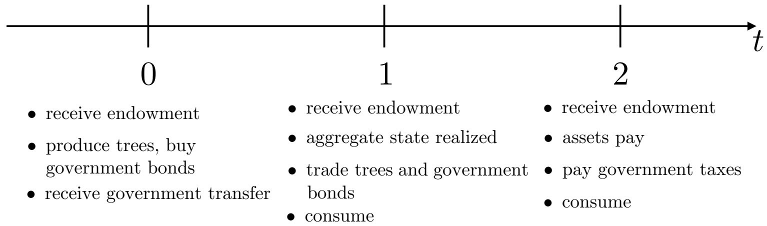

Figure 1: Timing

Therefore, the relevant state in periods 1 and 2 is $X \equiv \{\alpha, \lambda_E, B_0\}$, where the total number of trees in the economy, $H \equiv H_G + H_B$, can be obtained as the unique solution to $H = W - C(\lambda_E H) + \lambda_E H$ for a given λ_E.

To summarize, the timing of the economy is as follows. Agents start period 0 with an endowment of the final consumption good W_0, they receive a lump-sum transfer T_0, and they decide how to allocate their wealth between the production of private assets (good and bad) and government bonds. In period 1, agents receive an endowment of the final consumption good W_1, the aggregate shock α is realized, and agents receive an idiosyncratic liquidity shock, μ. Agents choose between two possible uses of the consumption goods they hold, that is, their *liquid wealth*: to consume or to buy assets in the market (trees and government bonds). Finally, in period 2, agents receive an endowment W_2, all assets pay, and agents consume. Figure 1 summarizes the timing.

2.2 First Best

Let's consider first the (*ex-ante*) Pareto efficient allocation. Suppose that the planner can observe the individual agents' realization of μ, and it chooses each agent's consumption, $c_1(\mu, \alpha)$ and $c_2(\mu, \alpha)$, as a function of the agent's idiosyncratic shock μ and the aggregate state α. Moreover, assume that the planner can determine the production of tree quality in period 0. Therefore, the planner's problem is given by

$$\max_{\{c_1(\mu,\alpha), c_2(\mu,\alpha)\}, H_G, H_B} E\left[\mu c_1(\mu, \alpha) + c_2(\mu, \alpha)\right] \tag{FB}$$

subject to

$$H_B + C(H_G) = W_0$$

$$\int_1^{\mu^{\max}} c_1(\mu, \alpha) dG(\mu) = W_1$$

$$\int_1^{\mu^{\max}} c_2(\mu, \alpha) dG(\mu) = W_2 + ZH_G + \alpha Z H_B$$

The next proposition characterizes the solution.

Proposition 1 (First Best)**.** *In the Pareto optimal allocation, only good trees are produced and only the agents with the highest realization of* μ, $\mu = \mu^{\max}$, *consume in period* 1. *Any allocation of consumption in* *period 2 is consistent with Pareto optimality.*

There are two dimensions to the planner's problem. On the one hand, the planner seeks to achieve *production efficiency*; that is, it makes sure that only good trees are produced in period 0. On the other hand, it also aims for *consumption efficiency*, by allocating the endowment W_1 to the agents that value it the most in period 1.

Program (FB) is very demanding in terms of the information available to the planner. It assumes that the planner can observe μ and make transfers conditional on this information, while also choosing the quality of trees produced by the agents. In Appendix B, I revisit the planner's problem under different assumptions about the information restrictions to better understand the role of the frictions in this economy.

2.3 Agents' Problem

The agents' problem in period 2 is simple: they receive the endowment W_2, collect the dividends from the trees and government bonds they own, pay taxes and consume. Their value function is

$$V_2(h_G, h_B, b; X) = W_2 + Zh_G + \alpha Z h_B + b + T_2(X). \tag{P2}$$

Let's turn to period 1. Denote the purchases of trees in the market by m. If an agent buys m units of trees, a fraction λ_M of them is good, while a fraction $1 - \lambda_M$ is bad.[13] Let s_G and s_B denote the sales of good and bad trees, respectively. The agents' problem in state X is given by:

$$V_1(h_G, h_B, b; \mu, X) = \max_{\substack{c, m, s_G, s_B, \\ h'_G, h'_B, b'}} \mu c + V_2(h'_G, h'_B, b'; X), \tag{P1}$$

subject to

$$c + P_M(X)(m - s_G - s_B) + Q_B(X)(b' - b) \leq W_1, \tag{1}$$

$$h'_G = h_G + \lambda_M(X)m - s_G, \tag{2}$$

$$h'_B = h_B + (1 - \lambda_M(X))m - s_B, \tag{3}$$

$$c \geq 0, \quad m \geq 0, \quad b' \geq 0, \quad s_G \in [0, h_G], \quad s_B \in [0, h_B],$$

where P_M is the price of one unit of a tree and Q_B is the price of government bonds in period 1. Constraint (1) is the agent's budget constraint, which states that consumption plus net purchases of assets (trees and government bonds) cannot be larger than the endowment W_1. Constraints (2) and (3) are the laws of motion of good and bad trees, respectively, which are given by the agents' initial holdings of trees plus a fraction of the purchases they make (where the fraction is given by the market composition of each type of tree) minus the sales they make.

[13] More formally, λ_M should denote the agents' beliefs about the quality of the trees in the market. Since I focus on *Rational Expectations Equilibria*, λ_M will coincide with the actual quality in the market. To save on notation, I have already imposed this equilibrium condition.

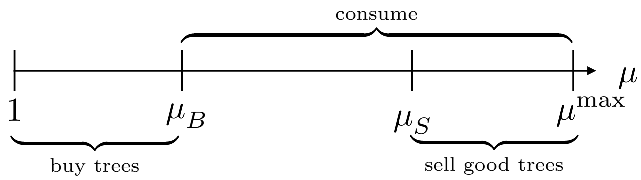

Figure 2: Agents' choices in period 1

The linear structure of the problem implies that we can characterize the agents' decisions in period 1 by two thresholds on μ: μ_B, which determines whether to consume or buy assets, and μ_S, which determines whether to sell good trees. The intuition is simple. The return from buying assets in the market is given by $\frac{\lambda_M Z+(1-\lambda_M)\alpha Z}{P_M}$ for trees and $\frac{1}{Q_B}$ for government bonds, which is the same for all agents. In equilibrium, market clearing requires that $\frac{\lambda_M Z+(1-\lambda_M)\alpha Z}{P_M} = \frac{1}{Q_B} \equiv r_M$. Because the utility from consuming in period 1 and the return from the market are both linear, agents simply compare μ and r_M to decide whether to use their liquid wealth to consume or to buy assets. Thus, the threshold for consumption satisfies $\mu_B = r_M$.

The decision to sell good trees involves similar calculations. In equilibrium, the market price of trees is always below the fundamental value of good trees, $\frac{Z}{r_M}$. Hence, the only reason the agent would sell her good trees is if the utility derived from consuming in period 1 instead of period 2 compensates for the loss. This happens if $\mu > \mu_S$, where $\mu_S \equiv \frac{Z}{P_M} \geq \mu_B$. Note that, in this economy, all agents sell their bad trees. Figure 2 summarizes these choices.

An important result that will significantly simplify the analysis that follows is the linearity of the agents' value function with respect to their holdings of each type of tree and government bonds.

Lemma 1. *The agents' value function in period* 1, $V_1(h_G, h_B, b; \mu, X)$, *is linear:*

$$V_1(h_G, h_b, b; \mu, X) = \widetilde{\gamma}(\mu, X)W_1 + \widetilde{\gamma}_G(\mu, X)h_G + \widetilde{\gamma}_B(\mu, X)h_B + \widetilde{\gamma}_{GB}(\mu, X)b + W_2 + T_2(X),$$

where

$$\widetilde{\gamma}(\mu, X) = \max\{\mu, \mu_B(X)\}, \quad \widetilde{\gamma}_{GB}(\mu, X) = \frac{\widetilde{\gamma}(\mu, X)}{\mu_B(X)},$$
$$\widetilde{\gamma}_G(\mu, X) = \max\{\mu P_M(X), Z\}, \quad \widetilde{\gamma}_B(\mu, X) = \widetilde{\gamma}(\mu, X)P_M(X). \tag{4}$$

Lemma 1 follows directly from the linearity of the objective function and the constraints. The agents' marginal utility of an extra unit of consumption good in period 1 is given by $\widetilde{\gamma}(\mu, X)$, which compares the marginal utility of consumption with the return from the market. Then, the value of a unit of government bond is $\widetilde{\gamma}(\mu, X)Q_B(X) = \frac{\widetilde{\gamma}(\mu,X)}{\mu_B(X)}$. Let's turn to the value of trees. Since bad trees are always sold, holding one unit of bad tree delivers $\widetilde{\gamma}(\mu, X)P_M(X)$. In contrast, good trees are sold only if the utility from selling in period 1, $\mu P_M(X)$, is higher than the utility from keeping it until maturity, Z. Note that the value of bad trees does not directly depend on its payoff in period 2, since no agent who *starts* the period owning bad trees holds them until maturity.

Finally, the problem of an agent in period 0 is given by

$$V_0 = \max_{h_G, h_B, b} E[V_1(h_G, h_B, b; \mu, X)], \tag{P0}$$

subject to

$$\begin{aligned} h_B + C(h_G) + Q_0^B b &\leq W_0 + T_0, \qquad (5)\\ h_G \geq 0, \quad h_B \geq 0, &\quad b \geq 0, \end{aligned}$$

where Q_0^B is the price of government bonds in period 0, T_0 is a lump-sum transfer, and E denotes the expectation operator with respect to μ and α. Constraint (5) is the agents' budget constraint, which states that expenditures in the production of trees and purchases of government bonds cannot be larger than the endowment plus transfers, $W_0 + T_0$.

Before solving the agents' problem in period 0, it is useful to define the key objects in the analysis that follows: the shadow value of trees.

Definition 1 (Shadow Value of Trees)**.** *The shadow value of good and bad trees are given by*

$$\begin{aligned} \gamma_G &\equiv E\left[\widetilde{\gamma}_G(\mu, X)\right] = E\left[\max\left\{\mu P_M(X), Z\right\}\right], \\ \gamma_B &\equiv E\left[\widetilde{\gamma}_B(\mu, X)\right] = E\left[\max\{\mu, \mu_B(X)\} P_M(X)\right]. \end{aligned}$$

The shadow value of trees is the expected value of the marginal utility of the trees in period 1, given by (4). To understand the intuition behind these expressions, I decompose them into three elements: a fundamental value, a liquidity premium, and an adverse selection tax/subsidy:

$$\gamma_G = E\left[\underbrace{Z}_{\text{fund. value}} + \underbrace{\left(\frac{\widetilde{\gamma}(\mu, X)}{r_M(X)} - 1\right) Z}_{\text{liq. premium}} - \min\left\{\widetilde{\gamma}(\mu, X) \underbrace{\left(\frac{Z}{r_M(X)} - P_M(X)\right)}_{\text{adv. sel. tax}}, \left(\frac{\widetilde{\gamma}(\mu, X)}{r_M(X)} - 1\right) Z\right\}\right], \tag{6}$$

$$\gamma_B = E\left[\underbrace{\alpha Z}_{\text{fund. value}} + \underbrace{\left(\frac{\widetilde{\gamma}(\mu, X)}{r_M(X)} - 1\right) \alpha Z}_{\text{liq. premium}} + \widetilde{\gamma}(\mu, X) \underbrace{\left(P_M(X) - \frac{\alpha Z}{r_M(X)}\right)}_{\text{adv. sel. subs.}}\right]. \tag{7}$$

First, the fundamental value is given by the dividend each type of tree pays in period 2, which is Z for good trees and αZ for bad trees.[14] Second, trees in this economy derive value from the fact that they can be traded in period 1, transforming a dividend in period 2 into resources in period 1, when they are potentially more valuable to the owner. The liquidity premium is a consequence of the *liquidity services* tradeable assets provide in economies with incomplete markets, as emphasized by Holmström and Tirole (2001). Note that in the first best, $\mu_B(X) = \mu^{\max}$ and, therefore, the liquidity premium would be equal to zero. A crucial feature of the analysis that fol-

[14] Recall that the marginal utility of consumption in period 2 is equal to 1 for all agents and there is no discounting.

lows, particularly the normative implications of Section 4, rely on the endogeneity of the liquidity premium.[15]

Finally, the asymmetric information problem in the market for trees introduces a wedge that is negative for good trees and positive for bad trees. Since the market price of trees is always between the fundamental value of good and bad trees, that is, $P_M(X) \in \left[\frac{\alpha Z}{r_M(X)}, \frac{Z}{r_M(X)}\right]$, the good trees feature an *adverse selection tax*. However, this tax is charged only if the tree is sold. Thus, the owners of good trees have a choice: sell the tree and pay the tax, generating a utility loss of $\widetilde{\gamma}(\mu, X)\left(\frac{Z}{r_M(X)} - P_M(X)\right)$, or keep the tree and give up the liquidity services associated with it, generating a utility loss of $\left(\frac{\widetilde{\gamma}(\mu,X)}{r_M(X)} - 1\right) Z$. The agents optimally choose the option that generates the smallest loss. In contrast, the pooling price implies an implicit *subsidy* for bad trees. It is the size of this cross-subsidization between good and bad trees, and the *option value* it generates on good trees, that shapes the incentives to produce different qualities.

A consequence of these expressions is that the shadow values have heterogeneous elasticities to market prices. Let $\gamma_i(P_M)$ be the shadow value of type $i \in \{G, B\}$ as a function of future prices $\{P_M(X)\}$, and let $\frac{\partial \gamma_i(P_M)}{\partial P_M(X)}$ be the associated derivative with respect to the market price in state X.[16] The next proposition presents a key result of the model.

Proposition 2 (Sensitivity of Shadow Values to Prices)**.** *The shadow value of trees satisfy*

$$\frac{\partial \gamma_B(P_M)}{\partial P_M(X)} > \frac{\partial \gamma_G(P_M)}{\partial P_M(X)} \geq 0.$$

Proposition 2 states that the shadow value of bad trees is more sensitive to changes in expected market prices than the shadow value of good trees. Or, put differently, that the private valuation of good trees is more insulated from changes in market conditions than that of bad trees. Good trees have the option value of being kept until maturity if market prices are not sufficiently high, or if liquidity needs are low, while this strategy is always dominated for bad trees. Bad trees are produced only to be sold in the future, that is, for *speculative motives*. Thus, while bad trees are always sold, there are states in which agents strictly prefer not to sell their good trees, insulating their value from price changes. This channel is at the core of the positive and normative analysis that follows.

Finally, I am ready to characterize the agents' choice in period 0. Due to the linearity of the value function in period 1, and using the definition of the shadow value of trees, there is a simple characterization of the agents' optimality conditions.

[15]The liquidity premium can be defined for period 1 as $\mu^{\max} - \mu_B(X)$, which reflects the difference between the consumption cutoff (and hence the rate of return) in the first best and in the laissez-faire economy.

[16]Since prices are a function of the state, the shadow values are *functionals*, so the appropriate concept to measure their change when prices change is the functional derivative. When the space of functions is a Banach space, the corresponding definition is the 'Fréchet" derivative. For an introduction to functional analysis, see Luenberger (1969).

Lemma 2. *Suppose* $\frac{\gamma_G}{C'(W_0)} < \gamma_B < \frac{\gamma_G}{C'(0)}$. *The agents' production decisions in period* 0 *satisfy*

$$\frac{\gamma_G}{\gamma_B} = C'(H_G) \quad \textit{and} \quad H_B = W - C(H_G). \tag{8}$$

Moreover,

$$\frac{\partial \lambda_E}{\partial P_M(X)} < 0 \quad \textit{and} \quad \frac{\partial H}{\partial P_M(X)} > 0.$$

Given the shadow value of trees, γ_G and γ_B, agents decide which quality of tree to produce by comparing the return per unit invested of each option (good or bad) at the margin. Importantly, the fraction of bad trees in the economy, λ_E, is decreasing in market prices, and the total number of trees, H, is increasing in market prices. The effect on λ_E is a corollary of Proposition 2: since the shadow value of bad trees is more sensitive to changes in prices than the shadow value of good trees, the average quality of trees in the economy decreases with market prices. Thus, the production of lemons is more elastic to *future* prices than the production of non-lemons. Moreover, as prices increase and more bad trees are produced, the total number of trees in the economy increases, since bad trees are cheaper to produce than good trees.

Next, I turn to the determination of equilibrium and present the positive analysis of the model.

3 Equilibrium and Market Fragility

In this section, I compute the equilibrium of the economy. First, I characterize the equilibrium in the market for trees for each realization of α, *conditional* on $\{\lambda_E, B_0\}$. Then I use the characterization of the agents' decisions in period 0 given their expectations about their liquidity needs and the market for trees in period 1, to define a *Rational Expectations Equilibrium*. Finally, I perform a comparative statics analysis to understand the sources of risk build-up in this economy.

3.1 Market for Trees

In period 1, agents have access to two markets: a market for trees and a market for government bonds. A no-arbitrage condition between these markets is given by

$$r_M(X) = \frac{\lambda_M(X)Z + (1 - \lambda_M(X))\alpha Z}{P_M(X)} = \frac{1}{Q_B(X)}.$$

That is, agents are indifferent about which asset they buy as long as they have the same (expected) return. We can characterize the equilibrium in the market for trees by the *net demand for trees* and a *supply of trees*. The net demand for trees is given by the demand of those agents who have a liquidity shock that is less than $\mu_B(X)$, net of the purchases of government bonds:

$$D(P_M; X) \equiv \frac{G(\mu_B(P_M; X))[W_1 + (1 - \lambda_E)HP_M] - [1 - G(\mu_B(P_M; X))]B_0 Q_B(P_M; X)}{P_M}, \tag{9}$$

where $G(\cdot)$ is the cumulative distribution function of μ. Note that buyers sell their bad trees in order to profit from the adverse selection subsidy. Moreover, only agents with $\mu \geq \mu_B(X)$ sell their government bonds, so market clearing implies that only a fraction $1 - G(\mu_B(P_M; X))$ of the total outstanding value of government bonds is traded in the market.

The supply of trees is given by the sum of the good and bad trees in the market, that is,

$$S(P_M; X) \equiv [1 - G(\mu_S(P_M; X))] \lambda_E H + (1 - \lambda_E) H. \tag{10}$$

While all agents sell their bad trees, only the fraction of agents with liquidity needs above $\mu_S(P_M; X)$ sell their good trees. Finally, we have that the fraction of good trees in the market is given by

$$\lambda_M(P_M; X) = \frac{[1 - G(\mu_S(P_M; X))] \lambda_E H}{S(P_M; X)}. \tag{11}$$

In order to organize the analysis of the equilibrium of the economy, it is useful to define a partial equilibrium of the market for trees in each state X.

Definition 2 (Partial Equilibrium in the Market for Trees)**.** *A partial equilibrium in the market for trees in state X is a price P_M, a fraction of good trees in the market λ_M, and a rate of return r_M, such that the demand for trees* (9) *equals the supply of trees* (10)*, the average quality of trees in the market is given by* (11)*, and*

$$\mu_B(X) = r_M(X).$$

The following assumption guarantees that a *financial collapse*, that is, the discontinuous decline in volume traded, can occur in equilibrium.

Assumption 2. *The cumulative distribution function G is weakly convex and weakly log-concave.*

Assumption 2 guarantees that at prices in the neighborhood of $P_M(X) = \frac{Z}{\mu^{\max}}$, the supply of trees reacts more strongly to price changes than the demand. As I show below, this implies a set of partial equilibria that is discontinuous in α.[17]

The next proposition characterizes the maximum volume of trade equilibrium in this economy.

Proposition 3. *Given a state X, a partial equilibrium always exists. In the unique maximum volume of trade equilibrium, P_M, λ_M and r_M are increasing in α and λ_E. Moreover, there exists $\alpha^*(\lambda_E, B_0) \in [\underline{\alpha}, \overline{\alpha}]$ such that if $\alpha > \alpha^*(\lambda_E, B_0)$, $\lambda_M > 0$ and if $\alpha < \alpha^*(\lambda_E, B_0)$, $\lambda_M = 0$. If Assumption 2 holds, the maximum volume of trade equilibrium is discontinuous at α^*, that is,*

$$\lim_{\alpha \to \alpha^{*+}} P_M(X) > \lim_{\alpha \to \alpha^{*-}} P_M(X).$$

[17] Assumption 2 is more demanding than what is necessary for the results that follow, but it greatly simplifies the analysis.

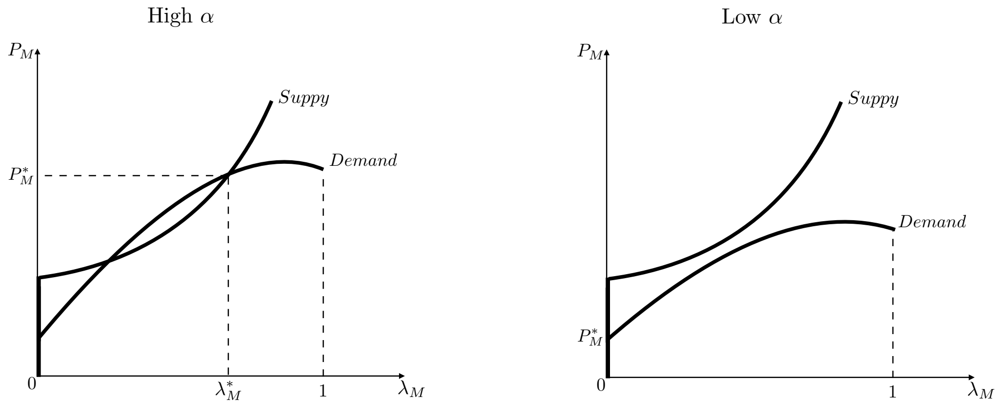

Figure 3: Market Equilibrium in period 1. (a) Multiple Equilibria: Maximal Volume of Trade Selected. (b) Unique Equilibrium: Financial Collapse.

Proposition 3 fully characterizes the market for assets in period 1. First, it shows that, in the maximum volume of trade equilibrium, market prices, the quality of trees in the market, and returns are all increasing in α. When α is high, the adverse selection problem is mild and the markets can perform their role of reallocating resources to the best use relatively well. When α is low, the adverse selection problem is severe, the price of trees is low and the price of government bonds is high (which is the inverse of $r_M(\alpha)$). The economy suffers from a high degree of resource misallocation, and bond prices increase, reflecting this problem.

Second, Proposition 3 states the conditions for a discontinuous change in market performance, or a financial collapse. If at every price greater than $\frac{Z}{\mu^{\max}}$ the demand for tree quality is lower than the supply, then the unique partial equilibrium has $\lambda_M = 0$. This characteristic of the market is emphasized in Kurlat (2013) as a description of the events leading to the collapse in the volume traded of mortgage-related assets at the onset of the Great Recession. However, the equilibrium of the economy can be smooth, that is, $\lim_{\alpha \to \alpha^{*+}} \lambda_M(\alpha) = 0$. In that case, the "collapse" of the market is not a particularly important event; the functioning of the financial markets is already severely impaired for values of α sufficiently close to α^*. However, under Assumption 2, the market equilibrium changes discretely at α^*. Figure 3 depicts the two scenarios in the space (P_M, λ_M).[18] Panel (a) shows a market in which the quality of bad trees is high and there are multiple partial equilibria. Following the literature, I select the equilibrium with the highest price (see Kurlat (2013), Chari et al. (2014), Bigio (2015)). As the quality of bad trees decreases, the demand moves down. When α is sufficiently low, the economy transitions to the market depicted in Panel (b). In this case, the interior intersection disappears, generating a discontinuous drop in the volume traded.

[18]The partial equilibrium is characterized by three equation: (9), (10) and (11). In order to obtain a two-dimensional representation of the market, I plot a supply and demand for *tree quality* as follows: $Supply: \ \lambda_M = \frac{1-G\left(\frac{Z}{P_M}\right)\lambda_E}{1-G\left(\frac{Z}{P_M}\right)\lambda_E+(1-\lambda_E)}$ and $Demand: \ P_M = \frac{\lambda_M Z+(1-\lambda_M)\alpha Z}{\mu_B(P_M)}$ where $\mu_B(P_M)$ is implicitly defined by the solution to $G(\mu_B)W_1 = \frac{1-\lambda_E}{1-\lambda_M} HP_M - (1-\lambda_E)HG(\mu_B)P_M + [1-G(\mu_B)]\frac{B_0}{\mu_B}$, given λ_M.

The previous discussion leads to the following definition of *market fragility*.

Definition 3. *Market fragility is defined as*

$$MF(\lambda_E, B_0) \equiv Prob(\alpha < \alpha^*(\lambda_E, B_0)).$$

Market fragility is the probability of a discontinuous drop in the volume traded in the market for trees. Even though market fragility is not a direct measure of welfare, it is a property that is tightly connected to the efficiency of the economy. The collapse of the market is the extreme case in which the flow of resources is severely impaired.

3.2 Equilibrium

A *Rational Expectations Equilibrium* in this economy consists of a maximum volume of trade partial equilibrium for every state α, agents' decision rules for the production of trees and consumption, and aggregate variables $\{\lambda_E, H\}$, such that the decision rules solve the agents' problem given the maximum volume of trade partial equilibria, and $\{\lambda_E, H\}$ are consistent with individual decisions.

In order to complete the characterization of the equilibrium, I have only to determine the fraction of good trees in period 1, λ_E, which is given by

$$\lambda_E = \frac{H_G}{H_G + W - C(H_G)}.$$

Note that the decision to produce trees in period 0 depends on the market prices in period 1. But the prices in period 1 depend on the fraction of good trees in the economy, which is in turn determined in period 0. It is useful to define the following function:

$$T(\lambda_E) = \frac{H_G(\lambda_E)}{H_G(\lambda_E) + W - C(H_G(\lambda_E))}.$$

An equilibrium of this economy requires that $T(\lambda_E) = \lambda_E$.[19] The function T is decreasing in λ_E, since higher λ_E implies higher expected prices, and the result follows from Lemma 2. When the distribution of α is continuous, γ_G and γ_B are continuous functions of λ_E, and hence T is continuous. Therefore, the equilibrium of the economy exists and is unique. The following lemma summarizes these results.

Lemma 3. *An equilibrium of the economy always exists and is unique. Moreover,* $\lambda_E \in (0, 1)$.

Figure 4 depicts a numerical example. The equilibrium features $\lambda_E = 0.84$ and $\alpha^* = 0.11$, that is, 84% of the trees in the economy are good and the probability of a crisis is 11%. The figure shows

[19]Note that T is a function of the agents' *expectation* of λ_E. A *Rational Expectations Equilibrium* requires that expected and actual λ_E coincide.

