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Abstract

In many search models of the labor market, unemployment insurance (UI) is conveniently interpreted as the value of leisure or home production and is, therefore, treated as a parameter. However, in reality, UI has to be funded through taxation that might be distortionary. In this paper, I analyze the welfare implications of different taxation systems within two equilibrium models of unemployment: random search and directed search. In a random search model without taxes, efficiency is typically not achieved, unless the so-called *Hosios condition* is satisfied. If the bargaining power of firms is large, a lump-sum tax can discourage firms from entering and improve welfare. In a directed search model without taxes, constrained efficiency is always achieved. Since firms “direct” workers to apply to them by posting wages, raising UI funds in a lump-sum manner always distorts the efficient allocation, as it gives firms an incentive to be excessively aggressive in their attempt to maximize the probability of filling up their vacancies. I discuss two ways through which this externality can be internalized and efficiency can be re-established.

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1 Introduction

In many search and matching models of the labor market, like the Mortensen-Pissarides model, unemployment insurance (henceforth UI) is interpreted as the value of leisure or home production.¹ This assumption is convenient because it allows authors to treat UI as a parameter of the model. However, in reality, UI has to be funded through taxes, and these taxes may distort the economy's allocations and affect welfare. In this paper, I study a search model of the labor market, in which unemployment arises as an equilibrium result, and the government wishes to guarantee an UI payment to all unemployed workers. I examine the optimal level of the unemployment benefit and the welfare implications of raising funds towards this benefit through various taxation systems, and under various specifications regarding the nature of the matching process.

The traditional approach of modeling frictions in the labor market assumes that firms search for workers, and *vice versa*, in a random way: an aggregate object, termed the “matching function”, brings together vacant firms and unemployed workers in random bilateral matches, and within all formed matches the terms of trade are determined through Nash bargaining. In this *random search* model, the incentive for firms to open a costly vacancy and search for workers is driven by their private benefit, which, in turn, is determined by bargaining. On the other hand, the social benefit of one more vacancy in the market is reflected by the consequent increase in the total number of successful matches (i.e. the elasticity of the matching function with respect to vacancies). Since the Nash sharing rule is based on an arbitrary parametric specification of the firms' and workers' bargaining power, there is no reason to expect that the marginal and social benefits described above will coincide. As a result, efficiency is typically not achieved in these types of models, unless the so-called *Hosios condition* is satisfied (Hosios (1990)).

A more recent approach to labor market frictions assumes that firms advertise their wage, and workers can direct their search towards specific wages. Equilibrium unemployment arises because firms only have a limited number of vacancies, and workers cannot coordinate their applications strategies. Hence, some firms can end up with a number of applications that exceeds their job openings, while others might receive no applications. This *directed search* model has become popular in the literature because it endogenizes the matching function, and it allows the study of competition among firms within markets that do not perfectly clear. In the core of the directed search model, there is a trade-off between a higher wage promise and a higher matching probability. Firms who contemplate opening a vacancy and attracting workers realize this trade-off, thus internalizing the search externality that governs the random search model. Consequently, in the baseline directed search model, the equilibrium allocation is always efficient.

Since the goal of this paper is to study the welfare effects of raising UI funds through different taxation systems, and since the baseline random and directed search models are characterized by such diverse predictions regarding efficiency, the following questions arise naturally. What is the optimal design of the taxation system, if the government wishes to guarantee a certain level of UI for all unemployed workers? Does the optimal design

¹ See for example Pissarides (2000).

of the taxation system depend on whether search in the labor market takes place in a directed or a random manner? In what follows, I attempt to provide answers to these questions.

The starting point of the analysis is a static model of the labor market with a given, finite number of identical firms and workers.² Initially, all agents are risk neutral, but I also discuss the interesting case of risk averse workers. Firms have only one job opening, and workers, who wish to maximize their expected utility, can apply only to one firm. The lack of coordination among workers means that, in equilibrium, excess supply and excess demand for labor will coexist. Hence, unemployment arises as an equilibrium result. Under random search, workers cannot observe any wage postings by the firms, hence, they choose to apply to one firm at random. Under directed search, on the other hand, workers observe all firms' advertisements, and they choose their application strategy realizing that firms promising a higher wage will be associated with higher queue lengths.