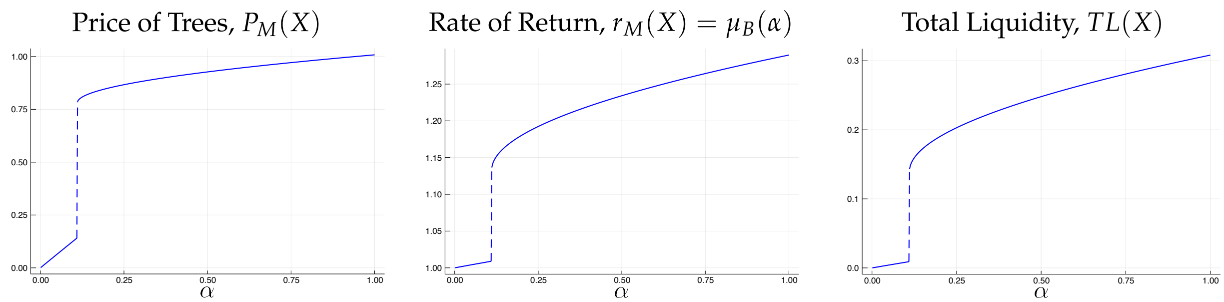

Figure 4: Equilibrium in period 1 as a function of the state α

Note: I assume $\alpha \sim U[0,1]$ and $\mu \sim U[1, \mu^{\max}]$ with $\mu^{\max} = 2$. Moreover, $C(H_G) = ae^{\eta H_G}$, with $a = 0.2$ and $\eta = 5$. The other parameters of the model are: $W_0 = 0.25$, $W_1 = W_2 = 1$, and $Z = 1.3$.

the price of trees, rate of return, and the total liquidity, which is defined as

$$TL(X) \equiv \underbrace{Q_B(X)B_0}_{\text{public liquidity}} + \underbrace{[[1 - G(\mu_S(X))]\lambda_E H + (1-\lambda_E)H]\, P_M(X)}_{\text{private liquidity}}. \tag{12}$$

That is, total liquidity is the market value of all the assets traded. Note that total liquidity depends on the supply of government bonds and on the performance of private markets. We can see that the price of trees, rate of return and total liquidity are all monotonically increasing in α. At $\alpha = \alpha^*$, the equilibrium is discontinuous. To the left of α^* the price of trees and total liquidity is low, which drives the rate of return of the economy down due to the increase in the liquidity premium. In particular, total liquidity to the left of α^* is 93.5% lower than to the right. Thus, a small difference in fundamentals can translate into a large change in outcomes.

Discussion of the model's ingredients. The model has three main ingredients: *i*) an asymmetric information problem with respect to the quality of private assets; *ii*) aggregate risk; *iii*) an endogenously determined liquidity premium. The combination of asymmetric information and aggregate risk introduces an endogenous probability of an abrupt collapse of the financial markets. Moreover, it opens the possibility of studying state-contingent policies. As I show in Section 4, the planner's trade-offs are very different if α is low than if α is high. Moreover, the endogenous liquidity premium provides a connection between the supply of public liquidity and the incentives to produce private assets, which can be exploited in the optimal policy design. From a technical point of view, the assumption that α has a continuous density is crucial for the existence of an equilibrium. If this were not true, the function T would be discontinuous. In that case, it can be shown that a *sunspot* equilibrium always exists.[20]

[20] See Caramp (2017) for the details in a similar setting.

3.3 Comparative Statics

Next, I study some comparative statics that highlight the sources of risk build-ups in this economy. Proposition 4 shows that positive shocks to fundamentals can distort the quality production decisions and increase market fragility. Proposition 5 studies the effects of changes in the supply of government bonds, which is one of the building blocks of the normative analysis in Section 4.

The Quality of Bad Trees

Consider the effects of an anticipated (from period 0's point of view) increase in the expected quality of bad trees (or an expected reduction of *default rates*). In particular, suppose that the distribution of α is indexed by a parameter $\theta : F(\alpha|\theta)$, where a higher θ means a better distribution in the First-Order Stochastic Dominance (FOSD) sense. An increase in θ is equivalent to an increase in prices for all states under the initial distribution. From Lemma 2, we know that the partial equilibrium effect is a reduction in the fraction of good trees in the economy, λ_E, and an increase in the total number of trees, H. While the reduction in λ_E feeds back into the prices, dampening the partial equilibrium result, the overall effect is a decline in the asset quality composition.[21]

Let's turn to the analysis of market fragility. Recall that market fragility is the probability that the quality of bad trees, α, is below the threshold α^*, that is, $MF(\lambda_E) = F(\alpha^*(\lambda_E)|\theta)$. Differentiating this expression with respect to θ, we get

$$\frac{d\text{MF}}{d\theta} = \underbrace{\frac{\partial F(\alpha^*|\theta)}{\partial \theta}}_{\leq 0} + f(\alpha^*;\theta) \underbrace{\frac{\partial \alpha^*(\lambda_E)}{\partial \lambda_E}}_{<0} \underbrace{\frac{\partial \lambda_E}{\partial \theta}}_{<0}.$$

For example, suppose that the change in F is concentrated at very high values of α, so that $\frac{\partial F(\alpha^*|\theta)}{\partial \theta} = 0$. Then, the effect of the endogenous adjustment mechanism of the economy dominates, and market fragility *increases*. In contrast, when the fraction of good trees in the economy is exogenously given, as in Eisfeldt (2004) and Kurlat (2013), $\frac{\partial \lambda_E}{\partial \theta} = 0$, and market fragility (weakly) *decreases* after the shock. The next proposition summarizes these results.

Proposition 4 (Increase in Bad Trees' Expected Quality)**.** *Consider an anticipated increase in θ, so that $F(\alpha|\theta)$ increases in the FOSD sense. Then,*

i. the total production of trees, H, increases, and the fraction of good trees in the economy, λ_E, decreases;

ii. market prices in period 1, P_M, decrease in every state;

iii. the threshold α^* increases;

iv. the effect on market fragility is ambiguous.

[21] There is also an effect on the rate of return which reinforces the price effect. See the proof for the details.

This is an important result because it states that a "positive" shock can endogenously increase the fragility of in the financial markets, in the sense of a higher probability of a market collapse. Thus, it formalizes the idea that positive shocks can set the stage for a financial crisis. Moreover, note that if the change in expectations does not reflect a change in the actual distributions (in the sense that it is just unfounded *optimism*), then fragility always increases after the shock.

Government Bonds

The previous analysis showed that it is the dual role that trees play that exposes the economy to financial risk. On the one hand, trees are a form of real investment, that is, a technology that transforms goods in one period into goods in other periods. On the other hand, trees facilitate trade in period 1, so that, in the context of incomplete markets, agents can obtain resources even when the trees do not pay any dividend. In reality, the government is an important provider of instruments that perform the second role, particularly through government bonds. Here, I study the channels through which the supply of government bonds can shape the incentives to produce tree quality and affect financial fragility from a positive perspective. In Section 4, I analyze the role of public liquidity from a normative perspective.

Consider the economy in period 1. Suppose that the supply of government bonds in the hands of agents increases exogenously, keeping $\{\lambda_E, H\}$ fixed. The idea is to isolate the market effect in period 1 from the incentives effect in period 0. Recall that the total liquidity in the economy depends on the supply of government bonds and on the performance of private markets. The next lemma shows that the price of trees always decreases with B_0.

Lemma 4. *Consider an economy in period* 1*. Assume that agents' holdings of government bonds increase uniformly, from* B_0 *to* $B_0 + dB_0$*, with* $dB_0 > 0$ *but small, keeping* $\{\lambda_E, H\}$ *fixed. The price of trees and private liquidity decrease in all* α*. There exists* $\tilde{\alpha} > \alpha^*$ *such that for all* $\alpha \in (\alpha^*, \tilde{\alpha})$*, total liquidity decreases. Market fragility increases.*

Keeping $\{\lambda_E, H\}$ fixed, an increase in B_0 reduces the net demand for trees, since government bonds and trees "compete" for the same funds. Thus, an increase in the supply of government bonds *exacerbates* the adverse selection in private markets, and the price of trees decreases. For states close to α^* this effect is sufficiently strong that it generates a private market collapse. Even though the available *public liquidity* increases, if dB_0 is small, the discrete drop in *private liquidity* reduces the total liquidity in the economy, which increases misallocation. Absent any change in the tree quality composition, government bonds increase the fragility in the private markets.

Anticipating the effects of a higher supply of government bonds on the market for trees, agents in period 0 react to higher sales of government bonds by adjusting their quality production.[22] Since the shadow value of bad trees is more sensitive to changes in market conditions than the

[22]Since all the proceeds from selling bonds in period 0 are rebated to the agents lump-sum, changes in the supply of government bonds affect the production of tree quality only through their effect on the market for trees in period 1.

shadow value of good trees, the quality of trees in the economy unambiguously increases. The next proposition summarizes these results.

Proposition 5 (The Supply of Government Bonds)**.** *Consider an increase in the supply of government bonds in period* 0*. The total production of trees decreases and the fraction of good trees in the economy increases. The effect on market fragility is ambiguous.*

Proposition 5 formalizes the idea that the scarcity of public safe (or liquid) assets increases the production of private substitutes. But in this model, that production is biased towards low-quality assets. In terms of market fragility, there are two competing forces at play. First, a lower supply of government bonds reduces the rate of return on assets, which pushes asset prices up. Second, it induces the production of low-quality assets, which depresses the price of private assets. If the endogenous production is sufficiently responsive to changes in market conditions, market fragility in the economy increases. However, note that the severity of a financial collapse always decreases with the supply of government bonds, as it provides a lower bound on the total liquidity available. Moreover, government bonds have a negative *beta*, as their price increases when the private market is in distress, providing additional liquidity in crises states.

This results may provide a narrative for some of the developments in the U.S. economy in the years leading to the Great Recession, in which the scarcity of safe assets due to sustained fiscal surpluses in the late 1990s, and the so-called global savings glut in the early 2000s, could have sowed the seeds of the financial crisis, as it put excessive pressure (i.e., generated perverse incentives) on the U.S. financial sector to produce safe assets.[23] This is a period in which the supply of asset quality was probably relatively elastic, as the supply of mortgage-related securities was increasing rapidly. Later on, the public provision of liquidity could have hindered the possibility of restoring the functioning of the private markets, as the production of new assets was low and, therefore, the asset quality composition in the economy was mostly fixed, so the effects of Lemma 4 may have dominated. Of course, this does not imply that the policy was suboptimal. As we shall see, the optimal policy requires an aggressive increase in the supply of public liquidity when restarting the private markets is too costly, but to limit the public provision of liquidity when trying to "jumpstart" the private markets.

4 Welfare and Optimal Policy

In the previous sections, I studied the dynamics of an economy in which market incompleteness and information frictions can lead to a financial crisis, and I analyzed the positive effects of policy changes, namely, the supply of public liquidity, on equilibrium outcomes. In this section, I analyze the model's normative implications by solving the problem of a social planner whose objective is to maximize the expected utility of the representative agent in period 0. The planner faces the

[23]See, for instance, Caballero (2006) for a narrative about safe asset shortages.

same constraints as the private economy; in particular: (i) agents' portfolios, idiosyncratic shock μ, and consumption are the agents' private information; (ii) the market for trees is anonymous; (iii) agents' unbacked promises are not enforceable.

I assume that the planner has access to two sets of tools. First, it can use lump-sum transfers to reallocate resources across agents, but it cannot force the agents to consume the transfers they receive because consumption is not observable. Moreover, I assume that these transfers can entail a deadweight loss. Second, the planner can create a wedge in the market for trees by setting a price paid by buyers that differs from the price received by sellers. Importantly, the planner cannot condition the transfers to the agents' participation in the market for trees, since the market is anonymous.[24]

The rest of this section sets up the planner's problem and solves for the optimal policy under different degrees of the planner's ability to commit to a policy in period 0. Since agents' type is their private information, I focus on direct mechanisms in which transfers are conditioned on the agents' announcement of their type.

4.1 Constrained Efficiency: The Planner's Problem

Let a plan $\mathcal{P} = \left(\{P_B(\alpha), P_S(\alpha)\}_{\forall\alpha}, \{T_1(\mu,\alpha), T_2(\mu,\alpha)\}_{\forall\mu,\alpha}\right)$ be a price paid by buyers of trees, $P_B(\alpha)$, a price received by sellers of trees, $P_S(\alpha)$, and transfers to the agents in periods 1 and 2 according to their announcement μ, $\{T_1(\mu,\alpha), T_2(\mu,\alpha)\}$, given the aggregate state α. For a given choice of plan $\mathcal{P}$, the characterization of the equilibrium of the economy is analogous to the one in *laissez-faire* studied in the previous section. Next, I briefly describe the set of constraints that the economy imposes on the planner's problem.

Consider first the equilibrium in the market for trees. As described in Section 2, agents' decisions are characterized by thresholds μ_B and μ_S, which partition the interval $[1, \mu^{\max}]$ into consumers, buyers and sellers. Given a plan $\mathcal{P}$, an active market equilibrium must satisfy

$$\mu_B(\alpha) = \frac{\lambda_M(\alpha)Z + (1-\lambda_M(\alpha))\alpha Z}{P_B(\alpha)}, \quad \mu_S(\alpha) = \min\left\{\frac{Z}{P_S(\alpha)}, \mu^{\max}\right\} \tag{13}$$

$$\lambda_M(\alpha) = \frac{[1-G(\mu_S(\alpha))]H_G}{S(\alpha)} \tag{14}$$

and

$$D(\alpha) = S(\alpha) \tag{15}$$

where

$$D(\alpha) = \begin{cases} \frac{\int_1^{\mu_B(\alpha)}[W_1 + P_S(\alpha)H_B + T_1(\mu,\alpha)]dG(\mu)}{P_B(\alpha)} & \text{if } P_S(\alpha) \geq \frac{\alpha Z}{\mu_B(\alpha)} \\ \frac{\int_1^{\mu_B(\alpha)}[W_1 + T_1(\mu,\alpha)]dG(\mu)}{P_B(\alpha)} & \text{if } P_S(\alpha) < \frac{\alpha Z}{\mu_B(\alpha)} \end{cases} \tag{16}$$

[24] This approach is similar to the one in Lorenzoni (2008), which solves a problem in which the planner takes the restrictions imposed by competitive markets as given.

$$S(\alpha) = \begin{cases} [1 - G(\mu_S(\alpha))]H_G + H_B & \text{if } P_S(\alpha) \geq \frac{\alpha Z}{\mu_B(\alpha)} \\ [1 - G(\mu_S(\alpha))]H_G + \left[1 - G\left(\frac{\alpha Z}{P_S(\alpha)}\right)\right] H_B & \text{if } P_S(\alpha) < \frac{\alpha Z}{\mu_B(\alpha)} \end{cases}. \tag{17}$$

These equations are analogous to those that characterize the equilibrium in Section 3, with three differences. First, government bonds are replaced by transfers in the market clearing condition. As I show in the implementation of the optimal policy of Section 4.4, transfers and bonds are tightly connected. Second, the formulas distinguish between the price paid by buyers and the price received by sellers. This means that there is a mismatch of resources flowing from buyers to sellers of

$$[P_S(\alpha) - P_B(\alpha)]S(\alpha).$$

This amount will enter into the planner's resource constraint, described below. For future reference, let

$$\omega(\alpha) \equiv \frac{P_S(\alpha)}{P_B(\alpha)} - 1$$

denote the *market wedge* induced by the planner's plan. A positive wedge implies a price received by sellers that is higher than the price paid by buyers, and vice versa.[25] Third, if $P_S(\alpha)$ is sufficiently low (such that $\omega(\alpha)$ is sufficiently negative), the adverse selection subsidy on bad trees becomes negative, so agents sell their bad trees only when their liquidity needs are sufficiently high (similar to the decision to sell good trees). Equations (16)-(17) reflect this possibility. Finally, note that the planner can choose to completely shut down the market by setting $P_S(\alpha) \leq \frac{\alpha Z}{\mu^{\max}}$ and $P_B(\alpha) \geq \alpha Z$, which has a zero resource cost.[26] Thus, the market is not a constraint in the planner's problem but rather an additional tool the planner can use to help reallocate resources across agents.

Consider next the production of tree quality. In an interior solution, production decisions are determined by

$$\frac{\gamma_G}{\gamma_B} = \frac{E\left[\max\{\mu P_S(\alpha), Z\}\right]}{E\left[\max\{\mu, \widetilde{\mu}_B(\alpha)\} P_S(\alpha)\right]} = C'(H_G) \tag{18}$$

$$H_B = W_1 - C(H_G), \tag{19}$$

where $\widetilde{\mu}_B \equiv \max\left\{\mu_B(\alpha), \frac{\alpha Z}{P_S(\alpha)}\right\}$. These are the analogous to the expression in (8) in Section 2, where P_M is replaced by the price received by sellers, P_S, and $\widetilde{\mu}_B(\alpha)$ replaces $\mu_B(\alpha)$. While the planner cannot observe the production of tree quality, it understands that its choices affect the shadow value of trees and therefore determines the incentives to produce tree quality. For example, if the planner chooses to shut down the market, we get that $\gamma_G = Z$ and $\gamma_B = E[\alpha]Z$, and, by Assumption 1, the equilibrium features $H_B = 0$. Moreover, as I show below, the transfers af-

[25] Interpreting a positive wedge as a transaction subsidy can be problematic, as transaction subsidies can induce spurious trades exclusively aimed at collecting the subsidy. I postpone the discussion about implementation until Section 4.4, and assume here that each tree can be traded only once per period.

[26] In that case, $S(\alpha) = 0$, so $\lambda_M(\alpha)$ is not well-defined.

fect $\mu_B(\alpha)$, which represents an additional way the planner can influence the agents' production decisions.

The feasibility of plan $\mathcal{P}$ requires that the transfers and market wedges satisfy a planner's resource constraint in each period. In period 1, the resource constraint is given by

$$\int_1^{\mu^{\max}} T_1(\mu,\alpha)dG(\mu) + [P_S(\alpha) - P_B(\alpha)]\, S(\alpha) = 0, \tag{20}$$

where I restrict $T_1(\mu,\alpha) \geq -W_1$ to prevent negative consumption. For period 2, I make two additional assumptions. First, since unbacked promises are not enforceable, I assume that transfers in period 2 take the form

$$T_2(\mu,\alpha) = \mathcal{T}_2(\mu,\alpha) - \mathbb{T}_2(\alpha), \quad \text{with} \quad \mathcal{T}_2(\mu,\alpha) \geq 0.$$

Transfers in period 2 have two components, one that depends on the agents' announcements and one that is common to all agents, and the agent-specific term has to be weakly positive. This assumption precludes the possibility of transfers that resemble loans to the agents, in which the planner offers a higher transfer in period 1 coupled with an agent-specific negative transfer in period 2. Second, I assume that $\mathbb{T}_2(\alpha)$ entails a deadweight loss. Following Tirole (2012), I assume a constant loss per unit of transfer, such that the planner's resource constraint in period 2 is given by

$$\mathbb{T}_2(\alpha) = (1+\chi)\int_1^{\mu^{\max}} \mathcal{T}_2(\mu,\alpha)dG(\mu), \tag{21}$$

for some $\chi \geq 0$. Moreover, I impose that $\mathbb{T}_2(\alpha) \geq -W_2$.

Finally, let $\{c_1(\mu,\hat{\mu},\alpha), c_2(\mu,\hat{\mu},\alpha)\}$ denote the consumption bundle of an agent of type μ who announces to be of type $\hat{\mu}$, in state α, given a plan $\mathcal{P}$. Recall that the planner cannot monitor agents' consumption, though it affects their decisions through the choice of market prices and transfers schedule. Plan $\mathcal{P}$ induces truth-telling if it satisfies the following *incentive compatibility* (IC) constraints:

$$\mu c_1(\mu,\alpha) + c_2(\mu,\alpha) \geq \mu c_1(\mu,\hat{\mu},\alpha) + c_2(\mu,\hat{\mu},\alpha), \quad \forall \mu,\hat{\mu} \in [1,\mu^{\max}], \forall \alpha \in [0,1], \tag{22}$$

where, with a slight abuse of notation, I set $c_t(\mu,\alpha) \equiv c_t(\mu,\mu,\alpha)$.

We are ready to state the planner's problem. Let a *truthfully-implementable* plan be a plan $\mathcal{P}$ that induces an equilibrium of the economy characterized by (13)-(21), and satisfies the IC constraints (22). Let $\mathbb{TIP}$ denote the set of truthfully-implementable plans. The optimal plan is a truthfully-implementable plan that maximizes the expected utility of the agents in period 0; that is, it solves

$$\mathbb{W} \equiv \max_{\mathcal{P} \in \mathbb{TIP}} E\left[\mu c_1(\mu,\alpha) + c_2(\mu,\alpha)\right]. \tag{PP}$$

4.2 Constrained Efficiency under Full Commitment

Implicit in the program (PP) is the assumption that the planner chooses a plan in period 0 and does not renege on it afterward. This is an important assumption: while the *expectation* about an intervention in period 1 affects the production of trees in period 0, the *actual* intervention does not.[27] Here, I solve for the optimal policy when the planner has full commitment. I explore how the solution changes when the planner has partial commitment in Section 4.3.

A simplified planner's problem. I begin the analysis by deriving some structure for the optimal transfers $\{T_1(\mu,\alpha), T_2(\mu,\alpha)\}_{\forall\alpha}$, which significantly reduces the dimensionality of the planner's problem.

Lemma 5. *In the constrained efficient allocation, transfers take the form*

$$T_1(\mu,\alpha) = \begin{cases} \overline{T}_1(\alpha) & \text{if } \mu > \widetilde{\mu}(\alpha) \\ \underline{T}_1(\alpha) & \text{if } \mu < \widetilde{\mu}(\alpha) \end{cases}$$

with $\overline{T}_1(\alpha) \geq \underline{T}_1(\alpha)$, *and*

$$\mathcal{T}_2(\mu,\alpha) = \begin{cases} 0 & \text{if } \mu > \widetilde{\mu}(\alpha) \\ \widetilde{\mu}(\alpha)\left[\overline{T}_1(\alpha) - \underline{T}_1(\alpha)\right] & \text{if } \mu < \widetilde{\mu}(\alpha) \end{cases}$$
$$\mathbb{T}_2(\alpha) = (1+\chi)G(\widetilde{\mu}(\alpha))\widetilde{\mu}(\alpha)\left[\overline{T}_1(\alpha) - \underline{T}_1(\alpha)\right]$$

If the market for trees is active, $\widetilde{\mu}(\alpha) = \mu_B(\alpha)$ *and*

$$\underline{T}_1(\alpha) = -\frac{1}{G(\mu_B(\alpha))}\left[[P_S(\alpha) - P_B(\alpha)]S(\alpha) + [1 - G(\mu_B(\alpha))]\overline{T}_1(\alpha)\right]$$

Lemma 5 effectively reduces the choice of transfers to a single value for each state α: the transfer to high-μ agents, $\overline{T}_1(\alpha)$. The reason for this result is that the planner classifies agents into only two groups: consumers and non-consumers.[28] Then, within each group, the planner treats all agents equally. After choosing how much to transfer to consumers in period 1, and given the market pricing policy, the transfer to non-consumers is pinned down by the planner's resource constraint. The transfers in period 2 are then determined by the IC constraints and the resource constraint in period 2. Since $\overline{T}_1(\alpha) \geq \underline{T}_1(\alpha)$, we have that $\mathbb{T}_2(\alpha) \geq 0$. Note that the deadweight loss associated with the transfers in period 2 affects both the direct transfers made in period 1, as well as any positive wedge induced in the market for trees. The difference between the two

[27] The standard restriction of a rational expectations equilibrium implies that in equilibrium, expected and actual interventions coincide. The argument points at the potential time inconsistency problem present in this environment.

[28] Note that this is not trivial, as the planner could have chosen $\widetilde{\mu}(\alpha) > \mu_B(\alpha)$, transferring resources only to very high-μ agents. Lemma 5 shows that this is not the case.

instruments is that the private market requires a smaller intervention in order to generate a given increase in total liquidity. However, a stronger private market exacerbates the production of bad trees, which the planner internalizes as a cost. Thus, the planner will trade off the costs associated with direct transfers with the perverse incentives generated by a stronger private market.

Using the planner's resource constraint in period 1, we can rewrite the market clearing condition as

$$G(\mu_B(\alpha))W_1 = [1 - G(\tilde{\mu}_B(\alpha))] P_S(\alpha)H_B + [1 - G(\mu_S(\alpha))P_S(\alpha)] H_G + [1 - G(\mu_B(\alpha))]\overline{T}_1(\alpha), \quad (23)$$

which is analogous to the market clearing condition in the *laissez-faire* economy, with P_S replacing P_M, $\overline{T}_1$ taking the place of the market value of bonds, $Q_B B_0$, and allowing for the possibility that not all bad trees are sold, represented by $\tilde{\mu}_B(\alpha) = \max\left\{\mu_B(\alpha), \frac{\alpha Z}{P_S(\alpha)}\right\}$.

Since the planner's problem is constrained by the equilibrium it induces, it is useful to define some basic equilibrium objects. In particular, let $\boldsymbol{\mu_B}(\alpha)$ denote the solution to equation (23), given $\{\alpha, P_S(\alpha), \overline{T}_1(\alpha), H_G\}$.[29] Moreover, note that $\boldsymbol{\lambda_M}(\alpha)$, as characterized by equation (14), only depends on $P_S(\alpha)$ and H_G (where $\boldsymbol{\mu_S}(\alpha) = \min\left\{\frac{Z}{P_S(\alpha)}, \mu^{\max}\right\}$). Thus, given H_G, the expression in (13) characterizes $\boldsymbol{P_B}(\alpha)$ as a function of the state α and the planner's choice of $P_S(\alpha)$ and $\overline{T}_1(\alpha)$. Finally, given $\boldsymbol{\mu_B}(\alpha)$, equation (18) defines an implicit function of $\boldsymbol{H_G}$ on $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$ (where $\boldsymbol{H_B} = W_1 - C(H_G)$).[30]

Define

$$\begin{aligned}
\boldsymbol{U_1}(\alpha) &\equiv \int_1^{\mu^{\max}} [\mu \boldsymbol{c_1}(\mu,\alpha) + \boldsymbol{c_2}(\mu,\alpha)]\, dG(\mu) \\
&= \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu \left[W_1 + \overline{T}_1(\alpha)\right] dG(\mu) + \int_{\tilde{\boldsymbol{\mu}}_{\boldsymbol{B}}(\alpha)}^{\mu^{\max}} \mu P_S(\alpha) \boldsymbol{H_B} dG(\mu) + \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu P_S(\alpha) H_G dG(\mu),
\end{aligned}$$

and

$$\mathbb{T}_{\boldsymbol{2}}(\alpha) \equiv (1+\chi)\boldsymbol{\mu_B}(\alpha)\left[\overline{T}_1(\alpha) + [P_S(\alpha) - \boldsymbol{P_B}(\alpha)]\, \boldsymbol{S}(\alpha)\right],$$

where $\boldsymbol{U_1}(\alpha)$ and $\mathbb{T}_{\boldsymbol{2}}(\alpha)$ are functions of $\{\alpha, P_S(\alpha), \overline{T}_1(\alpha), H_G\}$. The next lemma states a simplified planner's problem.