Taking as given (for now) that the government wishes to pay an UI, z , to every unemployed worker, I consider two taxation systems. The first is a lump-sum system, according to which every firm in the market pays an equal share of the UI bill (the product of z and the number of unemployed workers that arises in equilibrium). The second taxation system is personalized, and it dictates that each firm has to pay a tax that depends on how many applications it received. Since with directed search firms attract workers by posting public advertisements, for this model I also consider a third alternative: I allow firms to advertise not only a wage paid to the employed worker, but also a payment made to workers who apply and do not obtain the job. This third system is based on an idea by Jacquet and Tan (2012), who refer to it as a "wage-vacancy" contract.

Focusing on a small market with risk neutral agents first, allows me to study the strategic interaction among firms, and describe the economic forces that drive the equilibrium outcomes, under the various taxation systems. In fact, for the case of risk neutral workers, I can obtain closed form solutions for the equilibrium wage and profits. Then, I take the limit as the market grows large, and I assume that firms who wish to enter the labor market and search for workers have to pay an entry fee in advance. In this way, I determine the measure of active firms endogenously, through a zero-profit condition, and compare the decentralized equilibrium allocation with the Social Planner's solution.

It should be noted that, with risk neutral workers, the Social Planner's objective is to choose the measure of active firms in order to maximize the expected output, net of entry fees. In other words, insuring workers against the possibility of being unemployed does not improve welfare. However, this does not necessarily mean that the authorities might not wish to pay an UI to the unemployed. Perhaps the authorities want to achieve a more fair distribution of the output ex post, i.e. after the uncertainty regarding the workers' state has been resolved. This is especially true, since, as I show in what follows, the authorities can promise a positive UI to the unemployed without harming the economy's welfare (directed search model), and in some cases they can even improve welfare

² I focus on a one-shot game for simplicity. However, all the results of the paper are valid in dynamic environments, if one focuses on steady states.

by introducing UI and taxes (random search model).³ More importantly, as I point out above, the case of risk neutral workers is important for the analysis, because it provides a deeper intuition of how the economic forces at work affect equilibrium outcomes.

I now describe the main results of the paper. In the finite random search model, with risk neutral workers, the choice of a taxation system is inessential. Since firms do not make any non-trivial decisions (such as posting wages), personalized taxes do not affect a firm’s behavior. Hence, the equilibrium profits under the two systems are identical. Moving on to the large market, and examining the entry decision of firms, I find that there exist two distinct equilibria. The intuition is as follows. In any standard search model, the typical firm’s profit is decreasing in the number of active firms (more firms imply a lower matching probability). In the model with taxes, this variable is initially decreasing in the measure of firms, but eventually it becomes increasing because more firms imply fewer unemployed workers, and a smaller per firm share of the UI bill. As a result, profit is hump-shaped as a function of the measure of firms, leading to the existence of two equilibria: one with low entry, high taxes per firm, and a high matching probability, and another one with high entry, low taxes per firm, and a low matching probability.

Perhaps the most important result that arises in the random search model is that introducing UI and taxes can improve welfare, even when workers are risk neutral. In the baseline random search model, entry of firms is typically inefficient, and whether too many or too few firms enter in the market depends on whether the firms’ bargaining power is greater or smaller than the elasticity of the matching function with respect to an additional vacancy. If the firms’ bargaining power is greater than this critical level, too many firms enter, and unemployment will be inefficiently low. In this case, introducing taxes can improve welfare by decreasing the equilibrium measure of active firms.

In the directed search model, the taxation system choice becomes crucial. I show that in the finite model, under lump-sum taxation, the equilibrium wage is higher than in a model without taxes. This result might be surprising at first. Since in the model with UI firms know that, in equilibrium, they will have to pay positive taxes, one might expect that they should at least promise lower wages. This reasoning turns out to be wrong. With lump-sum taxation firms have an incentive to post high wages in order to attract many workers, and they only have to pay for a small part of this “aggressiveness”, since, at the end, all firms share equally the UI bill. Personalized taxes, that are Pigouvian in nature, induce firms to internalize this externality, but not fully: in the small market case, the equilibrium profit under personalized taxes and under no taxes do not coincide. I also study the case in which firms post “wage-vacancy” contracts, as in Jacquet and Tan (2012), and show that, in the finite market case, the sharing rule of the equilibrium surplus is indeterminate. This result is in the spirit of Coles and Eeckhout (2003).