Lemma 6. *The planner's problem* (PP) *is equivalent to*

$$\max_{\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}, H_G} E\left[\boldsymbol{U_1}(\alpha) + ZH_G + \alpha Z(W - C(H_G)) - \frac{\chi}{1+\chi}\mathbb{T}_{\boldsymbol{2}}(\alpha)\right] \quad \text{(PP')}$$

subject to

[29] Throughout this section, I adopt the following convention. I use a bold notation to represent functions that correspond to the equilibrium outcomes of the economy, while normal notation denotes standard variables. Moreover, I use a simplified notation for the function's arguments to avoid additional clutter.

[30] Note that I treat H_G as a choice variable, like $P_S(\alpha)$ and $\overline{T}_1(\alpha)$. Then, $H_G = \boldsymbol{H_G}$ is a constraint in the planner's problem. This approach will be useful to dissect the economic forces determining the planner's choices.

$$H_G = \boldsymbol{H_G}$$
$$0 \leq \mathbb{T}_{\mathbf{2}}(\alpha) \leq W_2$$

The program (PP′) greatly simplifies the planner's problem (PP) by use of Lemma 5. Unfortunately, a close inspection of the market equilibrium induced by the planner's policy reveals that the planner's objective is not everywhere differentiable. Consider, for example, the nature of the market equilibrium at $P_S(\alpha) = \frac{Z}{\mu^{\max}}$. Recall that $\frac{Z}{\mu^{\max}}$ is the threshold that determines whether there is any agent who chooses to sell their good trees in period 1. It is immediate to see that $\boldsymbol{\mu_B}(\alpha)$ is not differentiable at $P_S(\alpha) = \frac{Z}{\mu^{\max}}$, since the right derivative includes the effect that changes in the price have on the mass of agents that sell their good trees (which is proportional to $g(\mu^{\max}) > 0$), while the left derivative does not. A similar argument holds at $P_S(\alpha) = \frac{\alpha Z}{\mu^{\max}}$, where the supply of trees is discontinuous (see equation (16)). The analysis that follows will take these discontinuities into account.

Optimality conditions for an interior solution. In what follows, I focus on solutions in which the induced equilibrium features $\boldsymbol{\lambda_E} \in (0, 1)$. The next proposition characterizes the necessary conditions for such a solution.

Proposition 6. *Suppose $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$ is a solution to the planner's problem, and satisfies $0 < \mathbb{T}_{\mathbf{2}}(\alpha) < W_2$ for all α, $\boldsymbol{\lambda_E} \in (0, 1)$, and $\boldsymbol{U_1}(\alpha)$ and $\mathbb{T}_{\mathbf{2}}(\alpha)$ are differentiable at all $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$. Then, $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$ satisfy*

$$\underbrace{\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)}\right]}_{\text{liquidity effect}} f(\alpha) + \underbrace{E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + \left(1 - \alpha C'(\boldsymbol{H_G})\right) Z - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial H_G}\right]\frac{d\boldsymbol{H_G}}{dP_S(\alpha)}}_{\text{incentives effect}} = 0 \quad (24)$$

$$\underbrace{\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)}\right]}_{\text{liquidity effect}} f(\alpha) + \underbrace{E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + \left(1 - \alpha C'(\boldsymbol{H_G})\right) Z - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial H_G}\right]\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}}_{\text{incentives effect}} = 0 \quad (25)$$

for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$.

The necessary optimality conditions for an interior solution have two components: a *liquidity effect* in period 1, and an *incentives effect* in period 0. The liquidity effect captures the impact of the planner's plan on the reallocation of resources for a given composition of tree quality, $\{H_G, H_B\}$. This is the force that justifies government intervention in models in which the private sector fails to fully reallocate resources to the agents with the highest valuations, as in Woodford (1990) and Holmström and Tirole (1998). An increase in either P_S or $\overline{T}_1$ increases the flow of goods from low-μ to high-μ agents, so $\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}, \frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} > 0$. Such an increase also implies a change in the transfers the planner needs to collect. While $\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)} > 0$, implying that higher transfers always lead to a higher deadweight loss, the sign of $\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)}$ is ambiguous, as a *Laffer-curve* phenomenon in quality may arise, in which a higher $P_S(\alpha)$ disproportionally improves the quality of the trees in the market, so that $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} < 0$. However, for a sufficiently large $P_S(\alpha)$, we have $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} > 0$.

The liquidity effect would be present in my model, even if there were perfect information in private markets. However, the presence of asymmetric information has two novel implications. First, conditional on the tree quality composition, the choice of P_S and $\overline{T}_1$ determines the amount of liquidity in the economy and therefore indirectly affects the adverse selection problem in the private markets. Thus, the two instruments interact in nontrivial ways. Second, anticipating how the planner's choices affect the functioning of markets in period 1, agents respond by adjusting their production of tree quality in period 0. The planner internalizes these dynamics through the incentives effect.

The incentives effect reflects how (expected) changes in market liquidity in period 1 affect the determination of the tree quality composition in the economy in period 0. The incentives effect has two components. First, there is the direct change in the planner's objective for a given in H_G, which is given by

$$E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + \left(1 - \alpha C'(\boldsymbol{H_G})\right) Z - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial H_G}\right] > 0.$$

This effect is common to *all* states α and is independent of the instrument that triggered the change. An improvement in the asset quality composition of the economy is welfare-enhancing.

Second, the incentives effect depends on how the change in a particular instrument affects the determination of $\boldsymbol{H_G}$. Given a planner's plan, agents choose the quality of trees they produce according to the shadow values (see equation (18)). However, the shadow values not only depend on the planner's policy but also on the agents' choices, creating a complex feedback loop. The planner internalizes this effect when making its choices. The next lemma characterizes the *total* effect of policy on the determination of tree quality.[31]

Lemma 7. *Let* $\overline{\boldsymbol{\gamma}}_{\boldsymbol{G}}(\alpha) \equiv \int_1^{\mu^{\max}} \max\{\mu P_S(\alpha), Z\} dG(\mu)$ *and* $\overline{\boldsymbol{\gamma}}_{\boldsymbol{B}}(\alpha) \equiv \int_1^{\mu^{\max}} \max\{\mu, \widetilde{\boldsymbol{\mu}}_{\boldsymbol{B}}(\alpha)\} P_S(\alpha) dG(\mu)$. *Then, if* $\boldsymbol{H_G}$ *is differentiable at* $\{P_S(\alpha), \overline{T}_1(\alpha)\}$,

$$\frac{d\boldsymbol{H_G}}{dP_S(\alpha)} = \frac{\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)}\frac{1}{\gamma_G} - \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)}\frac{1}{\gamma_B}}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right]\frac{1}{\gamma_B}} f(\alpha) \quad \text{and} \quad \frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)} = -\frac{\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)}\frac{1}{\gamma_B}}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right]\frac{1}{\gamma_B}} f(\alpha). \tag{26}$$

where $\eta_G(\boldsymbol{H_G}) = \frac{C''(\boldsymbol{H_G})}{C'(\boldsymbol{H_G})}$. *If the solution to the planner's problem induces an equilibrium with* $\boldsymbol{\lambda_E} \in (0,1)$, *the incentives effect is negative.*

The expressions in (26) have two components. On the one hand, the numerator captures the direct impact of changes in the policy instruments on the shadow values. Note that changes in the instruments in a given state α *only* affect the valuation of trees in that state α.[32] A simple

[31] I use "determination of tree quality" as a synonym for "determination of $\boldsymbol{H_G}$." Recall that in this setting there is a one-to-one mapping between $\boldsymbol{H_G}$ and $\boldsymbol{\lambda_E}$, as $\boldsymbol{H_B} = W - C(\boldsymbol{H_G})$.

[32] For the sake of clarity, note that $\gamma_G = E[\overline{\gamma}_G(\alpha)]$ and $\gamma_B = E[\overline{\gamma}_B(\alpha)]$.

extension of Proposition 2 shows that $\frac{\partial \overline{\gamma}_{G}(\alpha)}{\partial P_{S}(\alpha)} \frac{1}{\gamma_{G}} - \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_{S}(\alpha)} \frac{1}{\gamma_{\boldsymbol{B}}} < 0$. Moreover, we have that $\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_{1}(\alpha)} > 0$, as increases in $\overline{T}_1(\alpha)$ increase $\boldsymbol{\mu_B}(\alpha)$, which, *given* $\{P_S(\alpha)\}_{\forall\alpha}$, increases $\boldsymbol{\gamma_B}$.[33] On the other hand, the denominator captures how changes in $\boldsymbol{H_G}$ affect the agents' choice of tree quality, which is mediated by two channels. First, $\eta_G(\boldsymbol{H_G})$ denotes the semi-elasticity of the marginal cost of good trees. Lower values of $\eta_G(\cdot)$ imply a stronger response of production to changes in the shadow values. Second, changes in $\boldsymbol{H_G}$ affect the production of trees through their effect on $\boldsymbol{\mu_B}(\alpha)$. Recall that $\boldsymbol{\mu_B}(\alpha)$ is the return of trees in the market. An increase in H_G *reduces* the number of trees in the market (both because only a fraction of good trees are sold, and because good trees are more expensive to produce), *reducing* the required return on trees and therefore the shadow value of bad trees. Thus, $E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right]\frac{1}{\gamma_{\boldsymbol{B}}} < 0$. This implies that the sign of $\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right]\frac{1}{\gamma_{\boldsymbol{B}}}$ is ambiguous. However, a negative value for this expression is inconsistent with an interior solution, as it would imply that *increases* in the price received by sellers *improves* the asset quality composition in the economy. Since my focus is on solutions that induce an equilibrium with $\boldsymbol{\lambda_E} \in (0,1)$, we have that the denominator is positive, which implies that the incentives effect is *negative*.

Finally, note the expressions in Proposition 6 do not hold at the kinks $P_S(\alpha) = \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$. It turns out that the optimality conditions are continuous at $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, which greatly simplifies the analysis.[34] However, that is not the case for the threshold $P_S(\alpha) = \frac{Z}{\mu^{\max}}$, which opens up the possibility of discontinuities in the optimal policy, and financial crises as part of the planner's optimal plan.

Optimal financial crises. The previous analysis provided a partial characterization of the solution to the planner's problem by studying the local (first-order) optimality conditions. However, as discussed above, the planner's problem features *kinks* that make these conditions necessary for an interior solution but not sufficient. A particularly important kink is at $P_S(\alpha) = \frac{Z}{\mu^{\max}}$, which determines whether agents trade their good trees. In the analysis of Section 3, I characterized the existence of a threshold α^* such that if $\alpha < \alpha^*$ then $\lambda_M(\alpha) = 0$, and showed that the volume traded in the market for trees is discontinuous at α^*. These were the states featuring a *financial collapse*. Here, I show the conditions that determine whether the planner chooses to induce financial crises along the equilibrium path. Later, I study how the planner mitigates the adverse effects of a financial crisis when it occurs.

Let $\tilde{\alpha}^*$ denote a threshold such that, if $\alpha < \tilde{\alpha}^*$, the planner induces $\boldsymbol{\lambda_M}(\alpha) = 0$, which I call the

[33]It might seem surprising that higher transfers increase the shadow value of bad trees, given that I showed in Section 3 that a higher supply of government bonds *reduces* the production of bad trees. The reason for the discrepancy is that here we are conditioning on the price that sellers face, while in Section 3 the price adjustment was an essential part of the result. In particular, we would get a similar result here if the planner chose the wedge $\omega(\alpha)$ instead of the price $P_S(\alpha)$.

[34]See the proof of Proposition 7 for the details.

crisis states, and if $\alpha \geq \tilde{\alpha}^*$ the planner induces $\boldsymbol{\lambda_M}(\alpha) > 0$, which I call the *normal* states.[35] Define

$$\boldsymbol{W}(\tilde{\alpha}) \equiv \int_{\underline{\alpha}}^{\tilde{\alpha}} \left[\boldsymbol{U_1}^C(\alpha) + Z\boldsymbol{H_G} + \alpha Z \boldsymbol{H_B} - \frac{\chi}{1+\chi} \mathbb{T}_{\mathbf{2}}{}^C(\alpha) \right] dF(\alpha) + \int_{\tilde{\alpha}}^{\overline{\alpha}} \left[\boldsymbol{U_1}^N(\alpha) + Z\boldsymbol{H_G} + \alpha Z \boldsymbol{H_B} - \frac{\chi}{1+\chi} \mathbb{T}_{\mathbf{2}}{}^N(\alpha) \right] dF(\alpha)$$

where $\boldsymbol{U_1}^C(\alpha)$ and $\mathbb{T}_{\mathbf{2}}{}^C(\alpha)$ denote $\boldsymbol{U_1}$ and $\mathbb{T}_{\mathbf{2}}$ evaluated at the optimal policy conditional on a crisis, and $\boldsymbol{U_1}^N(\alpha)$ and $\mathbb{T}_{\mathbf{2}}{}^N(\alpha)$ denote the corresponding values in normal states. Note that these policies need to be calculated jointly, as it is the full plan $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$ that determines $\boldsymbol{H_G}$. This involves solving the system of equations (24) and (25) *conditional* on α being greater or smaller than $\tilde{\alpha}^*$.[36] Note that the choice of $\tilde{\alpha}^*$ is a complex problem, as $\tilde{\alpha}^*$ affects the agents' choice of tree quality, which in turn affects the planner's optimal policy. However, many of these effects cancel out because of standard envelope arguments involving the optimality of $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$. The next lemma shows that there is a simple characterization of the planner's choice.

Lemma 8. *Suppose that the solution to the planner's problem induces an equilibrium with $\boldsymbol{\lambda_E} \in (0,1)$, and that $0 < \mathbb{T}_2(\alpha) < W_2$. The optimal policy features a unique threshold $\tilde{\alpha}^* \in [\underline{\alpha}, \overline{\alpha})$ such that if $\alpha < \tilde{\alpha}^*$, $\lambda_M(\alpha) = 0$ and if $\alpha \geq \tilde{\alpha}^*$, $\lambda_M(\alpha) > 0$. If $\tilde{\alpha}^* > \underline{\alpha}$, then $\boldsymbol{W}'(\tilde{\alpha}^*) = 0$, where*

$$\boldsymbol{W}'(\tilde{\alpha}) = \left[\boldsymbol{U_1}\left(\tilde{\alpha}^-\right) - \boldsymbol{U_1}\left(\tilde{\alpha}^+\right) - \frac{\chi}{1+\chi}\left[\mathbb{T}_{\mathbf{2}}\left(\tilde{\alpha}^-\right) - \mathbb{T}_{\mathbf{2}}\left(\tilde{\alpha}^+\right) \right] \right] f\left(\tilde{\alpha}\right) + E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + \left(1 - C'\left(\boldsymbol{H_G}\right)\alpha\right) Z - \frac{\chi}{1+\chi} \frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial H_G} \right] \frac{d\boldsymbol{H_G}}{d\tilde{\alpha}},$$

and $\tilde{\alpha}^+$ and $\tilde{\alpha}^-$ denote the limits of α to $\tilde{\alpha}$ from the left and the right, respectively.

The optimal threshold decision balances three forces: *i*) a direct liquidity benefit, given by $\boldsymbol{U_1}(\tilde{\alpha}^+) - \boldsymbol{U_1}(\tilde{\alpha}^-)$; *ii*) a direct liquidity cost, given by $\mathbb{T}_{\mathbf{2}}(\tilde{\alpha}^-) - \mathbb{T}_{\mathbf{2}}(\tilde{\alpha}^+)$; and *iii*) an incentives effect of the threshold, given by $E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + \left(1 - C'\left(\boldsymbol{H_G}\right)\alpha\right) Z - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial H_G}\right] \frac{d\boldsymbol{H_G}}{d\tilde{\alpha}}$. Notably, the planner's instruments affect these three channels differently. Consider, for example, a state $\alpha' = \alpha^* - \epsilon$, for some $\epsilon > 0$ but small (recall that α^* is the threshold of a financial crisis in the *laissez-faire* economy). The amount of liquidity in the economy is discontinuously lower at α' than at α^*, making the misallocation of resources discontinuously higher. If the planner wants to improve the outcome at α', it has two tools. On the one hand, it can set a small positive wedge $\omega(\alpha')$, which entails only a small deadweight but a large increase in the direct liquidity benefit. However, this policy

[35] In principle, there could be several thresholds such that the economy transitions from a crisis to a normal state. However, the optimal policy features a price schedule $\{P_S(\alpha)\}_{\forall\alpha}$ that is weakly increasing in α. Thus, the economy cannot transition from a normal to a crisis state as α increases.

[36] In particular, there exists a threshold $\hat{\alpha} > \frac{1}{\mu^{\max}}$ such that for all $\alpha \in [\underline{\alpha}, \hat{\alpha})$ and H_G, it is possible to find two pairs $\{P_S^C(\alpha), \overline{T}_1^C(\alpha)\}$ and $\{P_S^N(\alpha), \overline{T}_1^N(\alpha)\}$ that satisfy the optimality conditions in Proposition 6, with $P_S^C(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S^N(\alpha) \geq \frac{Z}{\mu^{\max}}$. Thus, for those states, the planner can choose a policy inducing a *crisis* or a *normal* state.

generates perverse incentives in the production of trees, inducing a reduction in the quality of the trees in the economy. On the other hand, the planner can provide liquidity through an increase in direct transfers. Keeping the market wedge $\omega(\alpha')$ fixed, this policy has a positive direct liquidity benefit, while at the same time it induces an increase in the production of good trees. However, for this policy to generate the same amount of liquidity benefit as the positive market wedge, it requires a larger increase in the deadweight loss from $\mathbb{T}_2$. In the characterization of the optimal policy below, I show an example in which the trade-off for the planner is not in terms of the liquidity benefit, as it sets $\boldsymbol{U_1}(\tilde{\alpha}^-) \simeq \boldsymbol{U_1}(\tilde{\alpha}^+)$, but between the cost of providing liquidity (since $\boldsymbol{T_2}(\tilde{\alpha}^-) > \boldsymbol{T_2}(\tilde{\alpha}^+)$) and the incentives to produce bad trees it generates.

Optimal policy. I am ready to characterize the optimal policy. The next proposition summarizes the planner's choice.

Proposition 7 (Optimal Policy with Commitment). *Suppose that the solution to the planner's problem induces an equilibrium with $\boldsymbol{\lambda_E} \in (0,1)$, and that $0 < \mathbb{T}_2(\alpha) < W_2$. Then, the optimal policy is characterized by two thresholds $\tilde{\alpha}^*, \tilde{\alpha}^{**}$, with $\underline{\alpha} \leq \tilde{\alpha}^* \leq \tilde{\alpha}^{**} \leq \overline{\alpha}$ such that:*

i. if $\alpha < \tilde{\alpha}^*$, then $\lambda_M(\alpha) = 0$, $\omega(\alpha) < 0$, $P_S(\alpha)$ is increasing in α, and $\overline{T}_1(\alpha)$ is decreasing in α;

ii. if $\alpha \in [\tilde{\alpha}^*, \tilde{\alpha}^{**})$, then $\lambda_M(\alpha) > 0$, $\omega(\alpha) > 0$, and $P_S(\alpha)$ and $\overline{T}_1(\alpha)$ are constant in α;

iii. if $\alpha \in (\tilde{\alpha}^{**}, \overline{\alpha}]$, then $\lambda_M(\alpha) > 0$, $\omega(\alpha) < 0$, $P_S(\alpha)$ is weakly increasing in α, and $\overline{T}_1(\alpha)$ is weakly decreasing in α.

If $\tilde{\alpha}^* > \underline{\alpha}$, then

$$\lim_{\alpha \to \tilde{\alpha}^{*-}} P_S(\alpha) < \lim_{\alpha \to \tilde{\alpha}^{*+}} P_S(\alpha) \quad \textit{and} \quad \lim_{\alpha \to \tilde{\alpha}^{*-}} \overline{T}_1(\alpha) > \lim_{\alpha \to \tilde{\alpha}^{*+}} \overline{T}(\alpha)$$

Proposition 7 completely characterizes the optimal policy when the planner has full commitment. Figure 5 depicts a numerical example of the optimal price, $P_S(\alpha)$, the optimal transfers, $\overline{T}_1(\alpha)$, and the corresponding market wedge, $\omega(\alpha)$, as functions of the aggregate state α. In this example, the optimal policy under full commitment induces an equilibrium with $\lambda_E = 0.9$, while the *laissez-faire* equilibrium has $\lambda_E = 0.84$. That is, the optimal policy generates an increase in the asset quality produced.

Proposition 7 and Figure 5 distinguish between three regions of intervention characterized by the level of α: *i*) a region of private market collapse and high transfers; *ii*) a region of support of the private market with a positive market wedge and low transfers; and *iii*) a region with high prices, negative market wedge, and low transfers. Moreover, there is a positive measure of α for which the planner completely stabilizes prices and transfers, and the market wedge is decreasing in α. Thus, it would seem that the optimal policy has two main characteristics: aggressive direct provision of liquidity in *crisis* states, and *leaning against the wind* in *normal* states. While this conclusion is broadly correct, there are some nuances that are worth exploring.

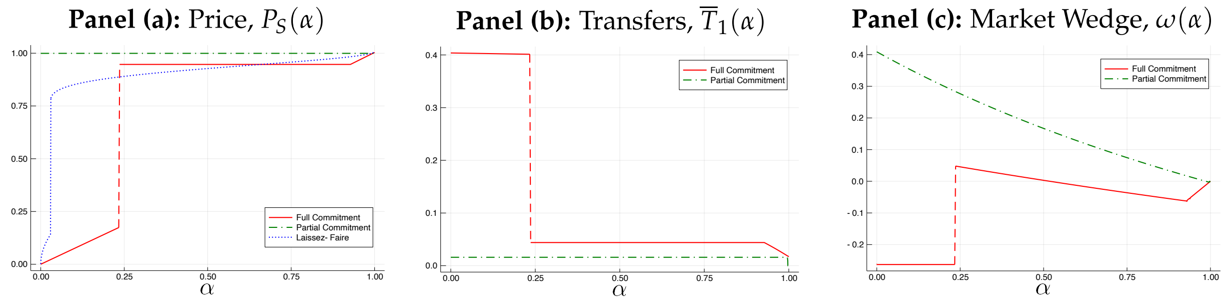

Figure 5: Optimal policy as a function of the aggregate state α

Note: I assume $\alpha \sim U[0,1]$ and $\mu \sim U[1, \mu^{\max}]$ with $\mu^{\max} = 2$. Moreover, $C(H_G) = ae^{\eta H_G}$, with $a = 0.2$ and $\eta = 5$. The other parameters of the model are: $W_0 = 0.25$, $W_1 = W_2 = 1$, and $Z = 1.3$. The deadweight loss of transfers is $\chi = 0.17$.

When α is low, the optimal policy induces a financial market collapse, in which only bad trees are traded. However, the planner does not allow the total liquidity in the economy to collapse, as it partially substitutes private liquidity with public liquidity. Figure 6 Panel (a) shows the total liquidity in the economy as a function of α. The figure shows that, in this example, total liquidity is *higher* in the crisis states. I will come back to this below. Interestingly, in the region of a private market collapse, the planner introduces a negative market wedge, such that $P_S(\alpha) < \boldsymbol{P_B}(\alpha)$, reducing the private liquidity even further (Figure 5 Panel (c)). To understand this result, note that, in a crisis state, the market for trees does not suffer from adverse selection. This implies that the planner can implement a small tax on trees and use the proceeds to increase $\overline{T}_1$, and the total liquidity in the economy would not change. However, by reducing the price received by sellers, the incentives to produce bad trees decreases, increasing overall welfare. That is, crisis states are a good time to tax a market in which only bad trees are traded. However, the planner does not set $\omega = -1$, as the previous argument required to use the revenues from taxation to pay for the transfers, implying a *Laffer-curve* phenomenon. That is, the planner never finds it optimal to completely shut down the private market.

At $\alpha = \tilde{\alpha}^*$, the planner changes its policy discontinuously: it reduces the direct provision of liquidity through transfers and increases the market wedge. The positive market wedge increases the price received by sellers, which induces them to sell more good trees, increasing the liquidity in the private market. Moreover, by reducing the amount of transfers, the planner makes liquidity more scarce, reducing the rate of return on assets, which, given a market wedge, increases the price of trees. Thus, both policies *increase* market prices and, therefore, contribute to stimulating the private markets. Note, however, that the planner chooses a probability of a private market collapse that is higher than the one in the *laissez-faire* equilibrium. Yet, Figure 6 Panel (a) shows that the planner does not allow the *total liquidity* in the economy to collapse. As mentioned before, total liquidity is higher in the low-α states. Interestingly, this is not reflected in $\boldsymbol{U_1}(\alpha)$, which is lower in the crisis states (Figure 6 Panel (b)). It might seem surprising that $\boldsymbol{U_1}(\alpha)$ is lower even though the total liquidity is higher. The reason for this result is that the private market transfers resources

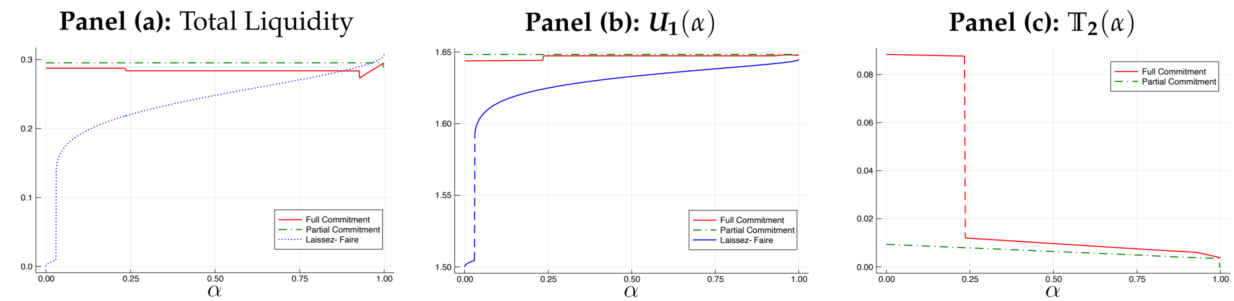

Figure 6: Optimal policy outcomes as a function of the aggregate state α

Note: I assume $\alpha \sim U[0,1]$ and $\mu \sim U[1, \mu^{\max}]$ with $\mu^{\max} = 2$. Moreover, $C(H_G) = ae^{\eta H_G}$, with $a = 0.2$ and $\eta = 5$. The other parameters of the model are: $W_0 = 0.25$, $W_1 = W_2 = 1$, and $Z = 1.3$. The deadweight loss of transfers is $\chi = 0.17$.

(mostly) to high-μ agents (recall that $\boldsymbol{\mu_S}(\alpha) > \boldsymbol{\mu_B}(\alpha)$, and the majority of trees in the economy are good). Thus, the planner's direct provision of liquidity would need to be substantially higher than what the market can achieve in order to generate a higher level of utility. But, as Figure 6 Panel (c) shows, the direct provision of liquidity is significantly more costly than supporting the private market. In fact, the difference in the deadweight loss around $\tilde{\alpha}^*$ is orders of magnitude larger than the difference in $\boldsymbol{U_1}(\alpha)$, showing that the choice of $\tilde{\alpha}^*$ is driven by a trade-off between the cost of direct liquidity provision and the incentives to produce tree quality that the policies generate, rather than about the amount of liquidity provided in the market, which is the main force in the policy analysis of Tirole (2012). Thus, the presence of multiple instruments can change the trade-offs faced by the planner, even when the economic forces are the same.

Finally, when α is sufficiently high, such that good and bad trees have similar payoffs, the planner chooses a negative market wedge but allows prices to increase in α, reducing the prevalence of direct transfers. Since good and bad trees are relatively similar in these states, the incentives benefit of high taxes are relatively small. Thus, while these states *always* feature a negative market wedge, the magnitude of the wedge can be increasing in α (i.e., less negative). This implies that while a *leaning against the wind* type of policy is mostly optimal in normal states, there are some considerations that might break the monotonicity.

Summary. The optimal policy balances a liquidity effect and an incentives effect, and it has two main characteristics. First, states with low levels of α feature a financial crisis, defined as a discontinuous reduction in the volume traded in the *private* financial markets. The planner mitigates the adverse effects of this drop by discontinuously increasing the direct transfers to the agents. Second, for higher levels of α, the level of direct transfers is low, and the market wedge can follow a *leaning against the wind* type of policy. However, since in high-α states good and bad trees are fairly similar, the incentives benefits of taxing the private markets might be non-monotonic.

Next, I study how the optimal policy changes when the planner has an imperfect ability to commit to a plan.

4.3 Constrained Efficiency under Partial Commitment

Now consider the case of a planner that can commit to a transfer schedule in period 0, but chooses the market price in period 1.[37] This implies that the choice of prices does not take into account the incentives effect they generate in period 0. However, the planner anticipates how its choice of prices in period 1 responds to changes in transfers, opening the possibility of using the transfers as a way to manipulate the decisions of its future self. As with the commitment case, I focus on interior solutions in which $\boldsymbol{\lambda_E} \in (0,1)$.

Consider first the choice of the prices in state α, *conditional* on $\{\overline{T}_1(\alpha), H_G\}$. In a solution with an active market for trees, a necessary condition for the optimality of $P_S(\alpha)$ is given by

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)} = 0. \tag{27}$$

Not surprisingly, the choice depends only on the liquidity effect. In period 1, a planner with no commitment takes the production of tree quality in period 0 as given, and therefore, for a given level of $\overline{T}_1(\alpha)$ and H_G, the price is higher than with full commitment. However, the characterization of the optimal policy is qualitatively similar: there are three regimes depending on the value of α, such that low levels of α lead to a financial collapse; for middle levels of α the market wedge is positive; and for high levels of α the wedge is negative.

Since transfers are chosen in period 0, before the market intervention in period 1, the planner internalizes that its choices can influence its future-self's actions. Thus, an important dimension of the planner's choice in period 1 is how the optimal price reacts to different levels of the transfers. Let

$$\boldsymbol{FOC}(P_S(\alpha), \overline{T}_1(\alpha), H_G) \equiv \frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)}.$$

Naturally, the optimality condition for $P_S(\alpha)$ is

$$\boldsymbol{FOC}(\boldsymbol{P_S}(\alpha), \overline{T}_1(\alpha), H_G) = 0,$$

where $\boldsymbol{P_S}(\alpha)$ denotes the optimal choice of P_S as a function of $\{\alpha, \overline{T}_1(\alpha), H_G\}$. The following lemma shows that the optimal price is decreasing in the transfers.

Lemma 9. *Suppose that the plan $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$ induces an equilibrium with $\boldsymbol{\lambda_E} \in (0,1)$, and that $0 < \mathbb{T}_2(\alpha) < W_2$. The optimal price when the planner has partial commitment satisfies*

$$\frac{\partial \boldsymbol{P_S}(\alpha)}{\partial \overline{T}_1(\alpha)} = -\frac{\frac{\partial \boldsymbol{FOC}(\boldsymbol{P_S}(\alpha), \overline{T}_1(\alpha), H_G)}{\partial \overline{T}_1(\alpha)}}{\frac{\partial \boldsymbol{FOC}(\boldsymbol{P_S}(\alpha), \overline{T}_1(\alpha), H_G)}{\partial P_S(\alpha)}} < 0.$$

[37] I do not study the case in which the planner has no commitment in either instrument for two reasons. First, the problem is not particularly interesting, as the planner would only care about the liquidity effect and would dismiss the incentives effect entirely. Second, in Section 4.4 I show that a natural implementation of the optimal policy involves government debt playing the role of transfers, making the commitment problem for this instrument less pervasive.

The intuition for this result is as follows. An increase in transfers increases the liquidity in the economy, which increases the rate of return of assets, $\boldsymbol{\mu_B}(\alpha)$. This has two effects. First, the marginal benefit of an additional unit of liquidity (in the form of higher prices) decreases. Second, the increase in the rate of return of assets increases the cost of transfers, $\mathbb{T}_\mathbf{2}(\alpha)$, thus increasing the marginal cost of supporting the market. These two forces generate a negative relation between P_S and $\overline{T}_1$, given H_G. Of course, $\boldsymbol{H_G}$ will also react to changes in $\overline{T}_1$ and P_S, so the overall effect will need to take this into account.

Consider now the choice of transfers. The necessary optimality condition for $\overline{T}_1(\alpha)$ is given by

$$\underbrace{\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_\mathbf{2}(\alpha)}{\partial \overline{T}_1(\alpha)} f(\alpha)}_{\text{liquidity effect}} - \underbrace{\overbrace{\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_\mathbf{2}(\alpha)}{\partial P_S(\alpha)}\right]}^{=0}\frac{\partial \boldsymbol{P_S}(\alpha)}{\partial \overline{T}_1(\alpha)}}_{\text{direct policy effect}} f(\alpha) +$$

$$\underbrace{E\left[\frac{\partial \boldsymbol{U_1}(\alpha')}{\partial H_G} + (1-\alpha' C'(\boldsymbol{H_G}))Z - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_\mathbf{2}(\alpha')}{\partial H_G}\right]\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}}_{\text{incentives effect}} = 0.$$

This condition is similar to the one we obtained in the case of full commitment, with two differences. First, note the presence of the *direct policy effect*, which accounts for the direct impact that transfers have on the choice of prices. However, this effect is equal to zero for standard envelope arguments: the price in period 1 is chosen optimally, so any (small) changes have a zero first-order effect on the objective. The second difference is more subtle, and it involves the change of $\boldsymbol{H_G}$ when $\overline{T}_1(\alpha)$ changes, present in the incentives effect. The expressions (26) fully characterized the change in the production of good trees for independent changes in P_S and $\overline{T}_1$. To avoid confusion, denote those expressions by $\frac{d\boldsymbol{H_G}}{dP_S(\alpha)}\Big|^{commit}$ and $\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}\Big|^{commit}$, respectively. We now need to take into account that the choice of $\overline{T}_1$ affects the choice of P_S, and its subsequent impact on $\boldsymbol{H_G}$. The next lemma characterizes $\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}$ for this case.

Lemma 10. *Suppose that the plan $\{P_S(\alpha), \overline{T}_1(\alpha)\}_{\forall\alpha}$ induces an equilibrium with $\boldsymbol{\lambda_E} \in (0,1)$, $0 < \mathbb{T}_2(\alpha) < W_2$ and that $P_S(\alpha)$ satisfies (27). There exists $\overline{\chi}$ such that if $\chi > \overline{\chi}$, and if $\boldsymbol{H_G}$ is differentiable at $\overline{T}_1(\alpha)$, we have*

$$\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)} = \underbrace{\frac{\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}\Big|^{commit}}{1 - \int_{\underline{\alpha}}^{\overline{\alpha}} \frac{d\boldsymbol{H_G}}{dP_S(\alpha')}\Big|^{commit}\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G}d\alpha'}}_{\textit{direct incentives effect}<0} + \underbrace{\frac{\frac{d\boldsymbol{H_G}}{dP_S(\alpha)}\Big|^{commit}\frac{\partial \boldsymbol{P_S}(\alpha)}{\partial \overline{T}_1(\alpha)}}{1 - \int_{\underline{\alpha}}^{\overline{\alpha}} \frac{d\boldsymbol{H_G}}{dP_S(\alpha')}\Big|^{commit}\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G}d\alpha'}}_{\textit{indirect policy effect}>0}$$

The total effect is the sum of two terms. First, there is the direct effect of transfers on $\boldsymbol{H_G}$, which is given by the total effect of transfers for a *given* level of $P_S(\alpha)$, divided by $1 - \int_0^1 \frac{\partial \boldsymbol{H_G}}{\partial P_S(\alpha')}\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G}dF(\alpha')$. The latter term corresponds to an adjustment that incorporates the joint determination of prices, $\{\boldsymbol{P_S}(\alpha')\}_{\forall\alpha'}$, and good trees, $\boldsymbol{H_G}$. Since an increase in transfers increases the market return on trees, which in turn increases the incentives to produce bad trees, the direct effect is, once again, negative.

The new, and most interesting, term is the indirect policy effect. It is given by the impact that transfers have on $\boldsymbol{H_G}$ that is mediated through the (expectation of the) choice of prices in period 1. Since $\frac{\partial \boldsymbol{H_G}}{\partial P_S(\alpha)}\Big|^{commit} < 0$ and $\frac{\partial \boldsymbol{P_S(\alpha)}}{\partial \overline{T}_1(\alpha)} < 0$, the indirect policy effect is positive. The difference between the commitment case of Section 4.2 and this partial commitment problem is that with partial commitment, the planner does not take into account the effect that its choice of prices has on incentives. The choice of transfers in period 0 partially remedies this problem. By anticipating the effect that transfers have on the choice of prices, the planner distorts its choices in period 0 to better align its policies in period 1 to its objectives in period 0. The next proposition summarizes these results.

Proposition 8 (Optimal Policy under Partial Commitment)**.** *Suppose that the solution to the planner's problem induces an equilibrium with* $\boldsymbol{\lambda_E} \in (0,1)$*, and that* $0 < \mathbb{T}_2(\alpha) < W_2$*. In the optimal policy under commitment, the market intervention in period* 1 *is determined by the liquidity effect. The transfers policy in period* 0 *balances three forces: the liquidity effect, the incentives effect, and a (indirect) policy effect. The policy effect is strictly positive; thus, the policy in period* 0 *is partly aimed at reducing the intervention in period* 1*.*

From the previous analysis one might conclude that the planner would issue more transfers when it has imperfect commitment, as the benefits of transfers seem to be higher. Figure 5 shows that this is not always the case. The reason is the following. *Conditional* on $P_S(\alpha)$, the transfers chosen by the planner are higher with imperfect commitment, as the optimality condition is strictly higher due to the policy effect. However, *conditional* on $\overline{T}_1(\alpha)$, the choice of $P_S(\alpha)$ is higher with partial commitment, as it neglects the (negative) incentives effect. A higher price in period 1 *reduces* the liquidity benefits of intervention in period 0, a force towards lower transfers. Thus, these two forces go in opposite directions. In my numerical example, the latter force dominates, and the policy with partial commitment features a lower level of transfers than with full commitment. Of course, the price chosen in period 1 is higher, and, in this example, the planner chooses complete stabilization of the financial markets, independent of the realization of α (in particular, the probability of a financial crisis is zero). Not surprisingly, the partial commitment optimal policy induces an equilibrium with a fraction of good trees in the economy, which is lower than in the *laissez-faire* equilibrium ($\lambda_E = 0.78$).

4.4 Policy Implementation

Finally, I present an implementation of the optimal policy that involves standard government instruments. The main result of this section is that the optimal policy can be implemented with three sets of instruments: state-contingent one-period government bonds, asset purchase programs, and transaction (or *Tobin*) taxes.

From the previous analysis, it is straightforward to see that the optimal policy could, in principle, be implemented with just two instruments: proportional (*ad-valorem*) transaction taxes (or subsidies) in the market for trees, and state-contingent one-period government bonds. However, the implementation of transaction subsidies might be problematic, as they are likely to generate spurious trades exclusively aimed at collecting the subsidy.[38] Thus, it would be useful to have an alternative instrument that could be mapped to the positive market wedge but that does not suffer from this problem. The next lemma shows that, in a market in which good trees are traded, i.e., $\boldsymbol{\lambda_M}(\alpha) > 0$, asset purchase programs are equivalent to a positive market wedge.

Lemma 11 (Instrument Equivalence)**.** *Consider an economy in period* 1 *with* $\boldsymbol{\lambda_M}(\alpha) > 0$ *and* $\omega(\alpha) > 0$*. There exists an asset purchase program by which the government purchases* $S_B(\alpha) > 0$ *units of bad trees, which induces the same equilibrium in period* 1 *and generates the same deadweight loss from transfers. Both instruments generate the same incentives in period* 0*.*

Two implications of Lemma 11 are worth noting. First, for the asset purchase program to have the desired effect, the government should only buy bad trees.[39] Second, while the two instruments are equivalent when $\boldsymbol{\lambda_M}(\alpha) > 0$, they differ if the resulting equilibrium has $\boldsymbol{\lambda_M}(\alpha) = 0$. The reason is that when $\boldsymbol{\lambda_M}(\alpha) = 0$, asset purchase programs have no impact on the equilibrium prices since they not affect the asset quality composition in the market, and any positive wedge needs to be financed with negative transfers to low-μ agents, thus the total liquidity in the economy does not change. In contrast, subsidies do have an impact, as the liquidity in the private markets effectively increases. However, optimal policy never prescribes a positive wedge when $\boldsymbol{\lambda_M}(\alpha) = 0$. Thus, we can map the positive wedge to asset purchase programs.

The next proposition characterizes an implementation of the optimal policy.

Proposition 9 (Implementation)**.** *Suppose that the solution to the planner's problem (with full or partial commitment) induces an equilibrium with* $\boldsymbol{\lambda_E} \in (0,1)$*, and that* $0 < \mathbb{T}_2(\alpha) < W_2$*. The optimal policy can be implemented with three sets of instruments: state-contingent government debt, asset purchase programs, and transaction (or "Tobin") taxes.*

One last remark with respect to the state-contingent bonds. An alternative implementation could map the transfers to state-contingent *systemic* bailouts. That is, positive transfers to *all* agents that does not condition on any characteristic that can be correlated to their holdings of bad trees. The bailout would work as follows. All agents receive a transfer in period 1 (the same way that all agents receive the payments of the one-period bonds), and the planner finances these transfers with one-period bonds bought by low-μ agents. While this policy generates the same incentives as the state-contingent government bonds, it presents two potential drawbacks. First, since bailouts

[38] This was ruled out in the previous analysis by assuming that each tree could be traded only once per period. In reality, a large number of trades could happen in a short period of time.

[39] That the optimal asset purchase program requires the government to buy the low-quality assets had been emphasized in Tirole (2012). However, the equivalence result is new.

are an *ex-post* policy, they are subject to the same time-consistency problems as the market interventions. Second, government bonds provide more information to the planner than bailouts, making bailouts potentially more costly. Since government bonds carry a premium, agents adjust their purchases to their expected needs. Thus, agents *self-select* into the system. On the contrary, all agents have incentives to claim the need of a bailout, as there is no cost of participation. Thus, government bonds are likely to be the dominant instrument in this type of economies.

5 Conclusion

I have presented a model in which the *ex-ante* production of assets interacts with the *ex-post* adverse selection in financial markets, exposing the economy to episodes which feature a sudden collapse in the volume traded in private markets, i.e., a *financial crisis*. Assets in the economy derive value from the dividend they pay and the liquidity services they provide. As a consequence, the supplies of privately produced assets and government bonds (i.e., private and public liquidity) interact through an endogenously determined liquidity premium.

This model provides a useful laboratory to study the optimal policy *mix*, in which the planner can directly intervene in the private markets or it actively manage the amount of public liquidity it provides. I showed that the optimal policy can be implemented with three instruments: state-contingent one-period government bonds, asset purchase programs, and transaction (or *Tobin*) taxes. Notably, optimal policy does not rule out the possibility of a financial crisis, but it aggressively increases the supply of public liquidity in such an event, in order to mitigate sharp variations in the total amount of liquidity. When the optimal policy prescribes supporting the private market, the planner chooses a combination of a high market wedge and a low rate of return (by providing a low level of public liquidity), which boosts asset prices. Moreover, if the planner has imperfect commitment, it finds it optimal to distort its choices in period 0 to better align its incentives in period 1.

An essential feature of the solution is the need of a state-contingent provision of public liquidity, which I model as state-contingent one-period government bonds. An interesting alternative would be to determine whether a state-contingent monetary policy that manages the market value of non-contingent bonds can play the same role. I conjecture that *conventional* policy alone would not be sufficient, but policies like "Operation Twist," which affects the *composition* of government debt, might provide the additional degree of freedom necessary for the implementation. Studying this would require to build a model with government debt of multiple maturities, and frictions that make these different assets imperfect substitutes. I plan to tackle these issues in future work.

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References

Akerlof, George A, "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 1970, *84* (3), 488–500.

Angeletos, George-Marios, Fabrice Collard, and Harris Dellas, "Public Debt as Private Liquidity: Optimal Policy," 2016.

Ashcraft, Adam B and Til Schuermann, "Understanding the Securitization of Subprime Mortgage Credit," *Foundations and Trends® in Finance*, 2008, *2* (3), 191–309.

Bank for International Settlements, "Collateral in Wholesale Financial Markets: Recent Trends, Risk Management and Market Dynamics," *Committee on the Global Financial System*, 2001.

Barclays Capital, "Equity Gilt Study 2012," 2012.

Bewley, Truman, "A difficulty with the optimum quantity of money," *Econometrica: Journal of the Econometric Society*, 1983, pp. 1485–1504.

Bianchi, Javier, "Efficient bailouts?," *American Economic Review*, 2016, *106* (12), 3607–59.

Bigio, Saki, "Endogenous Liquidity and The Business Cycle," *American Economic Review*, 2015, *105* (6), 1883–1927.

Caballero, Ricardo and Emmanuel Farhi, "The safety trap," *The Review of Economic Studies*, 2018, *85* (1), 223–274.

Caballero, Ricardo J, "On the Macroeconomics of Asset Shortages," *The Role of Money: Money and Monetary Policy in the Twenty-First Century, The Fourth European Central Banking Conference 9-10 November 2006, Andreas Beyer and Lucrezia Reichlin, editors.*, 2006, pp. 272–83.

_ , "The "Other" Imbalance and the Financial Crisis," *NBER Working Paper No. 15636*, 2010.

_ **, Emmanuel Farhi, and Pierre-Olivier Gourinchas**, "The safe assets shortage conundrum," *Journal of Economic Perspectives*, 2017, *31* (3), 29–46.

Calvo, Guillermo, "Puzzling over the Anatomy of Crises: Liquidity and the Veil of Finance," *Working Paper*, 2013.

Caramp, Nicolas, "Sowing the Seeds of Financial Crises: Endogenous Asset Creation and Adverse Selection," *Mimeo*, 2017.

Chari, V V, Ali Shourideh, and Ariel Zetlin-Jones, "Reputation and Persistence of Adverse Selection in Secondary Loan Markets," *American Economic Review*, 2014, *104* (12).

Chemla, Gilles and Christopher A Hennessy, "Skin in the game and moral hazard," *The Journal of Finance*, 2014, *69* (4), 1597–1641.

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki, "The great escape? A quantitative evaluation of the Fed's liquidity facilities," *American Economic Review*, 2017, *107* (3), 824–57.

Demiroglu, Cem and Christopher James, "How Important is Having Skin in the Game? Originator-Sponsor Affiliation and Losses on Mortgage-Backed Securities," *Review of Financial Studies*, 2012, *25* (11), 3217–58.

Downing, Chris, Dwight Jaffee, and Nancy Wallace, "Is the Market for Mortgage-Backed Securities a Market for Lemons?," *Review of financial Studies*, 2009, *22* (7), 2457–2494.

Eisfeldt, Andrea L, "Endogenous Liquidity in Asset Markets," *The Journal of Finance*, 2004, *59* (1), 1–30.

Fukui, Masao, "Asset Quality Cycles," *Journal of Monetary Economics*, 2018, *95*, 97–108.

Geanakoplos, John, "The leverage cycle," in "NBER Macroeconomics Annual 2009, Volume 24," University of Chicago Press, 2010, pp. 1–65.

Geromichalos, Athanasios, Juan Manuel Licari, and José Suárez-Lledó, "Monetary policy and asset prices," *Review of Economic Dynamics*, 2007, *10* (4), 761–779.

Gorton, Gary, "The history and economics of safe assets," *Annual Review of Economics*, 2017, *9*, 547–586.

_ and Guillermo Ordoñez, "The Supply and Demand for Safe Assets," *NBER Working Paper No. 18732*, 2013.

_ , Stefan Lewellen, and Andrew Metrick, "The Safe-Asset Share," *American Economic Review: Papers & Proceedings*, 2012, *102* (3), 101–106.

Greenwood, Robin, Samuel G Hanson, and Jeremy C Stein, "A Comparative-Advantage Approach to Government Debt Maturity," *The Journal of Finance*, 2015, *70* (4), 1683–1722.

Holmström, Bengt and Jean Tirole, "Private and Public Supply of Liquidity," *Journal of Political Economy*, 1998, *106* (1), 1–40.

_ and _ , "LAPM: A Liquidity-Based Asset Pricing Model," *The Journal of Finance*, 2001, *56* (5), 1837–67.

Keys, Benjamin, Tanmoy Mukherjee, Amit Seru, and Vikrant Vig, "Did Securitization Lead to Lax Screening? Evidence from Subprime Loans," *The Quarterly Journal of Economics*, 2010, *125* (1), 307–62.

Kiyotaki, Nobuhiro and John Moore, "Liquidity, Business Cycles, and Monetary Policy," *NBER Working Paper No. 17934*, 2012, pp. 1–59.

Krainer, John and Elizabeth Laderman, "Mortgage Loan Securitization and Relative Loan Performance," *Journal of Financial Services Research*, 2014, *45* (1), 39–66.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 2012, *120* (2), 233–67.

_ and _ , "The Impact of Treasury Supply on Financial Sector Lending and Stability," *Journal of Financial Economics*, 2015, *118* (3), 571–600.

Kurlat, Pablo, "Lemons Markets and the Transmission of Aggregate Shocks," *American Economic Review*, June 2013, *103* (4), 1463–89.

Lorenzoni, Guido, "Inefficient Credit Booms," *The Review of Economic Studies*, 2008, *75* (3), 809–33.

Luenberger, David G, "Optimization by vector space methods," 1969.

Neuhann, Daniel, "Macroeconomic Effects of Secondary Market Trading," *Working Paper*, 2017.

Parlour, Christine A and Guillaume Plantin, "Loan sales and relationship banking," *The Journal of Finance*, 2008, *63* (3), 1291–1314.

Philippon, Thomas and Vasiliki Skreta, "Optimal Interventions in Markets with Adverse Selection," *The American Economic Review*, 2012, *102* (1), 1–28.

Piskorski, Tomasz, Amit Seru, and James Witkin, "Asset Quality Misrepresentation by Financial Intermediaries: Evidence from the RMBS Market," *The Journal of Finance*, 2015, *70* (6), 2635–78.

Simsek, Alp, "Belief Disagreements and Collateral Constraints," *Econometrica*, 2013, *81* (1), 1–53.

Sunderam, Adi, "Money Creation and the Shadow Banking System," *Review of Financial Studies*, 2015, *28* (4), 939–77.

Tirole, Jean, "Overcoming Adverse Selection: How Public Intervention can Restore Market Functioning," *American Economic Review*, 2012, *102* (1), 29–59.

Vanasco, Victoria, "The downside of asset screening for market liquidity," *The Journal of Finance*, 2017, *72* (5), 1937–1982.

Woodford, Michael, "Public Debt as Private Liquidity," *American Economic Review: Papers & Proceedings*, 1990, *80* (2), 382–88.

A Proofs

Proof of Proposition 1. The first-order conditions associated to the first best program are:

$$\begin{aligned} (c_1(\mu,\alpha)): &\quad \mu \le \gamma_1(\alpha) \\ (c_2(\mu,\alpha)): &\quad 1 \le \gamma_2(\alpha) \\ (H_G): &\quad E[Z\gamma_2(\alpha)] \le \gamma_0 C'(H_G) \\ (H_B): &\quad E[\alpha Z\gamma_2(\alpha)] \le \gamma_0 \end{aligned}$$

where γ_0, $\gamma_1(\alpha)$ and $\gamma_2(\alpha)$ are the Lagrange multipliers associated to the resource constraint in periods 0, 1 and 2, respectively. Thus,

$$\begin{aligned} \gamma_0 &= \frac{Z}{C'(C^{-1}(W))} > E[\alpha Z], \\ \gamma_1(\alpha) &= \mu^{\max} \\ \gamma_2(\alpha) &= 1 \end{aligned}$$

where the inequality holds by Assumption 1. Thus, in the first best allocation only good trees are produced and agents with $\mu = \mu^{\max}$ consume all the endowment in period 1. Any allocation of the consumption good in period 2 is consistent with first best.

□

Proof of Lemma 1. Agents have only two uses for their endowment in period 1: either they consume, which delivers a utility of μ per unit, or they buy assets in the market, which delivers a return of r_M per unit spent. Thus, agents' valuation of a unit of the consumption good in period 1, state α, is $\max\{\mu, r_M(X)\}$, or, using that $\mu_B(X) = r_M(X)$, $\max\{\mu, \mu_B(X)\}$.

Since bad trees are always sold, a unit of bad trees delivers a utility of $\max\{\mu, \mu_B(X)\}P_M(X)$. In contrast, good trees are only sold if the market price is sufficiently high relative to the agents' shock μ, so that the valuation is $\max\{\mu P_M(X), Z\}$. Finally, since there is no frictions in the market for government bonds, they are valued as if they were always sold, $\max\{\mu, \mu_B(X)\}Q_B(X) = \frac{\max\{\mu,\mu_B(X)\}}{\mu_B(X)}$.