Next, I consider the case of a large market with free-entry of firms. Under lump-sum taxation, firms that are contemplating to enter the market, and have rational expecta-

³ This statement needs to be clarified, given that earlier in this paragraph I have stated that “with risk neutral workers [...] insuring workers against the possibility of being unemployed does not improve welfare”. What I mean is that introducing UI does not make workers better off through the insurance channel, but it might improve welfare because the taxes that are necessary in order to raise UI funds might lead to a more efficient entry of firms.

tions, realize that their profit will be negatively affected by the externality (or price war) described above. Once again, the typical firm’s profit is hump-shaped as a function of the measure of active firms, and so this variable is not uniquely pinned down. Regardless of whether the economy ends up in the low or the high entry equilibrium, the measure of active firms is always suboptimal. Hence, the government cannot pay UI to the unemployed without hurting the economy’s welfare. However, personalized taxes fully resolve this inefficiency: in the large market case, the equilibrium profit under personalized taxes and under no taxes coincide. Hence, the equilibrium measure of active firms coincides with the Planner’s solution. Under this type of taxation, the authorities can support *any level of UI* without affecting the economy’s efficiency. What is perhaps more surprising is that the equilibrium allocation is also optimal when the government does not use taxation, but rather requires the firms to post a “wage-vacancy” contract.

Finally, I consider the case of a large market with directed search and risk averse workers. With risk averse workers, the Planner’s solution involves not only maximizing expected output (net of entry fees), but also insuring the workers against the possibility of an unsuccessful search. I characterize the unique, strictly positive optimal level of UI, and I show that, under lump-sum taxation, this level can never be achieved as an equilibrium outcome. With personalized taxes, efficiency can be attained, but this requires that the authorities can set the UI level “just right”, an assumption that might be questionable. I show that the most straightforward way to guarantee that the decentralized market allocation will coincide with the Planner’s solution, is if the authorities do not choose the UI level and the taxation system themselves, but instead they simply require firms to post “wage-vacancy” contracts, *a la* Jacquet and Tan (2012).

This paper is closely related to the pioneering work of Acemoglu and Shimer (1999). In that paper, the authors consider a labor market with risk averse workers, and show that moderate UI can not only increase welfare through risk sharing, but also by increasing output. My paper differs from Acemoglu and Shimer (1999) in that I discuss the important question of how various taxation systems affect welfare, and I also consider the case of random search. The present paper is also closely linked to Golosov, Maziero, and Menzio (2012). Their goal is to study the optimal redistribution of income inequality in a directed search model with heterogeneous firms and moral hazard.⁴ Again, what sets my paper apart is that it provides a comparison of different taxation systems in terms of efficiency. Also, this paper analyzes the case of a finite market, and the possibility of random search. Lastly, the present paper is closely related to Jacquet and Tan (2012), since one of the alternatives that the authorities can adopt in order to raise UI funds is inspired by their “wage-vacancy” contracts. However, Jacquet and Tan (2012) do not study UI or taxation. This paper highlights that, under directed search, their mechanism can replace UI and taxation, in fact, it can do so in the most efficient of ways.

The present paper is also related to a large literature on optimal unemployment insurance. Examples include Hansen and İmrohoroglu (1992), Hopenhayn and Nicolini (1997), Wang and Williamson (2002), Shimer and Werning (2008), and Rendahl (2012).

⁴ In Golosov, Maziero, and Menzio (2012) applying for a job is costly, and the authorities cannot distinguish between a worker who did not apply for a job and one who applied but was not offered one.

What differentiates my paper from this strand of the literature is that, here, the focus is mainly on the optimal way to raise funds towards unemployment insurance, rather than the optimal unemployment insurance itself (although I do discuss the optimal level of unemployment insurance as well). Finally, this paper is related to a number of works in the directed search literature that examine competition among sellers (firms) who can advertise general mechanisms rather than just a single price (wage). Examples of such papers include Julien, Kennes, and King (2000), Coles and Eeckhout (2003), Virag (2007), Eeckhout and Kircher (2010), and Geromichalos (2009). None of these papers study UI and/or taxes explicitly.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 studies equilibrium in a random search model with risk neutral workers. Section 4 focuses on the case of a directed search model with risk neutral workers, and Section 5 generalizes the analysis to include risk aversion. Section 6 provides some concluding remarks.

2 The Model

I consider a static model of the labor market with m firms and n workers, $n, m \geq 2$. In the baseline model m, n are finite, but I also examine the limit as $n, m \rightarrow \infty$. Firms are risk neutral, and I consider the cases of both risk neutral and risk averse workers. Each firm has one vacancy. Any match within the period produces output y . At the end of the period, matched vacancies make a profit equal to y net of the wage, w , that they pay to the worker. Unmatched workers get an unemployment benefit z , and unmatched vacancies get nothing. The matching process and the determination of w depend on the model in question, and I consider two environments: random search and directed search. The size of z is decided by the government in the beginning of the period. The UI funds can be raised through various taxation systems, which I describe in detail below.

I now turn to the description of the matching process and the wage determination. Since the starting point of the analysis is a finite market, I do not assume the existence of an aggregate matching function. Instead, I assume that workers rationally choose their application strategies in order to maximize expected utility. Workers can apply only to one firm, and if a firm receives multiple applications it chooses to offer the job to one of the applicants at random. Moreover, workers cannot coordinate their application strategies. Hence, some firms can end up with more than one applications while others receive none. In this sense, the model gives rise to *equilibrium unemployment*.

I consider two environments. In the first, the directed search model, each firm posts a wage, $w \in [0, y]$, taking as given the strategies of her competitors. Following this stage, workers observe all the wage advertisements and choose a probability of applying to each firm, taking as given the strategies of other workers. As it is common in the literature, I focus on symmetric equilibria in which workers play mixed strategies in the subgame. Equilibria in pure strategies do exist, but they are considered implausible since they require an unreasonable degree of coordination among the workers.⁵ It is assumed that

⁵ For a detailed discussion on pure strategy equilibria see Burdett, Shi, and Wright (2001).

firms commit to their wage announcements, and I focus on wage announcements that lead to non-negative expected profit and expected utility for firms and workers, respectively.

The second environment that I consider is one of random search. This model is a finite market version of the static random search model of Rogerson, Shimer, and Wright (2005). In this framework, firms cannot advertise their wage. This simply means that, from the point of view of a worker, all firms are identical and, therefore, workers choose to apply to one firm at random.⁶ The firms that receive at least one application, bargain with the employed worker over the wage, and β denotes the worker’s bargaining power.

Finally consider taxation. The government wants to guarantee an UI payment of z to each unemployed worker, i.e. to each worker who remains unmatched after either of the matching games described above is played. The level of z is known to all market participants before the game starts. I assume that $z \leq y$, which is typically a necessary, but not sufficient condition for existence of equilibrium. The first way to raise funds, which I refer to as Taxation System A, is a lump-sum tax. In this setup, once the matching game (of either random or directed search) has been completed, the number of unemployed workers is enumerated, and each firm has to pay an equal share of the UI bill.⁷ For example, if a total of $x < n$ workers are unmatched, each firm will pay a tax equal to xz/m . Alternatively, the government can adopt the personalized Taxation System B, under which, each firm pays z only to workers who applied to that specific firm and were not offered the position (because more than one workers applied).

In addition to Systems A and B, for the model of directed search, I also consider a third alternative. Since, firms can publicly post wage advertisements, I study an extension of the baseline model where each firm can post a general schedule (w, z) , where w is a payment (wage) made to the employed worker, and z is a payment (UI) made to workers who applied to the firm in question but did not get a job. As I mention above, this specification follows Jacquet and Tan (2012), who refer to it as “wage-vacancy” contracts. For brevity, I refer to this environment as System C. Notice that this third alternative is only relevant in the directed search framework, since with random search firms cannot make any public advertisements in order to attract workers.