□

Proof of Proposition 2. Using that $\mu_B(X)P_M(X) = \lambda_M(X)Z + (1-\lambda_M(X))\alpha Z$, we have

$$\frac{\partial \gamma_G(P_M)}{\partial P_M(X)} = \int_{\mu_S(X)}^{\mu^{\max}} \mu dG(\mu) f(\alpha)$$

$$\frac{\partial \gamma_B(P_M)}{\partial P_M(X)} = \left[G(\mu_B(X))(1-\alpha)Z\frac{\partial \lambda_M(X)}{\partial P_M(X)} + \int_{\mu_B(X)}^{\mu^{\max}} \mu dG(\mu) \right] f(\alpha)$$

where

$$\frac{\partial \lambda_M(X)}{\partial P_M(X)} = \frac{\lambda_M(X)(1-\lambda_M(X))}{P_M(X)} \frac{g\left(\frac{Z}{P_M(X)}\right)\frac{Z}{P_M(X)}}{1-G\left(\frac{Z}{P_M(X)}\right)} > 0.$$

Hence, $\frac{\partial \gamma_B(P_M)}{\partial P_M(X)} > \frac{\partial \gamma_G(P_M)}{\partial P_M(X)} \ge 0$. Since $\gamma_B < \gamma_G$, the result holds.

□

Proof of Lemma 2. The agents' problem in period 0 can be written as

$$\max_{h_G, h_B, b} \gamma_G h_G + \gamma_B h_B + \gamma_{GB} b$$

subject to

$$h_B + C(h_G) + Q_0^B b \leq W + T_0$$
$$h_G \geq 0, \quad h_B \geq 0, \quad b \geq 0$$

where $\gamma_j \equiv E[\widetilde{\gamma}_j(\mu, X)]$ for $j \in \{G, B, GB\}$. Let κ be the Lagrange multiplier associated to the budget constraint. The first-order conditions are given by

$$\gamma_G - \kappa C'(h_G) \leq 0$$
$$\gamma_B - \kappa \leq 0$$
$$\gamma_{GB} - \kappa Q_0^B \leq 0$$

Assuming that $\frac{\gamma_G}{C'(W_0)} < \gamma_B < \frac{\gamma_G}{C'(0)}$, we get

$$\frac{\gamma_G}{\gamma_B} = C'(H_G).$$

Moreover, since $T_0 = Q_0^B B$, we have

$$H_B = W - C(H_G).$$

Next, note that, if $H_B > 0$, we have

$$\frac{\partial H}{\partial P_M(X)} = \frac{\partial H_G}{\partial P_M(X)} + \frac{\partial H_B}{\partial P_M(X)}$$

where

$$\frac{\partial H_B}{\partial P_M(X)} = -C'(H_G) \frac{\partial H_G}{\partial P_M(X)}$$

and

$$\frac{\partial H_G}{\partial P_M(X)} = \frac{\frac{\partial \gamma_G}{\partial P_M(X)} - \frac{\partial \gamma_B}{\partial P_M(X)}}{\frac{C''(H_G)}{C'(H_G)}}$$

Since $\frac{\partial \gamma_G}{\partial P_M(X)} - \frac{\partial \gamma_B}{\partial P_M(X)} < 0$, $\frac{\partial H_G}{\partial P_M(X)} < 0$. Moreover, since $C'(H_G) = \frac{\gamma_G}{\gamma_B} > 1$, $\frac{\partial H}{\partial P_M(X)} > 0$. Finally,

$$\frac{\partial \lambda_E}{\partial P_M(X)} \propto \frac{\partial H_G}{\partial P_M(X)} - \lambda_E \frac{\partial H}{\partial P_M(X)} < 0.$$

□

Proof of Proposition 3. It will be useful to work with a market for *tree quality*. Define the *supply of tree quality* as

$$\lambda_M^S = \frac{\left[1 - G\left(\frac{Z}{P_M}\right)\right] H_G}{\left[1 - G\left(\frac{Z}{P_M}\right)\right] H_G + H_B}$$

and the *demand for tree quality* as

$$\lambda_M^D = \frac{\mu_B(P_M)P_M - \alpha Z}{(1-\alpha)Z}$$

where $\mu_B(P_M)$ is the implicit function defined as the solution to

$$G(\mu_B)W = [1-G(\mu_B)]H_B P_M + \left[1-G\left(\frac{Z}{P_M}\right)\right]H_G P_M + [1-G(\mu_B)]\frac{B_0}{\mu_B}$$

First note that from the supply of tree quality, we have

$$\lambda_M^S = 0 \iff P_M \leq \frac{Z}{\mu^{\max}} \quad \text{and} \quad \lambda_M^S = \lambda_E \iff P_M \geq Z$$

while from the demand for tree quality, we have

$$\lambda_M^D = 0 \iff P_M < \alpha Z \quad \text{and} \quad \lambda_M^D = \lambda_E \iff P_M < Z$$

Hence, an intersection always exists. Since λ_M^S and λ_M^D are continuous, the set of intersections is compact, so a maximum volume of trade equilibrium exists and is unique.

Next, I show that in the maximum volume of trade equilibrium, $\frac{\partial \lambda_M^D}{\partial P_M} \geq \frac{\partial \lambda_M^S}{\partial P_M}$. Suppose, to the contrary, that $\frac{\partial \lambda_M^D}{\partial P_M} < \frac{\partial \lambda_M^S}{\partial P_M}$. We know that $\left(\lambda_M^D\right)^{-1}(\lambda_E) < \left(\lambda_M^S\right)^{-1}(\lambda_E)$. This means that if $\frac{\partial \lambda_M^D}{\partial P_M} < \frac{\partial \lambda_M^S}{\partial P_M}$ at the equilibrium price P_M^*, there exists $\widetilde{P}_M > P_M^*$ such that $\lambda_M^D = \lambda_M^S$. But this would imply that P_M^* is not part of the maximum volume of trade equilibrium, a contradiction.

Next, note that

$$\frac{\partial \mu_B}{\partial P_M} = \frac{[1-G(\mu_B)]H_B + \left[1-G\left(\frac{Z}{P_M}\right)\right]H_G + g\left(\frac{Z}{P_M}\right)\frac{Z}{P_M}H_G}{g(\mu_B)\left[W + H_B P_M + \frac{B_0}{\mu_B} + \frac{1-G(\mu_B)}{g(\mu_B)\mu_B}\frac{B_0}{\mu_B}\right]} > 0$$

Thus, since λ_M^D is decreasing in α and $\frac{\partial \lambda_M^D}{\partial P_M} \geq \frac{\partial \lambda_M^S}{\partial P_M}$, P_M, λ_M and μ_B are increasing in α. Similarly, since μ_B is decreasing in λ_E as a function of P_M, λ_M^D is decreasing in λ_E while λ_M^S is increasing. Since $\frac{\partial \lambda_M^D}{\partial P_M} \geq \frac{\partial \lambda_M^S}{\partial P_M}$, P_M and λ_M are increasing in λ_E. To see that μ_B is also increasing in λ_E, rewrite the market clearing condition as

$$G(\mu_B)W_1 = \left[\frac{1}{1-\lambda_M} - G(\mu_B)\right]H_B P_M + [1-G(\mu_B)]\frac{B_0}{\mu_B}$$

The LHS is increasing in μ_B, while the RHS is decreasing in μ_B. Moreover, the RHS is increasing in λ_M and P_M, hence μ_B is increasing in λ_E.

Finally, I need to show that, under Assumption 2, $P_M(X)$ is discontinuous at α^*.

Note that

$$\frac{\partial \lambda_M^S}{\partial P_M} = \frac{\lambda_E(1-\lambda_E)g\left(\frac{Z}{P_M}\right)\frac{Z}{P_M^2}}{\left[\left[1-G\left(\frac{Z}{P_M}\right)\right]\lambda_E + (1-\lambda_E)\right]^2}$$

and

$$\frac{\partial^2 \lambda_M^S}{\partial P_M^2} = -\frac{\left[\frac{g'\left(\frac{Z}{P_M}\right)}{g\left(\frac{Z}{P_M}\right)}\frac{Z}{P_M^2} + 2\frac{Z}{P_M}\right]\left[\left[1 - G\left(\frac{Z}{P_M}\right)\right]\lambda_E + (1-\lambda_E)\right] + 2g\left(\frac{Z}{P_M}\right)\frac{Z}{P_M^2}\lambda_E}{\left[1 - G\left(\frac{Z}{P_M}\right)\right]\lambda_E + (1-\lambda_E)}\frac{\partial \lambda_M^S}{\partial P_M} < 0$$

since $g'(\cdot) \geq 0$. Thus, I just need to show that $\frac{\partial^2 \lambda_M^D}{\partial P_M^2} > 0$ as $P_M \to \frac{Z}{\mu^{\max}}^-$ for states that satisfy $\frac{\alpha Z}{\mu_B} < \frac{Z}{\mu^{\max}}$, with μ_B the solution to

$$G(\mu_B) W = [1 - G(\mu_B)]\left[H_B \frac{\alpha Z}{\mu_B} + \frac{B_0}{\mu_B}\right]$$

The rest follows from the fact that P_M is increasing in α. We have that

$$\frac{\partial \lambda_M^D}{\partial P_M} = \frac{\mu_B + \frac{\partial \mu_B}{\partial P_M} P_M}{(1-\alpha) Z}$$

For $P_M \to \frac{Z}{\mu^{\max}}^-$, we have

$$\frac{\partial \mu_B}{\partial P_M} = \frac{[1 - G(\mu_B)] H_B}{g(\mu_B)\left[W + H_B P_M + \frac{B_0}{\mu_B} + \frac{1-G(\mu_B)}{g(\mu_B)\mu_B}\frac{B_0}{\mu_B}\right]}$$

and therefore

$$\mu_B + \frac{\partial \mu_B}{\partial P_M} P_M = \mu_B + \frac{[1 - G(\mu_B)] H_B}{\frac{g(\mu_B)}{G(\mu_B)} H_B + \Omega(P_M)}$$

where $\Omega(P_M) \equiv \left[\frac{g(\mu_B)}{G(\mu_B)} + \frac{1-G(\mu_B)}{\mu_B}\right]\frac{B_0}{\mu_B P_M}$. Clearly, $\Omega'(P_M) < 0$. Hence

$$\frac{\partial\left(\mu_B + \frac{\partial \mu_B}{\partial P_M} P_M\right)}{\partial P_M} = \frac{\frac{g(\mu_B)}{G(\mu_B)} H_B - g(\mu_B) H_B + \Omega(P_M)}{\left[\frac{g(\mu_B)}{G(\mu_B)} H_B + \Omega(P_M)\right]}\frac{\partial \mu_B}{\partial P_M} - \frac{[1 - G(\mu_B)] H_B \frac{\partial\left(\frac{g(\mu_B)}{G(\mu_B)} H_B + \Omega(P_M)\right)}{\partial P_M}}{\left[\frac{g(\mu_B)}{G(\mu_B)} H_B + \Omega(P_M)\right]^2} > 0$$

since, by Assumption 2, $\frac{\partial\left(\frac{g(\mu_B)}{G(\mu_B)} H_B + \Omega(P_M)\right)}{\partial P_M} < 0$.

□

Proof of Lemma 3. Since the cdf of α, F, is continuous, the shadow values of good and bad trees are continuous in λ_E even if market prices are discontinuous in the state of the economy. Because H_G is a continuous function of the shadow values, the function T is continuous in λ_E. Moreover, since prices are increasing in λ_E, Lemma 2 implies that H_G is decreasing in λ_E, so T is decreasing in λ_E. Therefore, by Brouwer fixed-point theorem, a fixed-point of T exists and is unique.

That $\lambda_E \in (0, 1)$ follows from Assumption 1 and that $C'(W_0) > 1$. □

Proof of Proposition 4. Note that the change in the distribution of F has no effect on the equilibrium in period 1 as long as λ_E doesn't change, since α does not affect the decision to produce tree quality directly but only through its effect on the shadow values.

We have

$$\gamma_G = \int_{\underline{\alpha}}^{\overline{\alpha}} \left[G\left(\frac{Z}{P_M(X)}\right) Z + \int_{\frac{Z}{P_M(X)}}^{\mu^{\max}} \mu P_M(X) dG(\mu) \right] dF(\alpha|\theta)$$

$$\gamma_B = \int_{\underline{\alpha}}^{\overline{\alpha}} \left[G\left(\hat{\mu}_B(P_M(X), X)\right) \hat{\mu}_B(P_M(X), X) P_M(X) + \int_{\hat{\mu}_B(P_M(X),X)}^{\mu^{\max}} \mu P_M(X) dG(\mu) \right] dF(\alpha|\theta)$$

where $\hat{\mu}_B(P_M(X), X)$ is implicitly defined as the solution to

$$G(\mu_B)\left[W + (1-\lambda_E) H P_M(X)\right] = [1 - G(\mu_B)](1-\lambda_E) H P_M(X) + \left[1 - G\left(\frac{Z}{P_M(X)}\right)\right] \lambda_E H P_M(X) + [1 - G(\mu_B)] \frac{B_0}{\mu_B}$$

and $P_M(X)$ is then the solution to

$$P_M = \frac{\lambda_M(P_M, X) Z + (1 - \lambda_M(P_M, X) \alpha Z}{\hat{\mu}_B(P_M, X)}$$

with

$$\lambda_M(P_M, X) = \frac{\left[1 - G\left(\frac{Z}{P_M}\right)\right] \lambda_E}{\left[1 - G\left(\frac{Z}{P_M}\right)\right] \lambda_E + (1-\lambda_E)}$$

Thus, since P_M is increasing in α for any value of λ_E, the increase in $F(\cdot|\theta)$ to $F(\cdot|\theta')$ is mathematically equivalent to an increase in prices in each state α by $\phi(\alpha) \geq 0$. By Lemma 2, an increase in prices reduces $H_G(\lambda_E)$, so that the function $T(\lambda_E)$ decreases for all λ_E. Hence, the fixed point $\lambda_{E*} = T(\lambda_{E*})$ decreases.

Because bad trees are cheaper to produce than good trees, the total production of trees increases. Moreover, because λ_E decreases, equilibrium prices decrease in all states, so the threshold α^* increases. Finally, market fragility is ambiguous since the change in F reduces it but the endogenous change in λ_E increases it. □

Proof of Lemma 4. First, note that an increase in B_0 reduces the net demand for trees. For states with $\alpha < \alpha^*$, the result is an increase in $\mu_B = r_M$, a drop in the price of trees P_M, but an increase in total liquidity, since total demand for assets is higher. For states with $\alpha \geq \alpha^*$, the response depends on on whether α is in the neighborhood of α^*. Consider the state $\alpha = \alpha^*$. The increase in B_0 pushes the demand down, so that an equilibrium in the market for trees with $\lambda_M > 0$ ceases to exist. In that case, P_M discontinuously drops. If the change in B_0 is small, then total liquidity decreases (since private liquidity drops discretely). Finally, if α is sufficiently higher than α^*, the drop in demand does not trigger a market collapse, so $\mu_B = r_M$ increases and total liquidity increases. Since in all cases P_M decreases, λ_M also decreases.

□

Proof of Proposition 5. From Lemma 4 we know that, given $\{\lambda_E, H\}$, P_M and $\mu_B P_M = \lambda_M Z + (1 - \lambda_M)\alpha Z$ decrease in all states. Thus, $H_G(\lambda_E)$ increases and the function T increases. Therefore, equilibrium λ_E increases. The effect on market fragility is

$$\frac{dMF}{dB_0} \propto \underbrace{\frac{\partial \alpha^*}{\partial B_0}}_{>0} + \underbrace{\frac{\partial \alpha^*}{\partial \lambda_E}}_{<0} \underbrace{\frac{\partial \lambda_E}{\partial B_0}}_{>0}$$

If $\frac{\partial \lambda_E}{\partial B_0} \simeq 0$, then fragility increases. If $\frac{\partial \lambda_E}{\partial B_0} > 0$ and $\frac{\partial \alpha^*}{\partial B_0}$ is not too high, fragility decreases.

□

Proof of Lemma 5. Agents' consumptions in periods 1 and 2 are given by

$$\begin{aligned} c_1(\mu, \alpha) &= W + T_1(\mu, \alpha) - P_B(\alpha) m(\mu, \alpha) + P_S(\alpha) \left[s_B(\mu, \alpha) + s_G(\mu, \alpha) \right] \\ c_2(\mu, \alpha) &= W + T_2(\mu, \alpha) + Z h_G(\mu, \alpha) + \alpha Z h_B(\mu, \alpha) \end{aligned}$$

The IC constraints can then be written as

$$\begin{aligned} &\mu \left[T_1(\mu, \alpha) - P_B(\alpha) m(\mu, \alpha) \right] + T_2(\mu, \alpha) + Z h_G(\mu, \alpha) + \alpha Z h_B(\mu, \alpha) \geq \\ &\qquad \mu \left[T_1(\hat{\mu}, \alpha) - P_B(\alpha) m(\hat{\mu}, \mu, \alpha) \right] + T_2(\hat{\mu}, \alpha) + Z h_G(\hat{\mu}, \mu, \alpha) + \alpha Z h_B(\hat{\mu}, \mu, \alpha) \quad \forall \mu, \hat{\mu}, \alpha \end{aligned}$$

where $\{m(\hat{\mu}, \mu, \alpha), h_G(\hat{\mu}, \mu, \alpha), h_B(\hat{\mu}, \mu, \alpha)\}$ reflect the optimal choice of an agent of type μ that announces to be $\hat{\mu}$. Consider the agents with $\mu < \mu_B(\alpha)$. These agents do not consume in period 1. Therefore, their IC constraints are

$$Z \underbrace{\left[h_G(\mu, \alpha) - h_G(\hat{\mu}, \mu, \alpha) \right]}_{= \frac{\lambda_M(\alpha)}{P_B(\alpha)} \left[T_1(\mu, \alpha) - T_1(\hat{\mu}, \alpha) \right]} + \alpha Z \underbrace{\left[h_B(\mu, \alpha) - h_B(\hat{\mu}, \mu, \alpha) \right]}_{= \frac{1 - \lambda_M(\alpha)}{P_B(\alpha)} \left[T_1(\mu, \alpha) - T_1(\hat{\mu}, \alpha) \right]} \geq T_2(\hat{\mu}, \alpha) - T_2(\mu, \alpha)$$

where I used that these agents use all their transfers to buy trees. Noting that $\frac{\lambda_M(\alpha) Z + (1 - \lambda_M(\alpha)) \alpha Z}{P_B(\alpha)} = \mu_B(\alpha)$, we can rewrite the IC constraints as

$$\mu_B(\alpha) \left[T_1(\mu, \alpha) - T_1(\hat{\mu}, \alpha) \right] \geq T_2(\hat{\mu}, \alpha) - T_2(\mu, \alpha)$$

Thus, all agents with $\mu < \mu_B(\alpha)$ face the same IC constraints.

Consider now the agents with $\mu > \mu_B(\alpha)$. Their IC constraints simplify to

$$\mu \left[T_1(\mu, \alpha) - T_1(\hat{\mu}, \alpha) \right] \geq T_2(\hat{\mu}, \alpha) - T_2(\mu, \alpha)$$

since for these agents $m(\mu, \alpha) = 0$, $h_G(\mu, \alpha) = h_G(\hat{\mu}, \mu, \alpha)$ and $h_B(\mu, \alpha) = h_B(\hat{\mu}, \mu, \alpha)$. Note that these IC constraints imply that $T_1(\mu, \alpha)$ is increasing in μ and $T_2(\mu, \alpha)$ is decreasing in μ. To see this, take $\mu'' > \mu' \geq \mu_B(\alpha)$ (note that $\mu' < \mu_B(\alpha)$ is equivalent to $\mu' = \mu_B(\alpha)$). Then, the IC constraints for these agents imply

$$\begin{aligned} \mu'' \left[T_1(\mu'', \alpha) - T_1(\mu', \alpha) \right] &\geq T_2(\mu', \alpha) - T_2(\mu'', \alpha) \\ \mu' \left[T_1(\mu', \alpha) - T_1(\mu'', \alpha) \right] &\geq T_2(\mu'', \alpha) - T_2(\mu', \alpha) \end{aligned}$$

Adding up the two constraints, we get

$$(\mu'' - \mu') \left[T_1(\mu'', \alpha) - T_1(\mu', \alpha) \right] \geq 0,$$

thus, $T_1(\mu'', \alpha) \geq T_1(\mu', \alpha)$. Moreover,

$$T_2(\mu', \alpha) \geq T_2(\mu'', \alpha) + \mu' \left[T_1(\mu'', \alpha) - T_1(\mu', \alpha)\right] \geq T_2(\mu'', \alpha),$$

thus $T_2(\mu'', \alpha) \leq T_2(\mu', \alpha)$.

Next, I show that it is (weakly) optimal to assign the same transfers to all agents with $\mu < \mu_B(\alpha)$. Suppose to the contrary that there exists $\mu', \mu'' < \mu_B(\alpha)$ such that $T_1(\mu'', \alpha) > T_1(\mu', \alpha)$. The IC constraints imply that $T_2(\mu', \alpha) > T_2(\mu'', \alpha)$. Consider the following alternative transfers for both agents:

$$\tilde{T}_1(\alpha) = \frac{T_1(\mu', \alpha) + T_1(\mu'', \alpha)}{2}$$
$$\tilde{T}_2(\alpha) = \frac{T_2(\mu', \alpha) + T_2(\mu'', \alpha)}{2}$$

It is immediate to see that the utility of the agents and the resources involved do not change. Moreover, $\{\tilde{T}_1(\alpha), \tilde{T}_2(\alpha)\}$ relax the IC constraints for agents with $\mu > \mu_B(\alpha)$ since $\tilde{T}_1(\alpha) < T_1(\mu'', \alpha)$. Thus, equal treatment of agents with $\mu < \mu_B(\alpha)$ is optimal. Let $\underline{T}_1(\alpha) \equiv T_1(\mu, \alpha)$ for $\mu < \mu_B(\alpha)$.

Next, I show that there exists $\tilde{\mu}(\alpha) \geq \mu_B(\alpha)$ such that $T_1(\mu, \alpha) = T_1(\mu^{\max}, \alpha) \equiv \overline{T}_1(\alpha)$ and $T_2(\mu, \alpha) = T_2(\mu^{\max}, \alpha) \equiv \overline{T}_2(\alpha)$ for all agents with $\mu > \tilde{\mu}(\alpha)$. If $T_1(\mu^{\max}, \alpha) = \underline{T}_1(\alpha)$, the result is immediate from the fact that $T_1(\mu, \alpha)$ is increasing in μ. Now, consider the case with $T_1(\mu^{\max}, \alpha) > \underline{T}_1(\alpha)$. Without loss of generality, assume that $T_1(\mu, \alpha) > \underline{T}_1(\alpha)$ for all $\mu > \tilde{\mu}(\alpha)$ (if it wasn't, we just need to redefine $\tilde{\mu}(\alpha)$). Suppose that there exists $\mu' > \tilde{\mu}(\alpha)$ such that $T_1(\mu', \alpha) \in (\underline{T}_1(\alpha), \overline{T}_1(\alpha))$. This implies that $T_2(\mu', \alpha) = \overline{T}_2(\alpha) + \mu'[\overline{T}_1(\alpha) - T_1(\mu', \alpha)]$. Take $\mu'' \in (\tilde{\mu}(\alpha), \mu')$. Since $T_1(\mu, \alpha)$ is increasing in μ, we have that $T_1(\mu'', \alpha) \in (\underline{T}_1(\alpha), T_1(\mu', \alpha)]$. Consider the following alternative transfers:

$$\tilde{T}_1(\mu'', \alpha) = \underline{T}_1(\alpha)$$
$$\tilde{T}_1(\mu', \alpha) = T_1(\mu', \alpha) + \left[T_1(\mu'', \alpha) - \underline{T}_1(\alpha)\right]$$
$$\tilde{T}_2(\mu'', \alpha) = T_2(\mu'', \alpha) + \mu'' \left[T_1(\mu'', \alpha) - \underline{T}_1(\alpha)\right]$$
$$\tilde{T}_2(\mu', \alpha) = T_2(\mu', \alpha) - \mu' \left[T_1(\mu'', \alpha) - \underline{T}_1(\alpha)\right]$$

These transfers increase the objective function, since they transfer consumption from low- to high-μ agents, and they reduce the transfer cost in period 2, since $\mu'' - \mu' < 0$. Thus, as long as $T_1(\mu, \alpha) < \overline{T}_1(\alpha)$ for some $\mu > \tilde{\mu}$, there is a reallocation that increases the objective function. The transfers in period 2 are immediate from the IC constraints and the planner's resource constraints. Moreover, note that $\mathcal{T}_2(\tilde{\mu}, \alpha) > 0$ only increases the cost of the transfers without any benefit for the planner.

Finally, I show that $\tilde{\mu}(\alpha) = \mu_B(\alpha)$. Suppose to the contrary that $\tilde{\mu}(\alpha) > \mu_B(\alpha)$. From the previous results, this can only happen if $T_1(\mu, \alpha) = \underline{T}_1(\alpha)$ for all $\mu \in (\mu_B(\alpha), \tilde{\mu}(\alpha))$. An active market for trees requires a positive demand, so $\underline{T}_1(\alpha) > -W$. Take $\mu', \mu'' \in (\mu_B(\alpha), \tilde{\mu}(\alpha))$, with $\mu' < \mu''$. Consider the following transfers:

$$\tilde{T}_1(\mu', \alpha) = \underline{T}_1(\alpha) - \varepsilon$$
$$\tilde{T}_1(\mu'', \alpha) = \underline{T}_1(\alpha) + \varepsilon$$
$$\tilde{T}_2(\mu', \alpha) = \underline{T}_2(\alpha) + \tilde{\mu}(\alpha)\varepsilon$$
$$\tilde{T}_2(\mu'', \alpha) = \underline{T}_2(\alpha) - \tilde{\mu}(\alpha)\varepsilon$$

for some $\varepsilon > 0$ but small so that $\tilde{T}_1(\mu', \alpha) > -W$ and $\tilde{T}_1(\mu'', \alpha) < \overline{T}_1(\alpha)$. These transfers have no impact on the demand or the supply in the market for trees since they do not affect the transfers of the buyers. Moreover, it is immediate to see that the transfers satisfy incentive compatibility, and they do not change the total transfers in period 2, but they increase the planner's objective function. Thus, the planner can keep doing this reallocation as long as $\underline{T}_1(\alpha) > -W$ and $\tilde{\mu}(\alpha) > \mu_B(\alpha)$.

It is immediate that $\mathcal{T}_2(\mu, \alpha) = 0$ for all $\mu > \mu_B(\alpha)$, as $\mathcal{T}_2(\mu, \alpha) > 0$ would only increase the cost for the planner without any benefit. Then, the expressions for $\mathbb{T}_2(\alpha)$ and $\underline{T}_1(\alpha)$ follow from the resource constraints.

□

Proof of Lemma 6. Immediate from the the definitions of $\{\mu_B(\alpha), \mu_S(\alpha)\}$ and the resource constraints.

□

Proof of Proposition 6. Consider the planner's problem from Lemma 6. Denote the solution by $\{\hat{P}_S(\alpha), \hat{\overline{T}}_1(\alpha)\}$. Suppose the solution satisfies $0 > \mathbb{T}_\mathbf{2}(\alpha) > -W$ for all α, and $\boldsymbol{U_1}(\alpha)$ and $\mathbb{T}_\mathbf{2}(\alpha)$ are differentiable at $\{\hat{P}_S(\alpha), \hat{\overline{T}}_1(\alpha)\}$. Consider the alternative policy

$$\tilde{P}_S(\alpha, \varepsilon) = \hat{P}_S(\alpha) + \epsilon\eta_1(\alpha) \quad \text{and} \quad \tilde{\overline{T}}_1(\alpha, \varepsilon) = \hat{\overline{T}}_1(\alpha) + \epsilon\eta_2(\alpha)$$

for some arbitrary functions $\eta_1(\cdot)$ and $\eta_2(\cdot)$ and ε sufficiently small so that the constraints on $\mathbb{T}_2(\cdot)$ are satisfied with strict inequality. Moreover, note that the determination of H_G implies

$$H_G(\varepsilon) = \boldsymbol{H_G}(\{\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon)\}_{\forall\alpha})$$

Define

$$\boldsymbol{W}(\varepsilon) \equiv \int_0^1 \Big[\boldsymbol{U_1}(\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon), H_G(\varepsilon)) + ZH_G(\varepsilon) + \alpha Z(W - C(H_G(\varepsilon))) + \frac{\chi}{1+\chi}\mathbb{T}_\mathbf{2}(\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon), H_G(\varepsilon))\Big] \, dF(\alpha)$$

Naturally, $W(\cdot)$ is maximized at $\varepsilon = 0$. Note that for any arbitrary scalar λ, we have

$$\lambda \left[\boldsymbol{H_G}(\{\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon)\}_{\forall\alpha}) - H_G(\varepsilon)\right] = 0$$

Adding this last expression to $W(\varepsilon)$, we get

$$\boldsymbol{W}(\varepsilon) = \int_0^1 \Big[\boldsymbol{U_1}(\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon), H_G(\varepsilon)) + ZH_G(\varepsilon) + \alpha Z(W - C(H_G(\varepsilon))) + \frac{\chi}{1+\chi}\mathbb{T}_\mathbf{2}(\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon), H_G(\varepsilon))\Big] \, dF(\alpha) + \lambda \left[\boldsymbol{H_G}(\{\tilde{P}_S(\alpha, \varepsilon), \tilde{\overline{T}}_1(\alpha, \varepsilon)\}_{\forall\alpha}) - H_G(\varepsilon)\right]$$

Differentiating with respect to ε, we get

$$\boldsymbol{W}'(\varepsilon) = \int_0^1 \left[\frac{\partial \boldsymbol{U_1}(\alpha,\varepsilon)}{\partial P_S(\alpha)}\eta_1(\alpha) + \frac{\partial \boldsymbol{U_1}(\alpha,\varepsilon)}{\partial \overline{T}_1(\alpha)}\eta_2(\alpha) + \frac{\partial \boldsymbol{U_1}(\alpha,\varepsilon)}{\partial H_G}H_G'(\varepsilon) + (1-\alpha C'(H_G(\varepsilon)))ZH_G'(\varepsilon) + \frac{\chi}{1+\chi}\left[\frac{\partial \mathbb{T}_2(\alpha,\varepsilon)}{\partial P_S(\alpha)}\eta_1(\alpha) + \frac{\partial \mathbb{T}_2(\alpha,\varepsilon)}{\partial \overline{T}_1(\alpha)}\eta_2(\alpha) + \frac{\partial \mathbb{T}_2(\alpha,\varepsilon)}{\partial H_G}H_G'(\varepsilon)\right]\right] dF(\alpha) + \lambda\left[\int_0^1 \left[\frac{\partial \boldsymbol{H_G}(\alpha,\varepsilon)}{\partial P_S(\alpha)}\eta_1(\alpha) + \frac{\partial \boldsymbol{H_G}(\alpha,\varepsilon)}{\partial \overline{T}_1(\alpha)}\eta_2(\alpha)\right] d\alpha - H_G'(\varepsilon)\right]$$

or

$$\boldsymbol{W}'(\varepsilon) = \int_0^1 \left[\left[\frac{\partial \boldsymbol{U_1}(\alpha,\varepsilon)}{\partial P_S(\alpha)} + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha,\varepsilon)}{\partial P_S(\alpha)}\right] f(\alpha) + \lambda \frac{\partial \boldsymbol{H_G}(\alpha,\varepsilon)}{\partial P_S(\alpha)}\right]\eta_1(\alpha) d\alpha + \int_0^1 \left[\left[\frac{\partial \boldsymbol{U_1}(\alpha,\varepsilon)}{\partial \overline{T}_1(\alpha)} + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2|(\alpha,\varepsilon)}{\partial \overline{T}_1(\alpha)}\right] f(\alpha) + \lambda \frac{\partial \boldsymbol{H_G}(\alpha,\varepsilon)}{\partial \overline{T}_1(\alpha)}\right]\eta_2(\alpha) d\alpha + \left[\int_0^1 \left[\frac{\partial \boldsymbol{U_1}(\alpha,\varepsilon)}{\partial H_G} + (1-\alpha C'(H_G(\varepsilon)))Z + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha,\varepsilon)}{\partial H_G} - \right] dF(\alpha) - \lambda\right] H_G'(\varepsilon)$$

Optimality requires that

$$\boldsymbol{W}'(0) = 0.$$

Since the choice of λ was arbitrary, I specialize to the following expression

$$\lambda = E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + (1-\alpha C'(H_G))Z - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial H_G}\right]$$

Moreover, since the choice of the functions $\eta_1(\cdot)$ and $\eta_2(\cdot)$ was also arbitrary, we obtain $W'(0) = 0$ if and only if

$$\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)}\right] f(\alpha) + E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + (1-\alpha C'(H_G))Z + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial H_G}\right]\frac{\partial \boldsymbol{H_G}(\alpha)}{\partial P_S(\alpha)} = 0$$

and

$$\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial \overline{T}_1(\alpha)}\right] f(\alpha) + E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + (1-\alpha C'(H_G))Z + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial H_G}\right]\frac{\partial \boldsymbol{H_G}(\alpha)}{\partial \overline{T}_1(\alpha)} = 0$$

□

Proof of Lemma 7. For $\boldsymbol{\lambda_E} \in (0,1)$, the total derivative of $\boldsymbol{H_G}$ is given by

$$\eta(\boldsymbol{H_G})d\boldsymbol{H_G} = \frac{d\boldsymbol{\gamma_G}}{\boldsymbol{\gamma_G}} - \frac{d\boldsymbol{\gamma_B}}{\boldsymbol{\gamma_B}}$$

where $\eta_G(\boldsymbol{H_G}) = \frac{C''(\boldsymbol{H_G})}{C'(\boldsymbol{H_G})}$, and

$$d\boldsymbol{\gamma_G} = \int_0^1 \frac{\partial \overline{\gamma}_G(\alpha)}{\partial P_S(\alpha)} dP_S(\alpha)\, dF(\alpha) = \int_0^1 \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu)\, dP_S(\alpha)\, dF(\alpha)$$

$$d\boldsymbol{\gamma_B} = \int_0^1 \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} dP_S(\alpha) dF(\alpha) + \int_0^1 \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)} d\overline{T}_1(\alpha) dF(\alpha) + \int_0^1 \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} d\boldsymbol{H_G} dF(\alpha)$$
$$= \int_0^1 \left[\left[G\left(\boldsymbol{\mu_B}(\alpha)\right) P_S(\alpha) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)} dP_S(\alpha) + \int_0^{\mu^{\max}} \max\left\{\boldsymbol{\mu_B}\left(\alpha\right), \mu\right\} dG\left(\mu\right)\right] dP_S\left(\alpha\right) + G\left(\boldsymbol{\mu_B}(\alpha)\right) P_S(\alpha) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial \overline{T}_1(\alpha)} d\overline{T}_1(\alpha)\right] dF(\alpha) + \int_0^1 G\left(\boldsymbol{\mu_B}(\alpha)\right) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial H_G} P_S\left(\alpha\right) dF\left(\alpha\right) dH_G$$

for $\{P_S(\alpha), \overline{T}_1(\alpha)\}$ where $\overline{\gamma}_{\boldsymbol{G}}(\alpha)$ and $\overline{\gamma}_{\boldsymbol{B}}(\alpha)$ are differentiable. Putting everything together, we get

$$d\boldsymbol{H_G} = \frac{\int_0^1 \left[\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_{\boldsymbol{G}}} - \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_{\boldsymbol{B}}}\right] dP_S(\alpha) dF(\alpha)}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} \frac{1}{\gamma_{\boldsymbol{B}}}\right]} - \frac{\int_0^1 \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)} \frac{1}{\gamma_{\boldsymbol{B}}} d\overline{T}_1(\alpha) dF(\alpha)}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} \frac{1}{\gamma_{\boldsymbol{B}}}\right]},$$

hence

$$\frac{\partial \boldsymbol{H_G}}{\partial P_S\left(\alpha\right)} = \frac{\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_{\boldsymbol{G}}} - \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_{\boldsymbol{B}}}}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} \frac{1}{\gamma_{\boldsymbol{B}}}\right]} f\left(\alpha\right) \quad \text{and} \quad \frac{\partial \boldsymbol{H_G}}{\partial \overline{T}_1\left(\alpha\right)} = -\frac{\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)} \frac{1}{\gamma_{\boldsymbol{B}}}}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} \frac{1}{\gamma_{\boldsymbol{B}}}\right]} f\left(\alpha\right),$$

Suppose $\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right] \frac{1}{\gamma_{\boldsymbol{B}}} < 0$. Then, the incentives effect is positive: as long as $\boldsymbol{\lambda_E} \in (0,1)$, increasing the price $P_S(\alpha)$ increases the fraction of good trees. To complete the proof, I will show that, in this case, $\frac{d\omega(\alpha)}{dP_S(\alpha)} < 0$, which would imply that $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} < 0$, and, therefore, an interior solution cannot exist.

First, note that

$$dP_S\left(\alpha\right) = \boldsymbol{P_B}\left(\alpha\right) d\omega\left(\alpha\right) + \left(1 + \omega\left(\alpha\right)\right) dP_B\left(\alpha\right)$$

hence

$$\frac{d\omega\left(\alpha\right)}{dP_S\left(\alpha\right)} = \frac{1}{\boldsymbol{P_B}\left(\alpha\right)} \left[1 - \frac{P_S\left(\alpha\right)}{\boldsymbol{P_B}\left(\alpha\right)} \frac{d\boldsymbol{P_B}\left(\alpha\right)}{dP_S\left(\alpha\right)}\right]$$

I need to show that $\frac{P_S(\alpha)}{\boldsymbol{P_B}(\alpha)} \frac{d\boldsymbol{P_B}(\alpha)}{dP_S(\alpha)} > 1$. Recall that

$$\left[\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} \frac{1}{\gamma_{\boldsymbol{B}}}\right]\right] \frac{d\boldsymbol{H_G}}{dP_S\left(\alpha\right)} = \left[\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_{\boldsymbol{G}}} - \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_{\boldsymbol{B}}}\right] f\left(\alpha\right)$$

or

$$\left[\eta_G\left(\boldsymbol{H_G}\right) + \frac{\int_0^1 G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial H_G} P_S\left(\alpha\right) \boldsymbol{H_G} dF\left(\alpha\right)}{\gamma_B}\right] \frac{dH_G}{dP_S\left(\alpha\right)} \frac{1}{H_G} =$$
$$\frac{\int_{\mu_S(\alpha)}^{\mu^{\max}} \mu dG\left(\mu\right)}{\gamma_G} - \frac{\int_0^{\mu^{\max}} \max\left\{\mu_B\left(\alpha\right), \mu\right\} dG\left(\mu\right) + G\left(\mu_B\left(\alpha\right)\right) P_S\left(\alpha\right) \frac{\partial \mu_B(\alpha)}{\partial P_S(\alpha)}}{\gamma_B} f\left(\alpha\right)$$

Since $\frac{d\boldsymbol{H_G}}{dP_S(\alpha)} > 0$, we have

$$0 > \left[\frac{\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu)}{\gamma_G} - \frac{\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu)}{\gamma_B}\right] f(\alpha) >$$
$$\frac{G(\boldsymbol{\mu_B}(\alpha))\,\boldsymbol{\mu_B}(\alpha)\, f(\alpha) + G(\boldsymbol{\mu_B}(\alpha))\, P_S(\alpha) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)} f(\alpha) + \int_0^1 G(\boldsymbol{\mu_B}(\alpha')) \frac{\partial \boldsymbol{\mu_B}(\alpha')}{\partial H_G} \frac{d\boldsymbol{H_G}}{dP_S(\alpha)} P_S(\alpha')\, dF(\alpha')}{\gamma_B}$$

hence

$$G(\boldsymbol{\mu_B}(\alpha))\,\boldsymbol{\mu_B}(\alpha)\, f(\alpha) + G(\boldsymbol{\mu_B}(\alpha))\, P_S(\alpha) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)} f(\alpha) + \int_0^1 G(\boldsymbol{\mu_B}(\alpha')) \frac{\partial \boldsymbol{\mu_B}(\alpha')}{\partial H_G} \frac{d\boldsymbol{H_G}}{dP_S(\alpha)} P_S(\alpha')\, dF(\alpha') < 0$$

Since $\frac{\partial \boldsymbol{\mu_B}(\alpha')}{\partial H_G} \frac{d\boldsymbol{H_G}}{dP_S(\alpha)} > 0$, we have

$$\boldsymbol{\mu_B}(\alpha) + P_S(\alpha) \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)} < 0$$

or

$$\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)} < -\frac{\boldsymbol{\mu_B}(\alpha)}{P_S(\alpha)}$$

Moreover, we know that $\frac{d\boldsymbol{\lambda_M}(\alpha)}{dP_S(\alpha)} = \frac{d(\boldsymbol{\mu_B}(\alpha)\boldsymbol{P_B}(\alpha))}{dP_S(\alpha)} > 0$ for all α, hence

$$\boldsymbol{P_B}(\alpha) \frac{d\boldsymbol{\mu_B}(\alpha)}{dP_S(\alpha)} + \boldsymbol{\mu_B}(\alpha) \frac{d\boldsymbol{P_B}(\alpha)}{dP_S(\alpha)} > 0$$
$$\frac{d\boldsymbol{P_B}(\alpha)}{dP_S(\alpha)} > -\frac{\boldsymbol{P_B}(\alpha)}{\boldsymbol{\mu_B}(\alpha)} \frac{d\boldsymbol{\mu_B}(\alpha)}{dP_S(\alpha)} > \frac{\boldsymbol{P_B}(\alpha)}{P_S(\alpha)}$$
$$\frac{d\boldsymbol{P_B}(\alpha)}{dP_S(\alpha)} \frac{P_S(\alpha)}{\boldsymbol{P_B}(\alpha)} > 1$$

thus,

$$\frac{d\omega(\alpha)}{dP_S(\alpha)} = \frac{1}{\boldsymbol{P_B}(\alpha)} \left[1 - \frac{d\boldsymbol{P_B}(\alpha)}{dP_S(\alpha)} \frac{P_S(\alpha)}{\boldsymbol{P_B}(\alpha)}\right] < 0$$

□

Proof of Lemma 8. From the optimality conditions for $P_S(\alpha)$ and $\overline{T}_1(\alpha)$ (see proof of Proposition 7 for a complete characterization) it is immediate that the optimality conditions for $P_S(\alpha)$ are weakly increasing in α. Thus, there cannot exist a threshold $\hat{\tilde{\alpha}}$ such that in $\alpha \to \hat{\tilde{\alpha}}^-$ the economy is in a normal state and in $\alpha \to \hat{\tilde{\alpha}}^+$ the economy is in a crisis state, at this would imply a drop in the price P_S. Thus, the threshold $\tilde{\alpha}^*$ is unique, and if $\tilde{\alpha}^* > \underline{\alpha}$, the economy is in a crisis state for all $\alpha < \tilde{\alpha}^*$ and it is in a normal state for all $\alpha > \tilde{\alpha}^*$.

The first-order necessary condition of $\boldsymbol{W}$ with respect to $\tilde{\alpha}$ is

$$\boldsymbol{W}'(\tilde{\alpha}) = \left[\boldsymbol{U_1}^C(\tilde{\alpha}^-) - \boldsymbol{U_1}^N(\tilde{\alpha}^+) - \chi\left[\mathbb{T}_{\mathbf{2}}^C(\tilde{\alpha}^-) - \mathbb{T}_{\mathbf{2}}^N(\tilde{\alpha}^+)\right]\right] f(\tilde{\alpha}) +$$
$$E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \boldsymbol{H_G}} + (1 - C'(\boldsymbol{H_G})\,\alpha)\, Z - \frac{\chi}{1+\chi} \frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \boldsymbol{H_G}}\right] \frac{d\boldsymbol{H_G}}{d\tilde{\alpha}} = 0$$

where I used that the changes in policy induced by changes in $\boldsymbol{H_G}$ are zero by standard envelope arguments, $\tilde{\alpha}^+$ and $\tilde{\alpha}^-$ denote the limits to $\tilde{\alpha}$ from the left and the right, respectively, and

$$\frac{d\boldsymbol{H_G}}{d\tilde{\alpha}} = \frac{\partial \gamma_{\boldsymbol{G}}}{\partial \tilde{\alpha}} \frac{1}{C'(\boldsymbol{H_G})} - \frac{\partial \gamma_{\boldsymbol{B}}}{\partial \tilde{\alpha}}$$

where

$$\frac{\partial \gamma_{\boldsymbol{G}}}{\partial \tilde{\alpha}} = \left[\left[1 - G\left(\boldsymbol{\mu_S}\left(\tilde{\alpha}^+\right)\right)\right] Z - \int_{\boldsymbol{\mu_S}(\tilde{\alpha}^+)}^{\mu^{\max}} \mu P_S\left(\tilde{\alpha}^+\right) dG\left(\mu\right) \right] f\left(\tilde{\alpha}\right)$$

$$\frac{\partial \gamma_{\boldsymbol{B}}}{\partial \tilde{\alpha}} = \left[\left[G\left(\frac{\tilde{\alpha}Z}{P_S\left(\tilde{\alpha}^-\right)}\right) \boldsymbol{\mu_B}\left(\tilde{\alpha}^-\right) P_S\left(\tilde{\alpha}^-\right) + \int_{\frac{\tilde{\alpha}Z}{P_S(\tilde{\alpha}^-)}}^{\mu^{\max}} \mu P_S\left(\tilde{\alpha}^-\right) dG\left(\mu\right) \right] - \left[G\left(\boldsymbol{\mu_B}\left(\tilde{\alpha}^+\right)\right) \boldsymbol{\mu_B}\left(\tilde{\alpha}^+\right) P_S\left(\tilde{\alpha}^+\right) + \int_{\boldsymbol{\mu_B}(\tilde{\alpha}^+)}^{\mu^{\max}} \mu P_S\left(\tilde{\alpha}^+\right) dG\left(\mu\right) \right] \right] f\left(\tilde{\alpha}\right)$$

□

Proof of Proposition 7. To characterize the optimality conditions, I totally differentiate $\boldsymbol{U_1}$, $\mathbb{T}_{\boldsymbol{2}}$ and $\boldsymbol{H_G}$. Since these expression are not differentiable everywhere, we need to consider six separate *market regimes*:

i. $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) > \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$

ii. $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) < \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$

iii. $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$

iv. $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) > \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$

v. $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) < \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$

vi. $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$

Below, I derive Regime i in detail. For the rest of the cases, I simply present the optimality conditions.

Regime i: $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) > \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$. Starting with $\boldsymbol{U_1}$, we have

$$d\boldsymbol{U_1}\left(\alpha\right) = -\left[W + P_S\left(\alpha\right)\boldsymbol{H_B} + \overline{T}_1(\alpha)\right] g\left(\boldsymbol{\mu_B}\left(\alpha\right)\right) \boldsymbol{\mu_B}\left(\alpha\right) d\boldsymbol{\mu_B}(\alpha) + \left[\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG\left(\mu\right) \boldsymbol{H_B} + \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG\left(\mu_1\right) \boldsymbol{H_G} + g\left(\boldsymbol{\mu_S}\left(\alpha\right)\right) \left(\boldsymbol{\mu_S}\left(\alpha\right)\right)^2 \boldsymbol{H_G}\right] dP_S(\alpha) + \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu) d\overline{T}_1(\alpha) + \left[\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu) - C'(\boldsymbol{H_G}) \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu)\right] P_S(\alpha) d\boldsymbol{H_G}$$

where $\boldsymbol{H_B} = W - c(\boldsymbol{H_G})$, so that $d\boldsymbol{H_B} = -c'(\boldsymbol{H_G})d\boldsymbol{H_G}$. Differentiating the market clearing condition, we get

$$d\boldsymbol{\mu_B}(\alpha) = \frac{1}{g(\boldsymbol{\mu_B}(\alpha))\left[W + \boldsymbol{H_B}P_S(\alpha) + \overline{T}_1(\alpha)\right]}\{[[1 - G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G} + [1 - G(\boldsymbol{\mu_B}(\alpha))]\boldsymbol{H_B}$$
$$+ g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G}]dP_S(\alpha) + [1 - G(\boldsymbol{\mu_B}(\alpha))]d\overline{T}_1(\alpha) +$$
$$\left[[1 - G(\boldsymbol{\mu_S}(\alpha))] - C'(\boldsymbol{H_G})[1 - G(\boldsymbol{\mu_B}(\alpha))]\right]P_S(\alpha)d\boldsymbol{H_G}\}$$

Introducing $d\boldsymbol{\mu_B}(\alpha)$ into $d\boldsymbol{U_1}(\alpha)$, we get

$$d\boldsymbol{U_1}(\alpha) = \left[\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))\,dG(\mu)\boldsymbol{H_B} + \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)\boldsymbol{H_G} +\right.$$
$$(\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha))\,g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G}]\,dP_S(\alpha) + \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)d\overline{T}_1(\alpha) +$$
$$\left[\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu) - C'(\boldsymbol{H_G})\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)\right]P_S(\alpha)d\boldsymbol{H_G}$$

Thus,

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} = \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))\,dG(\mu)\boldsymbol{H_B} + \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)\boldsymbol{H_G} +$$
$$(\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha))\,g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G} > 0$$

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} = \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}}(\mu - \boldsymbol{\mu_B}(\alpha))\,dG(\mu) > 0$$

$$E\left[\frac{\partial \boldsymbol{U_1}(\alpha')}{\partial H_G} + \frac{\partial \boldsymbol{U_1}(\alpha')}{\partial \mu_B(\alpha')}\frac{\partial \boldsymbol{\mu_B}(\alpha')}{\partial H_G} + \left(1 - C'(\boldsymbol{H_G})\alpha\right)Z\right] =$$
$$E\left[[1 - G(\boldsymbol{\mu_S}(\alpha))]\underbrace{(Z - \boldsymbol{\mu_B}(\alpha)P_S(\alpha))}_{=(\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha))P_S(\alpha)} + C'(\boldsymbol{H_G})(\boldsymbol{\mu_B}(\alpha)P_S(\alpha) - \alpha Z)\right] > 0$$

where I used that if $\boldsymbol{H_G} \in (0,1)$ then $Z \geq E[\boldsymbol{\mu_B}(\alpha)P_S(\alpha)] \geq E[\alpha]Z$.