2.1 Efficiency Benchmark

In Sections 3 and 4, I consider the case of risk neutral workers. Under this assumption, I am able to provide closed form solutions for the equilibrium wage and profit, for any

⁶ This interpretation of random search for finite markets follows Lester (2011).

⁷ Since in this paper it is the firms that make the interesting decisions, I assume that only firms pay taxes. Alternatively, one can assume that both firms and workers pay taxes, or even that only firms and employed workers pay taxes. These different assumptions would, of course, affect the equilibrium outcomes of the paper, but they would not change the spirit of the results. Hence, I adopt the assumption that makes the analysis as simple as possible. On the contrary, in Golosov, Maziero, and Menzio (2012) there are moral hazard considerations, and workers choose whether to apply to a firm or not. Hence, in their paper, taxation on workers becomes non-trivial. In fact, many of the interesting results of that paper concern income taxation.

n, m , and under any taxation system. Since I focus on symmetric equilibria, with risk neutral workers and finite n, m , the expected surplus in the economy is always given by

$$S(n, m) \equiv m \left[1 - \left(1 - \frac{1}{m} \right)^n \right] y, \quad (1)$$

where $1 - (1 - 1/m)^n$ is the probability with which the typical firm receives at least one application, when workers apply to all firms with equal probability. Hence, a discussion about efficiency can only take place in a model with free-entry. To that end, I obtain the limit of equilibrium profits, under the various specifications, as $n, m \rightarrow \infty$, keeping the market tightness $b \equiv n/m$ fixed. Each firm is assumed to pay an entry fee in advance in order to enter the labor market, and the equilibrium measure of operating firms is pinned down by a zero-profit condition.⁸ Then, I compare the various equilibrium outcomes with the Social Planner's solution, which is now discussed.

It is a well-known result in the literature, that in the baseline directed search model (no taxes) with risk neutral workers and free-entry, the measure of firms operating in equilibrium coincides with the Social Planner's solution (see for example Shimer (1996), Moen (1997), and Rogerson, Shimer, and Wright (2005)). Hence, it is useful to shortly describe the equilibrium outcome of this benchmark model here. In a finite economy, the symmetric equilibrium wage in the directed search model with no taxes is⁹

$$w^* = \frac{n \left(1 - \frac{1}{m} \right)^n y}{m - \left(m + \frac{n}{m-1} \right) \left(1 - \frac{1}{m} \right)^n}, \quad (2)$$

and the equilibrium profit for each firm is given by

$$\pi^* = \left[1 - \left(1 - \frac{1}{m} \right)^n \right] (y - w^*). \quad (3)$$

Now consider the limiting market. Let $n, m \rightarrow \infty$, with $b = n/m$ held fixed. Noticing that $\lim_{n \rightarrow \infty} (1 - b/n)^n = e^{-b}$, the limiting profit can be written as

$$\bar{\pi}^* = (1 - e^{-b} - be^{-b})y.$$

Hence, if firms have to pay an entry fee $k > 0$ in advance in order to enter the labor market, the equilibrium measure of active firms will be (uniquely) given by

$$b^* \equiv \{b : (1 - e^{-b} - be^{-b})y = k\}.$$

⁸ I only study free-entry in the large market case. Free-entry in a finite market creates some complexities, which I wish to avoid here, since the focus of this paper is on UI and optimal taxation. For more details see Jacquet and Tan (2012) and Geromichalos (2009).

⁹ I do not explain equations (2) and (3) in detail, since they follow directly from the analysis in Burdett, Shi, and Wright (2001), if one replaces sellers with firms and buyers with workers.

For the reason explained above, in what follows I will refer to b^* as the Social Planner's solution for the environment with risk neutral workers.¹⁰

In the case of risk averse workers, it will not be possible to provide closed form solutions for the equilibrium wage and profits. In this case, I will focus directly on large markets, and compare equilibrium outcomes with the Social Planner's allocation. With risk aversion, the Planner's problem will be slightly more complicated, because total surplus depends not only on b but also on the z paid to unemployed workers. I relegate the discussion of the Planner's problem under risk aversion to Section 5, since it is only relevant to that section.