Turning to the transfers, totally differentiating $\mathbb{T}_{\mathbf{2}}$, we get

$$d\mathbb{T}_{\mathbf{2}}(\alpha) = \boldsymbol{\mu_B}(\alpha)d\overline{T}_1(\alpha) + \left\{\frac{\overline{T}_1(\alpha) + P_S(\alpha)\boldsymbol{H_B} + [1 - G(\boldsymbol{\mu_S}(\alpha))]P_S(\alpha)\boldsymbol{H_G}}{[1 - G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G} + \boldsymbol{H_B}}d\boldsymbol{\mu_B}(\alpha) +\right.$$
$$\left.\boldsymbol{\mu_B}(\alpha)dP_S(\alpha) - (1-\alpha)Zd\boldsymbol{\lambda_M}(\alpha) + [\boldsymbol{\mu_B}(\alpha)P_S(\alpha) - \boldsymbol{\lambda_M}(\alpha)(1-\alpha)Z - \alpha Z]\frac{g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)}{1 - G(\boldsymbol{\mu_S}(\alpha))}\frac{\boldsymbol{\lambda_M}(\alpha)}{P_S(\alpha)}dP_S(\alpha)\right\}$$
$$[[1 - G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G} + \boldsymbol{H_B}] + [\boldsymbol{\mu_B}(\alpha)P_S(\alpha) - \alpha Z - (1-\alpha)Z\boldsymbol{\lambda_M}(\alpha)]\left[[1 - G(\boldsymbol{\mu_S}(\alpha))] - C'(\boldsymbol{H_G})\right]d\boldsymbol{H_G}$$

Totally differentiating $\boldsymbol{\lambda_M}(\alpha)$, we get

$$d\boldsymbol{\lambda_M}(\alpha) = (1-\boldsymbol{\lambda_M}(\alpha))\frac{g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)}{1-G(\boldsymbol{\mu_S}(\alpha))}\frac{\boldsymbol{\lambda_M}(\alpha)}{P_S(\alpha)}dP_S(\alpha)+ \\ \boldsymbol{\lambda_M}(\alpha)(1-\boldsymbol{\lambda_M}(\alpha))\left(\frac{1}{\boldsymbol{H_G}}+C'(\boldsymbol{H_G})\frac{1}{\boldsymbol{H_B}}\right)d\boldsymbol{H_G}$$

Plugging into the previous equation, we get

$$d\mathbb{T}_{\mathbf{2}}(\alpha) = d\boldsymbol{\mu_B}(\alpha)d\overline{T}_1(\alpha) + \left\{\frac{\overline{T}_1(\alpha)+P_S(\alpha)\boldsymbol{H_B}+[1-G(\boldsymbol{\mu_S}(\alpha))]P_S(\alpha)\boldsymbol{H_G}}{[1-G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G}+\boldsymbol{H_B}}d\boldsymbol{\mu_B}(\alpha)+\boldsymbol{\mu_B}(\alpha)dP_S(\alpha)- \right. \\ \left.(\boldsymbol{\mu_S}(\alpha)-\boldsymbol{\mu_B}(\alpha))\frac{g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)}{1-G(\boldsymbol{\mu_S}(\alpha))}\boldsymbol{\lambda_M}(\alpha)dP_S(\alpha)\right\}[[1-G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G}+\boldsymbol{H_B}]+ \\ \left[[\boldsymbol{\mu_B}(\alpha)P_S(\alpha)-\alpha Z-(1-\alpha)Z\boldsymbol{\lambda_M}(\alpha)]\left[\boldsymbol{\lambda_M}(\alpha)\frac{1}{H_G}-c'(\boldsymbol{H_G})(1-\boldsymbol{\lambda_M}(\alpha))\frac{1}{\boldsymbol{H_B}}\right]d\boldsymbol{H_G}- \right. \\ \left.(1-\alpha)Z\boldsymbol{\lambda_M}(\alpha)(1-\boldsymbol{\lambda_M}(\alpha))\left(\frac{1}{\boldsymbol{H_G}}+C'(\boldsymbol{H_G})\frac{1}{\boldsymbol{H_B}}\right)d\boldsymbol{H_G}\right][[1-G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G}+\boldsymbol{H_B}]$$

Replacing for $d\boldsymbol{\mu_B}(\alpha)$, and using the market clearing condition

$$G(\boldsymbol{\mu_B}(\alpha))\left[W+P_S(\alpha)\boldsymbol{H_B}+\overline{T}_1(\alpha)\right] = P_S(\alpha)\boldsymbol{H_B}+[1-G(\boldsymbol{\mu_S}(\alpha))]P_S(\alpha)\boldsymbol{H_G}+\overline{T}_1(\alpha)$$

we get

$$d\mathbb{T}_{\mathbf{2}}(\alpha) = \left[\boldsymbol{\mu_B}(\alpha)+\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1-G(\boldsymbol{\mu_B}(\alpha))]\right]d\overline{T}_1(\alpha)+\left\{\left(\boldsymbol{\mu_B}(\alpha)+\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)[1-G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G}+ \right. \\ \left.\left(\boldsymbol{\mu_B}(\alpha)+\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1-G(\boldsymbol{\mu_B}(\alpha))]\right)\boldsymbol{H_B}-\left(\boldsymbol{\mu_S}(\alpha)-\boldsymbol{\mu_B}(\alpha)-\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G}\right\}dP_S(\alpha)- \\ \left[[Z-\boldsymbol{\mu_B}(\alpha)P_S(\alpha)][1-G(\boldsymbol{\mu_S}(\alpha))]+C'(\boldsymbol{H_G})[\boldsymbol{\mu_B}(\alpha)P_S(\alpha)-\alpha Z]+ \right. \\ \left.\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[[1-G(\boldsymbol{\mu_S}(\alpha))]+C'(\boldsymbol{H_G})[1-G(\boldsymbol{\mu_B}(\alpha))]]P_S(\alpha)\right]d\boldsymbol{H_G}$$

Therefore

$$\frac{\partial\mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)} = \left(\boldsymbol{\mu_B}(\alpha)+\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)[1-G(\boldsymbol{\mu_S}(\alpha))+g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)]\boldsymbol{H_G}+ \\ \left(\boldsymbol{\mu_B}(\alpha)+\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1-G(\boldsymbol{\mu_B}(\alpha))]\right)\boldsymbol{H_B}-\boldsymbol{\mu_S}(\alpha)g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G}$$

$$\frac{\partial\mathbb{T}_{\mathbf{2}}(\alpha)}{\partial\overline{T}_1(\alpha)} = \boldsymbol{\mu_B}(\alpha)+\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1-G(\boldsymbol{\mu_B}(\alpha))] > 0$$

$$\frac{\partial\mathbb{T}_{\mathbf{2}}(\alpha)}{\partial H_G} = -\left[[Z-\boldsymbol{\mu_B}(\alpha)P_S(\alpha)][1-G(\boldsymbol{\mu_S}(\alpha))]+C'(\boldsymbol{H_G})[\boldsymbol{\mu_B}(\alpha)P_S(\alpha)-\alpha Z]+ \right. \\ \left.[1-G(\boldsymbol{\mu_B}(\alpha))]\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}P_S(\alpha)\left[C'(\boldsymbol{H_G})-\frac{1-G(\boldsymbol{\mu_S}(\alpha))}{1-G(\boldsymbol{\mu_B}(\alpha))}\right]\right] < 0$$

Note in $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)}$, the first two terms are strictly positive, while the last term is be negative. For $\lambda_E \to 1$ and $P_S(\alpha) \to \frac{Z}{\mu^{\max}}$, we get $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} < 0$. As $P_S(\alpha) \to Z$,we have $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} > 0$.

Finally, recall that

$$\frac{\partial \boldsymbol{H_G}}{\partial P_S(\alpha)} \propto \left[\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)}\frac{1}{C'(\boldsymbol{H_G})} - \frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)}\right] f(\alpha) \quad \text{and} \quad \frac{\partial \boldsymbol{H_G}}{\partial \overline{T}_1(\alpha)} \propto -\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)}\frac{1}{\boldsymbol{\gamma_B}} f(\alpha),$$

where $\frac{\gamma_G}{\gamma_B} = C'(\boldsymbol{H_G})$, and

$$\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} = \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu)\, dF(\alpha) > 0$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} = \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\frac{[[1-G(\boldsymbol{\mu_S}(\alpha))]\boldsymbol{H_G} + [1-G(\boldsymbol{\mu_B}(\alpha))]\boldsymbol{H_B} + g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G}]P_S(\alpha)}{W + H_B P_S(\alpha) + \overline{T}_1(\alpha)} + G(\boldsymbol{\mu_B}(\alpha))\boldsymbol{\mu_B}(\alpha) + \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu) > 0$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)} = \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\frac{[1-G(\boldsymbol{\mu_B}(\alpha))]P_S(\alpha)}{W + \boldsymbol{H_B} P_S(\alpha) + \overline{T}_1(\alpha)} > 0$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} = -\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\frac{\left[c'(\boldsymbol{H_G}) - \frac{1-G(\boldsymbol{\mu_S}(\alpha))}{1-G(\boldsymbol{\mu_B}(\alpha))}\right][1-G(\boldsymbol{\mu_B}(\alpha))](P_S(\alpha))^2}{W + \boldsymbol{H_B} P_S(\alpha) + \overline{T}_1(\alpha)} < 0$$

It is immediate to see that, conditional on $P_S(\alpha)$ and $\overline{T}_1(\alpha)$, the optimality conditions are independent of α in Regime i.

Regime ii: $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) < \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$. For $\boldsymbol{U_1}(\alpha)$, we have

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} = \int_{\frac{\alpha Z}{P_S(\alpha)}}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)\boldsymbol{H_B} + \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)\boldsymbol{H_G} + (\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha))\boldsymbol{\mu_S}(\alpha)g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{H_G} + \left(\frac{\alpha Z}{P_S(\alpha)} - \boldsymbol{\mu_B}(\alpha)\right)\frac{\alpha Z}{P_S(\alpha)} g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\boldsymbol{H_B}$$

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} = \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)$$

For transfers $\mathbb{T}_2(\alpha)$, we have

$$\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} = \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)[1 - G(\boldsymbol{\mu_S}(\alpha)) + g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)]\boldsymbol{H_G} + \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)\left[1 - G\left(\frac{\alpha Z}{P_S(\alpha)}\right) + g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\frac{\alpha Z}{P_S(\alpha)}\right]\boldsymbol{H_B} - \boldsymbol{\mu_S}(\alpha)g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G} - \frac{\alpha Z}{P_S(\alpha)} g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\frac{\alpha Z}{P_S(\alpha)}\boldsymbol{H_B}$$

$$\frac{\partial \mathbb{T}_2(\alpha)}{\partial \overline{T}_1(\alpha)} = \boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1 - G(\boldsymbol{\mu_B}(\alpha))]$$

And for the shadow values, we have

$$\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} = \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu) > 0$$
$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} = \int_{\frac{\alpha Z}{P_S(\alpha)}}^{\mu^{\max}} \mu dG(\mu) > 0$$
$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)} = 0$$
$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} = 0$$

Note that

$$\begin{aligned}\frac{\partial^2 \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)\partial\alpha} = &-\left[\left[1-G(\mu_S(\alpha)) + g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\right]\boldsymbol{H_G} + \right.\\ &\left.\left[1-G\left(\frac{\alpha Z}{P_S(\alpha)}\right) + \frac{\alpha Z}{P_S(\alpha)} g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\right]\boldsymbol{H_B}\right]\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial\alpha} + \\ &\left[1+\left(\frac{\alpha Z}{P_S(\alpha)} - \boldsymbol{\mu_B}(\alpha)\right)\frac{g'\left(\frac{\alpha Z}{P_S(\alpha)}\right)}{g\left(\frac{\alpha Z}{P_S(\alpha)}\right)}\right]\frac{Z}{P_S(\alpha)} g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\frac{\alpha Z}{P_S(\alpha)}\boldsymbol{H_B} > 0\end{aligned}$$
$$\frac{\partial^2 \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)\partial\alpha} = -\left[1-G(\boldsymbol{\mu_B}(\alpha))\right]\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial\alpha} > 0$$

and

$$\begin{aligned}\frac{\partial^2 \mathbb{T}_{\boldsymbol{2}}(\alpha)}{\partial P_S(\alpha)\partial\alpha} = &\left[\left[1-G(\boldsymbol{\mu_S}(\alpha))\right]\boldsymbol{H_G} + \left[1-G\left(\frac{\alpha Z}{P_S(\alpha)}\right)\right]\boldsymbol{H_B} + g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G} + \right.\\ &\left. g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\frac{\alpha Z}{P_S(\alpha)}\boldsymbol{H_B}\right]\left(1+\frac{g(\boldsymbol{\mu_B}(\alpha))^2 - G(\boldsymbol{\mu_B}(\alpha))\,g'(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))^2}\right)\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial\alpha} - \\ &\left[\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} + \frac{\alpha Z}{P_S(\alpha)} + \left(\frac{\alpha Z}{P_S(\alpha)} - \mu_B(\alpha) - \frac{G(\mu_B(\alpha))}{g(\mu_B(\alpha))}\right)\left[1+\frac{g'\left(\frac{\alpha Z}{P_S(\alpha)}\right)}{g\left(\frac{\alpha Z}{P_S(\alpha)}\right)}\frac{\alpha Z}{P_S(\alpha)}\right]\right] \\ &g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\frac{Z}{P_S(\alpha)}\boldsymbol{H_B} < 0\end{aligned}$$
$$\frac{\partial^2 \mathbb{T}_{\boldsymbol{2}}}{\partial \overline{T}_1 \partial\alpha} = \left[1+\frac{g(\boldsymbol{\mu_B}(\alpha))^2 - G(\boldsymbol{\mu_B}(\alpha))\,g'(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))^2}\right]\left[1-G(\boldsymbol{\mu_B}(\alpha))\right]\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial\alpha} < 0$$

where I used that $\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial\alpha} = -\frac{g\left(\frac{\alpha Z}{P_S(\alpha)}\right)}{g(\boldsymbol{\mu_B}(\alpha))}\frac{Z\boldsymbol{H_B}}{W+\overline{T}_1(\alpha)} < 0$ and that $\frac{G(\mu)}{g(\mu)}$ is increasing in μ. Thus, the liquidity effect is increasing in α. Moreover

$$\frac{\partial^2 \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)\partial\alpha} = 0$$
$$\frac{\partial^2 \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)\partial\alpha} = -\frac{Z}{P_S(\alpha)}\frac{\alpha Z}{P_S(\alpha)} g\left(\frac{\alpha Z}{P_S(\alpha)}\right) < 0$$

so the incentives effect is (weakly) increasing in α. Thus, in Regime ii, $P_S(\alpha)$ and $\overline{T}_1(\alpha)$ are increasing in α.

Regime iii: $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$. This regime is different than the others in that $P_S(\alpha)$ is not chosen independently from $\overline{T}_1(\alpha)$, because of the constraint $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B(\alpha)}}$. However, it is possible to write the optimality conditions in this regime in terms of derivatives that connect to the other regimes.

Consider first $\boldsymbol{U_1}(\alpha)$. We have

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} = \left.\frac{\partial^{ii}\boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} + \left.\frac{\partial^{ii}\boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial \overline{T}_1(\alpha)}{\partial P_S(\alpha)}$$

where $\left.\frac{\partial^{ii}\boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$ and $\left.\frac{\partial^{ii}\boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$ denote the partial derivatives in Regime ii evaluated at $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, and

$$\frac{\partial \overline{T}_1}{\partial P_S(\alpha)} = -\frac{\frac{(\boldsymbol{\mu_B}(\alpha))^2}{\alpha Z} + \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)}}{\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial \overline{T}_1(\alpha)}} < 0$$

$$\frac{\partial \mu_B(\alpha)}{\partial P_S(\alpha)} = \frac{[1 - G(\boldsymbol{\mu_B}(\alpha))]\boldsymbol{H_B} P_S(\alpha) + [1 - G(\boldsymbol{\mu_S}(\alpha)]\boldsymbol{H_G} P_S(\alpha) + g(\boldsymbol{\mu_S}(\alpha))\boldsymbol{\mu_S}(\alpha)\boldsymbol{H_G}}{g(\boldsymbol{\mu_B}(\alpha))\left[W + \boldsymbol{H_B}\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} + \overline{T}_1(\alpha)\right]} > 0$$

$$\frac{\partial \mu_B(\alpha)}{\partial \overline{T}_1(\alpha)} = \frac{1 - G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))\left[W + \boldsymbol{H_B}\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} + \overline{T}_1(\alpha)\right]} > 0$$

For transfers $\mathbb{T}_{\mathbf{2}}(\alpha)$, we have

$$\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)} = \left.\frac{\partial^{ii}\mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} + \left.\frac{\partial^{ii}\mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial \overline{T}_1(\alpha)}{\partial P_S(\alpha)}$$

Finally, for the shadow values, we have

$$\frac{\partial \overline{\boldsymbol{\gamma}}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} = \left.\frac{\partial^{ii}\overline{\boldsymbol{\gamma}}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$$

$$\frac{\partial \overline{\boldsymbol{\gamma}}_{\boldsymbol{G}}(\alpha)}{\partial H_G} = \left.\frac{\partial^{ii}\overline{\boldsymbol{\gamma}}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial P_S(\alpha)}{\partial H_G}$$

$$\frac{\partial \overline{\boldsymbol{\gamma}}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} = \left.\frac{\partial^{ii}\overline{\boldsymbol{\gamma}}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$$

$$\frac{\partial \overline{\boldsymbol{\gamma}}_{\boldsymbol{B}}(\alpha)}{\partial H_G} = \left.\frac{\partial^{ii}\overline{\boldsymbol{\gamma}}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial P_S(\alpha)}{\partial H_G}$$

where

$$\frac{\partial P_S(\alpha)}{\partial H_G} = -\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial H_G} < 0$$

$$\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial H_G} = -\frac{\left[Cq'\left(\boldsymbol{H_G}\right) - \frac{1-G\left(\frac{\boldsymbol{\mu_B}(\alpha)}{\alpha}\right)}{1-G(\boldsymbol{\mu_B}(\alpha))}\right]\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}{g\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)\left[W + \boldsymbol{H_B}\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} + \overline{T}_1\left(\alpha\right)\right]} < 0$$

Regime iv: $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) > \frac{\alpha Z}{\boldsymbol{\mu_B}(\boldsymbol{\alpha})}$. Evaluating Regime i at $\boldsymbol{\mu_S}(\alpha) > \mu^{\max}$, we get Regime iv. In particular,

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} = \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)\boldsymbol{H_B}$$

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} = \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))dG(\mu)$$

and

$$\frac{\partial \mathbb{T}_{\boldsymbol{2}}(\alpha)}{\partial P_S(\alpha)} = \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1 - G(\boldsymbol{\mu_B}(\alpha))]\right)\boldsymbol{H_B}$$

$$\frac{\partial \mathbb{T}_{\boldsymbol{2}}(\alpha)}{\partial \overline{T}_1(\alpha)} = \boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}[1 - G(\boldsymbol{\mu_B}(\alpha))]$$

Moreover,

$$\frac{\partial \overline{\gamma}_{\boldsymbol{G}}\left(\alpha\right)}{\partial P_S\left(\alpha\right)} = 0$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}\left(\alpha\right)}{\partial P_S\left(\alpha\right)} = \frac{G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)}{g\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)}\frac{\left[1 - G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)\right]\boldsymbol{H_B}P_S\left(\alpha\right)}{W + \boldsymbol{H_B}P_S\left(\alpha\right) + \overline{T}_1\left(\alpha\right)} + G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)\boldsymbol{\mu_B}\left(\alpha\right) + \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG\left(\mu\right)$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}\left(\alpha\right)}{\partial \overline{T}_1\left(\alpha\right)} = \frac{G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)}{g\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)}\frac{\left[1 - G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)\right]P_S\left(\alpha\right)}{W + \boldsymbol{H_B}P_S\left(\alpha\right) + \overline{T}_1\left(\alpha\right)}$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}\left(\alpha\right)}{\partial H_G} = -\frac{G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)}{g\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)}\frac{c'\left(\boldsymbol{H_G}\right)\left[1 - G\left(\boldsymbol{\mu_B}\left(\alpha\right)\right)\right]\left(P_S\left(\alpha\right)\right)^2}{W + \boldsymbol{H_B}P_S\left(\alpha\right) + \overline{T}_1\left(\alpha\right)}$$

Regime v: $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) < \frac{\alpha Z}{\boldsymbol{\mu_B}(\boldsymbol{\alpha})}$. For $\boldsymbol{U_1}(\alpha)$, we have

$$\frac{\partial \boldsymbol{U_1}\left(\alpha\right)}{\partial P_S\left(\alpha\right)} = \int_{\frac{\alpha Z}{P_S(\alpha)}}^{\mu^{\max}} \left(\mu - \boldsymbol{\mu_B}\left(\alpha\right)\right)\boldsymbol{H_B}dG\left(\mu\right) + \left(\frac{\alpha Z}{P_S\left(\alpha\right)} - \boldsymbol{\mu_B}\left(\alpha\right)\right)\frac{\alpha Z}{P_S\left(\alpha\right)}\boldsymbol{H_B}g\left(\frac{\alpha Z}{P_S\left(\alpha\right)}\right)$$

$$\frac{\partial \boldsymbol{U_1}\left(\alpha\right)}{\partial \overline{T}_1\left(\alpha\right)} = \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \left(\mu - \boldsymbol{\mu_B}\left(\alpha\right)\right)dG\left(\mu\right)$$

For transfers $\mathbb{T}(\alpha)$, we have

$$\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)} = \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)\left[1 - G\left(\frac{\alpha Z}{P_S(\alpha)}\right)\right]\boldsymbol{H_B} -$$

$$\left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right)\left[\frac{\alpha Z}{P_S(\alpha)} - \boldsymbol{\mu_B}(\alpha) - \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\right] g\left(\frac{\alpha Z}{P_S(\alpha)}\right)\frac{\alpha Z}{P_S(\alpha)}\boldsymbol{H_B}$$

$$\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)} = \boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\left[1 - G(\boldsymbol{\mu_B}(\alpha))\right]$$

And for the shadow values, we have

$$\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial P_S(\alpha)} = 0$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} = \int_{\frac{\alpha Z}{P_S(\alpha)}}^{\mu^{\max}} \mu dG(\mu)$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial \overline{T}_1(\alpha)} = 0$$

$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} = 0$$

Regime vi: $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\boldsymbol{\alpha})}$. Similar to Regime iii, in this case it is not possible to choose $P_S(\alpha)$ independently of $\overline{T}_1(\alpha)$. Thus, for $\boldsymbol{U_1}(\alpha)$, we have

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} = \left.\frac{\partial^{v} \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} + \left.\frac{\partial^{v} \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)}\right|_{P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial \overline{T}_1(\alpha)}{\partial P_S(\alpha)}$$

where $\left.\frac{\partial^{v} \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$ and $\left.\frac{\partial^{v} \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)}\right|_{P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$ denote the partial derivatives in Regime v evaluated at $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, and

$$\frac{\partial \overline{T}_1}{\partial P_S(\alpha)} = -\frac{\frac{(\boldsymbol{\mu_B}(\alpha))^2}{\alpha Z} + \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial P_S(\alpha)}}{\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial \overline{T}_1(\alpha)}} < 0$$

$$\frac{\partial \mu_B(\alpha)}{\partial P_S(\alpha)} = \frac{\left[1 - G(\boldsymbol{\mu_B}(\alpha))\right]\boldsymbol{H_B} P_S(\alpha)}{g(\boldsymbol{\mu_B}(\alpha))\left[W + \boldsymbol{H_B}\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} + \overline{T}_1(\alpha)\right]} > 0$$

$$\frac{\partial \mu_B(\alpha)}{\partial \overline{T}_1(\alpha)} = \frac{1 - G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))\left[W + \boldsymbol{H_B}\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} + \overline{T}_1(\alpha)\right]} > 0$$

For transfers $\mathbb{T}_{\mathbf{2}}(\alpha)$, we have

$$\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)} = \left.\frac{\partial^{v} \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} + \left.\frac{\partial^{v} \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)}\right|_{P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial \overline{T}_1(\alpha)}{\partial P_S(\alpha)}$$

Finally, for the shadow values, we have

$$\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial \overline{T}_1(\alpha)} = 0$$
$$\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial H_G} = 0$$
$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)} = \left.\frac{\partial^v \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}}$$
$$\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G} = \left.\frac{\partial^v \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial P_S(\alpha)}\right|_{P_S(\alpha)=\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}} \frac{\partial P_S(\alpha)}{\partial H_G}$$

where

$$\frac{\partial P_S(\alpha)}{\partial H_G} = -\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} \frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial H_G} < 0$$
$$\frac{\partial \boldsymbol{\mu_B}(\alpha)}{\partial H_G} = -\frac{c'(H_G)\left[1 - G(\mu_B(\alpha))\right]\frac{\alpha Z}{\mu_B(\alpha)}}{g(\boldsymbol{\mu_B}(\alpha))\left[W + \boldsymbol{H_B}\frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)} + \overline{T}_1(\alpha)\right]} < 0$$

Optimal Policy. The rest of the proof identifies which regime the planner chooses for different values of α. First, note that since the planner's problem is discontinuous at $P_S(\alpha) = \frac{Z}{\mu^{\max}}$, we will have to study the optimality of $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ separately.

Moreover, it will be useful to define

$$\tilde{\Omega} \equiv \frac{E\left[\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial H_G} + (1 - \alpha c'(H_G))Z + \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial H_G}\right]}{\eta_G(\boldsymbol{H_G})\boldsymbol{\gamma_B} + E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right] - C'(\boldsymbol{H_G})E\left[\frac{\partial \overline{\gamma}_{\boldsymbol{G}}(\alpha)}{\partial H_G}\right]}$$

$\tilde{\Omega}$ is a common factor in the incentives effect.

i. First, note that Regime iv cannot happen. In this regime, we have that

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)} = \left(\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_2(\alpha)}{\partial \overline{T}_1(\alpha)}\right)\boldsymbol{H_B}$$

while

$$\frac{\partial \boldsymbol{H_G}}{\partial P_S(\alpha)} < \frac{\partial \boldsymbol{H_G}}{\partial \overline{T}_1(\alpha)}\boldsymbol{H_B}.$$

Thus, transfers dominate a positive market wedge.

ii. The optimality condition for $\overline{T}_1(\alpha)$ in Regime i is

$$\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))\, dG(\mu) - \chi\left[\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\left[1 - G(\boldsymbol{\mu_B}(\alpha))\right]\right] - \tilde{\Omega}\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))}\frac{\left[1 - G(\boldsymbol{\mu_B}(\alpha))\right]P_S(\alpha)}{W + \boldsymbol{H_B}P_S(\alpha) + \overline{T}_1(\alpha)} = 0$$

while in Regime ii is

$$\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))\, dG(\mu) - \chi \left[\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} [1 - G(\boldsymbol{\mu_B}(\alpha))] \right] = 0$$

Thus, $\boldsymbol{\mu_B}(\alpha)$ is higher in Regime ii than in Regime i. Thus, exists $\tilde{P}_S(\alpha), \tilde{\tilde{P}}_S(\alpha)$ with $\tilde{P}_S(\alpha) < \tilde{\tilde{P}}_S(\alpha)$ such that if $P_S(\alpha) < \tilde{P}_S(\alpha)$ then $P_S(\alpha) < \frac{\alpha Z}{\boldsymbol{\mu_B}(\boldsymbol{\alpha})}$ (which corresponds to Regime ii) and if $P_S(\alpha) > \tilde{\tilde{P}}_S(\alpha)$ then $P_S(\alpha) > \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$ (which corresponds to Regime i).

iii. The optimality condition for $P_S(\alpha)$ in Regime ii at $P_S(\alpha) \to \tilde{P}_S(\alpha)$ is strictly greater than the optimality condition for $P_S(\alpha)$ in Regime i at $P_S(\alpha) \to \tilde{\tilde{P}}_S(\alpha)$. To see this, note that the optimality condition for $P_S(\alpha)$ in Regime i is

$$\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))\, \boldsymbol{H_B} dG(\mu) + \int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha)) \boldsymbol{H_G} dG(\mu) + (\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha))\, \boldsymbol{\mu_S}(\alpha) \boldsymbol{H_G} g(\boldsymbol{\mu_S}(\alpha)) -$$
$$\chi \left[\left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) [[1 - G(\boldsymbol{\mu_S}(\alpha))] + g(\boldsymbol{\mu_S}(\alpha))\, \boldsymbol{\mu_S}(\alpha)]\, \boldsymbol{H_G} + \right.$$
$$\left. \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} [1 - G(\boldsymbol{\mu_B}(\alpha))] \right) \boldsymbol{H_B} - \boldsymbol{\mu_S}(\alpha) g(\boldsymbol{\mu_S}(\alpha))\, \boldsymbol{\mu_S}(\alpha)\, \boldsymbol{H_G} \right] +$$
$$\tilde{\Omega} \left[\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu) - \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \frac{[[1 - G(\boldsymbol{\mu_S}(\alpha))]\, \boldsymbol{H_G} + [1 - G(\boldsymbol{\mu_B}(\alpha))]\, \boldsymbol{H_B} + g(\boldsymbol{\mu_S}(\alpha))\, \boldsymbol{\mu_S}(\alpha)\, \boldsymbol{H_G}]\, P_S(\alpha)}{W + H_B P_S(\alpha) + \overline{T}_1(\alpha)} - \right.$$
$$\left. G(\boldsymbol{\mu_B}(\alpha))\, \boldsymbol{\mu_B}(\alpha) - \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu) \right] = 0$$

while in Regime ii is

$$\int_{\frac{\alpha Z}{P_S(\alpha)}}^{\mu^{\max}} (\mu - \mu_B(\alpha))\, H_B dG(\mu) + \int_{\mu_S(\alpha)}^{\mu^{\max}} (\mu - \mu_B(\alpha))\, H_G dG(\mu) + (\mu_S(\alpha) - \mu_B(\alpha))\, \mu_S(\alpha)\, H_G g(\mu_S(\alpha)) -$$
$$\chi \left[\left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) [1 - G(\boldsymbol{\mu_S}(\alpha))]\, \boldsymbol{H_G} + \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} [1 - G(\boldsymbol{\mu_B}(\alpha))] \right) \boldsymbol{H_B} - \right.$$
$$\left. \left(\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha) - \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) g(\boldsymbol{\mu_S}(\alpha))\, \boldsymbol{\mu_S}(\alpha)\, \boldsymbol{H_G} \right] + \tilde{\Omega} \left[\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu) - \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu) \right] = 0$$