3 The Random Search Model

3.1 The Finite Market Case

I start by analyzing equilibrium in a model with random search and risk neutral workers. First, I describe a model with finite market participants. Later, I consider the case of a large market and study free-entry.

Suppose that the government wants to guarantee an UI payment z to all unemployed workers. If the authorities adopt Taxation System A, the expected profit for a typical firm is given by

$$\pi_R^A = \left[1 - \left(1 - \frac{1}{m} \right)^n \right] (1 - \beta)(y - z) - \frac{1}{m} \left\{ n - m \left[1 - \left(1 - \frac{1}{m} \right)^n \right] \right\} z. \quad (4)$$

The expression for π_R^A is very intuitive. For the typical firm, the probability of receiving at least one application is $1 - (1 - 1/m)^n$. In this case, a net surplus of $y - z$ will be generated, and a fraction $1 - \beta$ of that surplus will stay with the firm. The second term in (4) represents the expected taxes paid by each firm. The term in curly brackets is the total number of unemployed workers. Each of these workers receives an UI payment equal to z , and each firm pays a share $1/m$ of the total UI bill.

Now consider the case in which the government adopts Taxation System B. The expected profit for the typical firm is given by

$$\pi_R^B = \sum_{i=1}^n \binom{n}{i} \left(1 - \frac{1}{m} \right)^{n-i} \left(\frac{1}{m} \right)^i [(1 - \beta)(y - z) - (i - 1)z]. \quad (5)$$

The term $\binom{n}{i} (1 - 1/m)^{n-i} (1/m)^i$ is the probability with which the firm receives i applications. For any $i \geq 1$ the firm hires one worker and receives $(1 - \beta)(y - z)$. If $i > 1$ the

¹⁰ The fact that the Planner's solution and the equilibrium value of b in the baseline directed search model coincide, is very standard in the literature. For the reader who might not be familiar with this result, consider a large market with a measure of workers normalized to the unit, and, since $b = n/m$, a measure of firms equal to $1/b$. It follows from (1), that the total surplus (or production) in this economy is $(1 - e^{-b})y/b$. The Planner's objective is to maximize the surplus net of entry fees, i.e. $\max_b \{(1 - e^{-b})y/b - k/b\}$, and one can easily verify that b^* is the unique maximizer of this problem.

firm also has to pay an amount z to all $i - 1$ workers who were not offered the job. The expression in (5) looks quite different than the one in (4). However, one can show that¹¹

$$\sum_{i=1}^n \binom{n}{i} \left(1 - \frac{1}{m}\right)^{n-i} \left(\frac{1}{m}\right)^i = 1 - \left(1 - \frac{1}{m}\right)^n, \quad (6)$$

$$\sum_{i=1}^n \binom{n}{i} \left(1 - \frac{1}{m}\right)^{n-i} \left(\frac{1}{m}\right)^i i = \frac{n}{m}. \quad (7)$$

Substituting (6) and (7) into (5) reveals that

$$\pi_R^B = \pi_R^A = \left[1 - \left(1 - \frac{1}{m}\right)^n\right] (1 - \beta)(y - z) - \frac{1}{m} \left\{n - m \left[1 - \left(1 - \frac{1}{m}\right)^n\right]\right\} z \equiv \pi_R. \quad (8)$$

One can conclude that in the random search framework, raising UI funds through lump-sum or personalized taxes makes no difference. This result is very intuitive. With random search firms do not have access to advertisements that could help them attract workers. Personalized taxes cannot affect the firms' behavior because the firms do not make any non-trivial choices. Hence, the equilibrium profits under the two systems are identical. This conclusion is specific to the random search environment. As it will be shown in the next section, in the directed search model, whether the government chooses System A or B in order to raise UI funds will critically affect the equilibrium outcome.

3.2 The Large Market Case

In this subsection, I assume that $n, m \rightarrow \infty$, and the market tightness $b \equiv n/m$ remains constant. In the large market, I normalize the measure of workers to the unit, so that b represents the inverse of the measure of firms. Moreover, I assume that all firms who wish to enter into the labor market have to pay an entry fee $k > 0$ in advance. Then, I use a zero-profit condition in order to pin down the measure of active firms, $1/b$, in the labor market and compare it to the Social Planner's solution.

Recall that in the finite market case, the profit of the firm is given by π_R , described in (8), and it is independent of the taxation system adopted by the authorities. Using the definition of b , one can write

$$\pi_R = \left[1 - \left(1 - \frac{b}{n}\right)^n\right] (1 - \beta)(y - z) - bz + \left[1 - \left(1 - \frac{b}{n}\right)^n\right] z,$$

and since $\lim_{n \rightarrow \infty} (1 - b/n)^n = e^{-b}$, we have

$$\bar{\pi}_R(b) = \lim_{n \rightarrow \infty} \pi_R = (1 - e^{-b}) (1 - \beta)(y - z) - \left(1 - \frac{1 - e^{-b}}{b}\right) bz, \quad (9)$$

¹¹ The expression on the left-hand side of (6) is the sum of the probabilities of receiving one or more applications, so it is equal to 1 minus the probability of receiving no applications. The expression on the left-hand side of (7) is just the expected number of applications per firm. Since workers randomize among all firms, this expression has to be equal to the market tightness n/m .

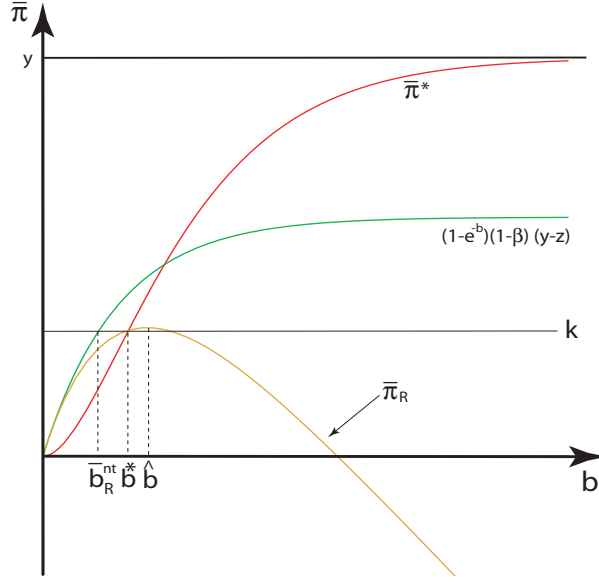


Figure 1: Equilibrium in the random search model.

where $\bar{\pi}_R(b)$ denotes the limiting profit of each firm, net of the entry fee. Equation (9) admits an interpretation similar to (4): $1 - e^{-b}$ is the probability with which a firm receives at least one application. Also, the total measure of unemployed workers is given by $1 - (1 - e^{-b})/b$, and the total UI bill is divided equally among the $1/b$ firms.

The (inverse of the) measure of firms that decide to enter the market is given by $\bar{b}_R \equiv \{b : \bar{\pi}_R(b) = k\}$. The function $\bar{\pi}_R$ is strictly concave in b , since $\bar{\pi}_R''(b) = -e^{-b}(1 - \beta)y - \beta ze^{-b} < 0$. Moreover, $\bar{\pi}_R$ attains a global maximum at the point $b = \hat{b} = \ln[\beta + (1 - \beta)y/z]$. Therefore, some interesting results emerge. First, a necessary and sufficient condition for the existence of a labor market equilibrium is

$$\bar{\pi}_R(\hat{b}) = (1 - \beta)(y - z) - \ln \left[\beta - \frac{(1 - \beta)y}{z} \right] > k.$$

The term $\bar{\pi}_R(\hat{b})$ is decreasing in z , thus, as k increases, the set of values of z under which a market equilibrium exists shrinks.

Another notable result that follows from the curvature of $\bar{\pi}_R$ is that, if equilibria exist, they always come in pairs, except from the non-generic case in which $\bar{\pi}_R(\hat{b}) = k$. The introduction of taxes gives rise to some interesting complementarities. In the baseline model (without taxes), the typical firm's equilibrium profit is always decreasing in the number of firms in the market (more firms make competition for workers harder). In the model with taxes, however, the typical firm's profit can be increasing in the number of firms for two reasons: more firms imply a lower number of unemployed workers, and a smaller per firm share of the UI bill. This new force endows $\bar{\pi}_R$ with its hump-shape, and leads to the existence of multiple equilibria.