Using the optimality conditions for transfers, these expressions become

$$\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))\, \boldsymbol{H_G} dG(\mu) + (\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha))\, \boldsymbol{\mu_S}(\alpha)\, \boldsymbol{H_G} g(\boldsymbol{\mu_S}(\alpha)) -$$
$$\chi \left[\left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) [1 - G(\boldsymbol{\mu_S}(\alpha))]\, \boldsymbol{H_G} - \left(\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha) - \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) g(\boldsymbol{\mu_S}(\alpha))\, \boldsymbol{\mu_S}(\alpha)\, \boldsymbol{H_G} \right] +$$
$$\tilde{\Omega} \left[\int_{\boldsymbol{\mu_S}(\alpha)}^{\mu^{\max}} \mu dG(\mu) - \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \frac{[[1 - G(\boldsymbol{\mu_S}(\alpha))]\, \boldsymbol{H_G} + g(\boldsymbol{\mu_S}(\alpha))\, \boldsymbol{\mu_S}(\alpha)\, \boldsymbol{H_G}]\, P_S(\alpha)}{W + \boldsymbol{H_B} P_S(\alpha) + \overline{T}_1(\alpha)} - G(\boldsymbol{\mu_B}(\alpha))\, \boldsymbol{\mu_B}(\alpha) - \right.$$
$$\left. \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu) \right] = 0$$

and

$$\int_{\mu_S(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha)) \boldsymbol{H_G} dG(\mu) + (\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha)) \boldsymbol{\mu_S}(\alpha) \boldsymbol{H_G} g(\boldsymbol{\mu_S}(\alpha)) -$$
$$\chi \left[\left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) [1 - G(\boldsymbol{\mu_S}(\alpha))] \boldsymbol{H_G} - \left(\boldsymbol{\mu_S}(\alpha) - \boldsymbol{\mu_B}(\alpha) - \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \right) g(\boldsymbol{\mu_S}(\alpha)) \boldsymbol{\mu_S}(\alpha) \boldsymbol{H_G} \right] -$$
$$\tilde{\Omega} \left[\int_{\mu_S(\alpha)}^{\mu^{\max}} \mu dG(\mu) - \int_{\mu_B(\alpha)}^{\mu^{\max}} \mu dG(\mu) \right] = 0$$

Since

$$\frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} \frac{[[1 - G(\boldsymbol{\mu_S}(\alpha))] \boldsymbol{H_G} + g(\boldsymbol{\mu_S}(\alpha)) \boldsymbol{\mu_S}(\alpha) \boldsymbol{H_G}] P_S(\alpha)}{W + H_B P_S(\alpha) + \overline{T}_1(\alpha)} + G(\boldsymbol{\mu_B}(\alpha)) \boldsymbol{\mu_B}(\alpha) > 0$$

and $\tilde{P}_S(\alpha) < \tilde{\tilde{P}}_S(\alpha)$, then the optimality condition in Regime ii is higher than in Regime i at their respective boundaries.

iv. The optimality condition for $P_S(\alpha)$ in Regime iii is strictly higher than the optimality condition for $P_S(\alpha)$ in Regime ii for all $P_S(\alpha) < \tilde{P}_S(\alpha)$, and they coincide at $P_S(\alpha) \to \tilde{P}_S(\alpha)$. To see this, note that the partial derivates characterizing the optimality conditions in these regimes coincide. Thus, the only difference is in the determination of the transfers $\overline{T}_1(\alpha)$. But since Regime *ii* has $P_S(\alpha) < \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$ and Regime iii has $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, it is immediate that the optimality condition for $P_S(\alpha)$ is higher in Regime iii. Moreover, it is immediate to see that in Regime ii, $\boldsymbol{\mu_B}(\alpha)$ is independent of $P_S(\alpha)$. Thus, $\tilde{P}_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, and the optimality conditions in Regimes ii and iii coincide at $P_S(\alpha) \to \tilde{P}_S(\alpha)$.

v. The optimality condition for $P_S(\alpha)$ in Regime iii is strictly lower than the optimality condition for $P_S(\alpha)$ in Regime i for all $P_S(\alpha) < \tilde{P}_S(\alpha)$, and they coincide at $P_S(\alpha) \to \tilde{\tilde{P}}_S(\alpha)$. To see this, note that the liquidity effect in these regimes coincide but the incentives effect is lower in Regime i. Since Regime *i* has $P_S(\alpha) > \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$ and Regime iii has $P_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, it is immediate that the optimality condition for $P_S(\alpha)$ is lower in Regime iii. Moreover, from the optimality condition for $\overline{T}_1(\alpha)$, we get that $\boldsymbol{\mu_B}(\alpha)$ in Regime i is increasing in $P_S(\alpha)$. Thus, $\tilde{\tilde{P}}_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$, which implies that the optimality conditions for $P_S(\alpha)$ coincide in Regimes i and iii.

vi. Figure 7 depicts the optimality conditions for $P_S(\alpha)$, given the optimal transfer $\overline{T}_1(\alpha)$ (which is itself a function of $P_S(\alpha)$). Next, I show that in Panel (a), Regime i dominates Regime iii. Analogously, Regime ii dominates Regime iii if both intersect zero.

vii. Consider the two optimality candidates in Panel (a). We know that Regimes i and iii coincide at $P_S(\alpha) \to \tilde{\tilde{P}}_S(\alpha)$. Thus, the objective function increases by more when moving in the direction of Regime i than in the direction of iii. A similar argument shows the optimality of Regime ii over Regime iii when they both intersect zero.

viii. Finally, note that the optimality condition in Regime i is independent of α while the optimality condition in Regime ii is increasing in α, while the thresholds $\tilde{P}_S(\alpha)$ and $\tilde{\tilde{P}}_S(\alpha)$ are increasing in α. Thus, there exists α'' such that if $\alpha < \alpha''$, the candidate optimal policy is in Regime i, and if $\alpha > \alpha''$ the candidate optimal policy is in Regimes ii or iii.

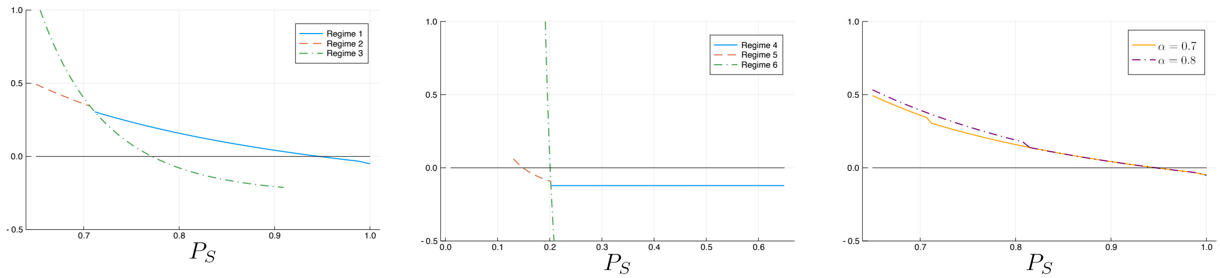

Figure 7: Optimality conditions for P_S

ix. Let's switch to $P_S(\alpha) < \frac{Z}{\mu^{\max}}$. We already know that Regime iv cannot be part of the optimal policy. Next I show that only Regime v can be part of the planner's solution.

x. First, note that there exists $\hat{P}_S(\alpha)$ such that $\hat{P}_S(\alpha) = \frac{\alpha Z}{\boldsymbol{\mu_B}(\alpha)}$ in Regime vi. It is immediate that Regimes v and vi coincide at $P_S(\alpha) \to \hat{P}_S(\alpha)$, since the optimality conditions coincide. Taking the optimality conditions of Regime v in the limit $P_S(\alpha) \to \hat{P}_S(\alpha)$, we get

$$\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))\, dG(\mu)\, \boldsymbol{H_B} - \chi \left(\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} [[1 - G(\boldsymbol{\mu_B})]] \right) \boldsymbol{H_B} - \tilde{\Omega} \int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} \mu dG(\mu) = 0$$

and

$$\int_{\boldsymbol{\mu_B}(\alpha)}^{\mu^{\max}} (\mu - \boldsymbol{\mu_B}(\alpha))\, dG(\mu) - \chi \left[\boldsymbol{\mu_B}(\alpha) + \frac{G(\boldsymbol{\mu_B}(\alpha))}{g(\boldsymbol{\mu_B}(\alpha))} [1 - G(\boldsymbol{\mu_B}(\alpha))] \right] = 0$$

thus the optimality condition is negative at $P_S(\alpha) \to \hat{P}_S(\alpha)$. Similar arguments as in the case with $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ imply that Regime v always dominates Regime vi (see Figure 7 Panel (b)).

xi. Finally, given the optima choices for $P_S(\alpha) > \frac{Z}{\mu^{\max}}$ and $P_S(\alpha) < \frac{Z}{\mu^{\max}}$, the optimal *crises* thresholds is chosen by the expression in Lemma 8.

□

Proof of Lemma 9. First, note that $\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}$ and $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)}$ depend on $\overline{T}_1(\alpha)$ only through $\boldsymbol{\mu_B}(\alpha)$, independently of the regime. Second, $\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)}$ is always decreasing in $\boldsymbol{\mu_B}(\alpha)$. Third, given Assumption 2, $\frac{\partial \mathbb{T}_2(\alpha)}{\partial P_S(\alpha)}$ is increasing in $\boldsymbol{\mu_B}(\alpha)$. Thus, $\frac{\partial \boldsymbol{FOC}(P_S(\alpha), \overline{T}_1(\alpha), H_G)}{\partial \overline{T}_1(\alpha)} < 0$. Moreover, $\frac{\partial \boldsymbol{FOC}(\boldsymbol{P_S}(\alpha), \overline{T}_1(\alpha), H_G)}{\partial P_S(\alpha)} < 0$ because it is the second-order condition of the problem.

□

Proof of Lemma 10. We know that

$$\frac{d\boldsymbol{H_G}}{dP_S(\alpha)}\Big|^{commit} = \frac{\frac{\partial \overline{\gamma}_G(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_G} - \frac{\partial \overline{\gamma}_B(\alpha)}{\partial P_S(\alpha)} \frac{1}{\gamma_B}}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial \overline{\gamma}_B(\alpha)}{\partial H_G}\right] \frac{1}{\gamma_B}} f(\alpha)$$

and

$$\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}|^{commit} = -\frac{\frac{\partial\overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial\overline{T}_1(\alpha)}\frac{1}{\gamma_{\boldsymbol{B}}}}{\eta_G(\boldsymbol{H_G}) + E\left[\frac{\partial\overline{\gamma}_{\boldsymbol{B}}(\alpha)}{\partial H_G}\right]\frac{1}{\gamma_{\boldsymbol{B}}}}f(\alpha).$$

Moreover,

$$d\boldsymbol{P_S}(\alpha') = \frac{\partial \boldsymbol{P_S}(\alpha')}{\partial\overline{T}_1(\alpha')}d\overline{T}_1(\alpha') + \frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G}d\boldsymbol{H_G}$$

Hence

$$d\boldsymbol{H_G} = \frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}|^{commit}d\overline{T}_1(\alpha) + \int_{\underline{\alpha}}^{\overline{\alpha}}\frac{d\boldsymbol{H_G}}{dP_S(\alpha')}|^{commit}\left[\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial\overline{T}_1(\alpha)}d\overline{T}_1(\alpha) + \frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G}d\boldsymbol{H_G}\right]d\alpha'$$

Since $\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial\overline{T}_1(\alpha)} = 0$ for all $\alpha \neq \alpha'$, we get

$$\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)} = \frac{\frac{d\boldsymbol{H_G}}{d\overline{T}_1(\alpha)}|^{commit} + \frac{d\boldsymbol{H_G}}{dP_S(\alpha)}|^{commit}\frac{\partial \boldsymbol{P_S}(\alpha)}{\partial\overline{T}_1(\alpha)}}{1 - \int_{\underline{\alpha}}^{\overline{\alpha}}\frac{d\boldsymbol{H_G}}{dP_S(\alpha)}|^{commit}\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G}d\alpha'}$$

It is immediate to check that $\frac{\partial^2\mathbb{T}_2}{\partial P_S(\alpha)\partial H_G} < 0$. Since $\frac{d\boldsymbol{H_G}}{dP_S(\alpha)}|^{commit} < 0$ and, for sufficiently large χ, $\frac{\partial \boldsymbol{P_S}(\alpha')}{\partial H_G} > 0$, the denominator is positive. □

Proof of Proposition 8. Immediate from the formulas in the text. □

Proof of Proposition 11. The asset purchase program consists of a quantity $S_B(\alpha)$ of bad trees bought by the planner at the market price. The resource constraint of the planner in period 1 becomes

$$\underline{T}_1 = -\frac{1}{G(\mu_B(\alpha))}\left[P_M(\alpha)S_B(\alpha) + [1 - G(\mu_B)]\overline{T}_1(\alpha)\right]$$

and hence

$$\mathbb{T}_2(\alpha) = (1+\chi)\mu_B(\alpha)\left[P_M(\alpha)S_B(\alpha) + \overline{T}_1(\alpha)\right]$$

The equilibrium in the market for trees is given by

$$P_M(\alpha) = \frac{\lambda_M(\alpha)Z + (1-\lambda_M(\alpha))\alpha Z}{\mu_B(\alpha)}$$

$$\lambda_M(\alpha) = \frac{\left[1 - G\left(\frac{Z}{P_M(\alpha)}\right)\right]H_G}{\left[1 - G\left(\frac{Z}{P_M(\alpha)}\right)\right]H_G + H_B - S_B(\alpha)}$$

$$G(\mu_B(\alpha))W = [1 - G(\mu_B(\alpha))]H_B P_M(\alpha) + [1 - G(\mu_S(\alpha))] + [1 - G(\mu_B(\alpha))]\overline{T}_1(\alpha)$$

Note that $S_B(\alpha)$ does not appear in the market clearing condition as it cancels out with the transfers collected by the planner. Thus, if $P_M(\alpha) = P_S(\alpha)$, the two instruments generate the same $\mu_B(\alpha)$.

Therefore, if a positive wedge and the asset purchase program generate the same $P_M(\alpha) = P_S(\alpha)$ and they have the same cost, they are equivalent. Let's begin with the price. The two instruments generate the same price if and only if

$$P_S(\alpha) = \frac{\frac{\left[1-G\left(\frac{Z}{P_M(\alpha)}\right)\right]H_G}{\left[1-G\left(\frac{Z}{P_M(\alpha)}\right)\right]H_G+H_B-S_B(\alpha)}Z + \frac{H_B}{\left[1-G\left(\frac{Z}{P_M(\alpha)}\right)\right]H_G+H_B-S_B(\alpha)}\alpha Z}{\mu_B(\alpha)}$$

Hence, we need to choose $S_B(\alpha)$ as the solution to

$$S_B(\alpha) = \left[1-G\left(\frac{Z}{P_S(\alpha)}\right)\right]H_G + H_B - \frac{\left[1-G\left(\frac{Z}{P_M(\alpha)}\right)\right]H_G Z + H_B \alpha Z}{\mu_B(\alpha) P_S(\alpha)}$$

with $\mu_B(\alpha)$ being the solution to

$$G(\mu_B(\alpha))W = [1-G(\mu_B(\alpha))]H_B P_S(\alpha) + [1-G(\mu_S(\alpha))] + [1-G(\mu_B(\alpha))]\overline{T}_1(\alpha)$$

Thus, we only need to show that the two instruments have the same cost for the planner. The cost of the asset purchase program is

$$\mathbb{T}_2^{AP} = (1+\chi)\mu_B(\alpha)\left[P_S(\alpha)S_B(\alpha) + \overline{T}_1(\alpha)\right]$$

while the cost of the positive wedge is

$$\mathbb{T}_2^{W} = (1+\chi)\mu_B(\alpha)\left[\left[P_S(\alpha) - \frac{\frac{[1-G(\mu_S(\alpha))]H_G Z + H_B\alpha Z}{[1-G(\mu_S(\alpha))]H_G + H_B}}{\mu_B(\alpha)}\right][[1-G(\mu_S(\alpha))]H_G + H_B] + \overline{T}_1(\alpha)\right]$$

And the two costs are equal if and only if

$$P_S(\alpha)S_B(\alpha) = \left[P_S(\alpha) - \frac{\frac{[1-G(\mu_S(\alpha))]H_G Z + H_B\alpha Z}{[1-G(\mu_S(\alpha))]H_G + H_B}}{\mu_B(\alpha)}\right][[1-G(\mu_S(\alpha))]H_G + H_B]$$

which is true if we replace with the expression for $S_B(\alpha)$

$$P_S(\alpha)[[1-G(\mu_S(\alpha))]H_G + H_B] - \frac{[1-G(\mu_S(\alpha))]H_G Z + H_B\alpha Z}{\mu_B(\alpha)} = \left[P_S(\alpha) - \frac{\frac{[1-G(\mu_S(\alpha))]H_G Z + H_B\alpha Z}{[1-G(\mu_S(\alpha))]H_G + H_B}}{\mu_B(\alpha)}\right][[1-G(\mu_S(\alpha))]H_G + H_B]$$

so both instruments have the same cost.

□

Proof of Proposition 9. That transaction taxes can play the role of negative wedges is immediate. Given the results in Lemma 11, any policy that prescribes weakly negative wedges in the case of a financial collapse can be implemented with asset purchase programs. This is always true with full commitment from Proposition 7. For the optimal policy under imperfect commitment, recall that when $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $\omega(\alpha) > 0$ (i.e., Regime iv), we have

$$\frac{\partial \boldsymbol{U_1}(\alpha)}{\partial P_S(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial P_S(\alpha)} \propto \frac{\partial \boldsymbol{U_1}(\alpha)}{\partial \overline{T}_1(\alpha)} - \frac{\chi}{1+\chi}\frac{\partial \mathbb{T}_{\mathbf{2}}(\alpha)}{\partial \overline{T}_1(\alpha)}$$

Hence, the planner will never find it optimal (in an interior solution) to induce a policy in period 1 with $P_S(\alpha) < \frac{Z}{\mu^{\max}}$ and $\omega(\alpha) > 0$. Finally, one-period bonds can be mapped to transfers as follows: the planner sells state contingent government bonds in period 0 at price $Q_0^B(\alpha)$ with face value $\overline{T}_1(\alpha)$, which it rebates to the agents. All agents then receive $\overline{T}_1(\alpha)$ in period 1. The planner then sells new one-period bonds with face value $\mu_B(\alpha)[\overline{T}_1(\alpha) + [[P_S(\alpha) - P_B(\alpha)]S(\alpha)]$ at price $\mu_B(\alpha)$. It is immediate to see that these transfers satisfy the planner's resources constraints and implement the optimal allocation.

□

B Planner's problem under different degrees of information incompleteness

I first find the solution to a relaxed version in which the planner can observe agents' μ but not the agents' portfolios, trades or production decisions. It turns out that by observing μ, the planner can implement the (*ex-ante*) first best allocation. Then, I show that first best is not feasible when μ is unobservable. These exercises clarify the role of the different information frictions in this economy.

Observable μ

Consider the program (PP), but assume that the planner can observe agents' type μ, so that the optimal plan does not need to satisfy the IC constraints. The tree quality production and the agents' portfolios are still their private information, and trade of trees takes place in an anonymous market. Under these assumptions, the planner can implement the *ex-ante* first best allocation.

Lemma 12. *Consider a planner that can observe the agents' individual type μ, but cannot observe agents' tree quality production or their portfolios, and trade of trees takes place in an anonymous market. The solution to the planner's problem is a solution to the first best program* (FB). *The volume traded in the market for trees is zero.*

Proof. Consider the following plan:

$$P_S(\alpha) = 0, P_B(\alpha) > Z$$

$$T_1(\mu,\alpha) = \begin{cases} -W & \text{if } \mu < \tilde{\mu} \\ \frac{G(\tilde{\mu})W}{1-G(\tilde{\mu})} & \text{if } \mu \geq \tilde{\mu} \end{cases}$$

$$T_2(\mu,\alpha) = 0$$

for some $\tilde{\mu} > 1$. With these prices and transfers, the volume traded in the market for trees is zero. Thus, $\gamma_G = Z$ and $\gamma_B = E[\alpha]Z$. By Assumption 1, this implies $H_B = 0$. Moreover, the transfers are feasible since

$$\int_1^{\mu^{\max}} T_1(\mu,\alpha)dG(\mu) = -G(\tilde{\mu})W + [1-G(\tilde{\mu})]\frac{G(\tilde{\mu})W}{1-G(\tilde{\mu})} = 0$$

Setting $\tilde{\mu} = \mu^{\max}$, the plan implements the first best.

Suppose there is a plan that implements the first best with positive volume traded. For trade to happen, it must be that $P_S(\alpha) \geq \frac{Z}{\mu^{\max}}$, as only good trees are produced in the first best allocation. Since first best implies that $\mu_B(\alpha) = \mu^{\max}$, we get

$$\gamma_B \geq \gamma_G \geq Z$$

which contradicts that only good trees are produced. Thus, the first best implementation requires to shut down the market for trees. □

The economy features two different sources of information asymmetry. First, agents have private information about the quality of the trees they produce in period 0 and they trade in period 1. Second, they have private information about their liquidity needs in period 1, given by μ. These two frictions, combined with the anonymity in the market for trees, leads to a *laissez-faire* equilibrium that features inefficient production of trees, and the exposure of the economy to abrupt financial crises. Lemma 12 shows, however, that by observing agents' type μ, the planner can achieve a first best allocation, even in the presence of the other sources of information asymmetry. The optimal plan is as follows: the planner sets $T_1(\mu,\alpha) = -W_1$ for all $\mu < \mu^{\max}$ and transfers all the resources to the agents with $\mu = \mu^{\max}$. This way, the allocation of consumption is optimal and the demand for trees is zero, effectively shutting down the private market, and making the information asymmetry about tree quality inconsequential. Since bad trees are produced only to be sold in the secondary market, the induced equilibrium features only production of good trees. That is, the optimal policy separates the liquidity value of assets from their dividend value, so that trees are produced only for fundamental reasons. Crucial to this solution is the planner's "taxation" power (through transfers), which allows it to bypass the agents' limited commitment problem.[40]

In contrast, when μ is unobservable, the resource requirements of the IC constraints makes the first best allocation infeasible.

Unobservable μ

Now, assume that the planner cannot observe μ. Even if the deadweight loss from transfers is zero, i.e., $\chi = 0$, and the production of trees is observable, the planner cannot achieve a first best allocation.

Lemma 13. *Consider the problem of a planner that cannot observe the agents' individual type μ, but it can observe the agents' tree quality production and portfolios. Suppose that the deadweight loss from transfers is zero (i.e., $\chi = 0$). The solution to the planner's problem is not a solution of the first best program* (FB)*.*

Proof. First, suppose that the market for trees is inactive. From incentive compatibility, the transfers in period 1 that implement the first best require the following transfers in period 2:

$$\mathcal{T}_2(\mu,\alpha) = \tilde{\mu}\,[T_1(\tilde{\mu},\alpha) + W] \quad \forall\, \mu < \tilde{\mu}$$

(see proof of Lemma 5 below for a detailed derivation of these transfers). Therefore

$$\mathbb{T}_2(\alpha) = -(1+\chi)G(\tilde{\mu})\tilde{\mu}\,[T_1(\tilde{\mu},\alpha) + W] = -(1+\chi)\frac{G(\tilde{\mu})\tilde{\mu}}{1-G(\tilde{\mu})}W.$$

But

$$\lim_{\tilde{\mu}\to\mu^{\max}} \mathbb{T}_2(\alpha) = -\infty$$

So the transfers are not feasible. It is immediate to see that any other transfer schedule different than the one in Lemma 12 is either infeasible or does not reallocate all the endowment to the agents with $\mu = \mu^{\max}$.

Now, suppose the market for trees is active. Since the planner can observe the production of trees, it can implement $\lambda_E = 1$. Moreover, a positive demand for trees in state α requires $T_1(\mu,\alpha) > -W$ for some $\mu < \mu^{\max}$. In that case, market clearing implies

$$\int_1^{\mu_B(\alpha)} [W + T_1(\mu,\alpha)]\,dG(\mu) = [1 - G(\mu_B(\alpha))]\,H_G P_B(\alpha)$$

[40] Note, however, that this policy does not implement an *interim* Pareto improvement, since agents with $\mu < \mu^{\max}$ are worse off after the planner's intervention.

Given that $\lambda_M(\alpha) = 1$, $P_B(\alpha) = \frac{Z}{\mu_B(\alpha)}$, and we get

$$\frac{1 - G(\mu_B)}{G(\mu_B)\mu_B} = \frac{W + \frac{\int_1^{\mu_B} T_1(\mu,\alpha)dG(\mu)}{G(\mu_B)}}{ZH_G} > 0$$

since $\frac{\int_1^{\mu_B} T_1(\mu,\alpha)dG(\mu)}{G(\mu_B)} > -W$ by construction. Hence, $\mu_B < \mu^{\max}$ and the first best is not implemented.

□

Recall that first best requires that the planner transfer all the endowment to the agents with $\mu = \mu^{\max}$. When the planner cannot observe agents' type μ, any transfer intended for one type has to be made available to all other types. Because of the assumption that agent-specific transfers in period 2 have to be non-negative, the IC constraints imply that all agents with $\mu < \mu^{\max}$ have to be promised at least $\mu^{\max}[T_1(\mu^{\max}, \alpha) + W_1]$ in period 2, which is what the marginal agent is giving up by not choosing the transfers intended for $\mu = \mu^{\max}$. It turns out that this promise requires an infinite amount of resources. This argument resembles Bewley (1983), who finds that in a model with idiosyncratic risk and infinitely lived agents, the quantity of "money" (which pays a positive interest, so it resembles government bonds) necessary for agents to self-insure is infinite, so first best is not feasible. I extend Bewley's result to a more general transfer scheme and finitely lived agents. Moreover, the result does not rely on a positive deadweight loss of transfers.