These results are summarized in Figure 1, where I also plot the Planner's solution, b^* ,

and the equilibrium in the random search model with no taxes, $\bar{b}_R^{nt}$. In the model with taxes and UI, the equilibrium measure of firms is always lower compared to the model without taxes. How close the equilibrium b , $\bar{b}_R$, will be to b^* depends on the parameter β . In the baseline random search model, constraint efficiency will be achieved only if the bargaining power of the firm is equal to the elasticity of the matching function with respect to vacancies, a requirement known as the Hosios condition.¹² If β is greater than this critical level, too many firms enter, and unemployment will be inefficiently low. In this case, introducing taxes can improve welfare by increasing $\bar{b}_R$. Figure 1 represents the extreme case in which z has been chosen so that $\bar{b}_R$ exactly coincides with the Planner's solution b^* (assuming that the lower equilibrium b prevails).

4 The Directed Search Model

4.1 The Finite Market Case

First, I consider the case in which the government wishes to guarantee an UI payment of z to every unemployed worker, and adopts the Taxation System A in order to raise the necessary funds. Unlike the random search model, where firms take as given the probability with which they receive applications, here they can affect these probabilities through publicly posting the wage that they will pay to the employed worker. Since, the goal is to construct symmetric equilibria, I focus on the behavior of the typical firm j , and I assume that all other firms post the same wage $\tilde{w}$. Firm j chooses its wage w^j rationally, realizing that, in the second stage of the game, workers will observe $(w^j, \tilde{w})$ (and the z announced by the government), and they will determine the probabilities with which they visit each firm, so that they are indifferent among all firms.

Let θ denote the probability with which the typical worker applies to firm j , and let $\tilde{\theta} = (1 - \theta)/(m - 1)$ represent the probability with which she applies to any of the other $m - 1$ firms. Then, the problem solved by firm j is

$$\begin{aligned} \max_{w^j} \left\{ \pi^j(w^j, \theta) \right\} &= [1 - (1 - \theta)^n] (y - w^j) - \tau_D^A, \\ \text{s.t. } \frac{1 - (1 - \theta)^n}{n\theta} w^j + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta} \right] z &= \frac{1 - (1 - \tilde{\theta})^n}{n\tilde{\theta}} \tilde{w} + \left[1 - \frac{1 - (1 - \tilde{\theta})^n}{n\tilde{\theta}} \right] z, \\ \tau_D^A &= \frac{1}{m} \left\{ n\theta - 1 + (1 - \theta)^n + (m - 1) \left[n\tilde{\theta} - 1 + (1 - \tilde{\theta})^n \right] \right\} z. \end{aligned}$$

¹² From the definition of b^* and the equilibrium condition $(1 - e^{-b})(1 - \beta)y = k$, it turns out that efficiency in the baseline random search model requires $1 - \beta = (1 - e^{-b} - be^{-b})/(1 - e^{-b})$. To see why this term is equal to the elasticity of the matching function with respect to vacancies, let the measure of vacancies be denoted by v and recall that here $v = 1/b$. Given that the measure of workers has been normalized to the unit, the matching function can be written as $m(1, v) = v(1 - e^{-1/v})$. It is then straightforward to show that $(\partial m / \partial v)(v/m) = (1 - e^{-b} - be^{-b})/(1 - e^{-b})$.

A few comments are in order. The term $1 - (1 - \theta)^n$ represents the probability with which a firm receives one or more applications. Therefore, when worker i applies to a firm which is chosen by other workers with probability θ , the probability with which she will get the job is given by $[1 - (1 - \theta)^n]/(n\theta)$. Now consider the tax, τ_D^A , that firm j has to pay. This should equal a fraction $1/m$ of the total UI bill, which, in turn, equals z times the number of unmatched workers in the economy. To see why the term in curly brackets, in the problem above, is the aggregate unemployment, notice that when workers apply to a firm with probability θ , the number of workers to whom that firm cannot offer a job is

$$\sum_{i=1}^n \binom{n}{i} (1 - \theta)^{n-i} \theta^i (i - 1) = n\theta - 1 + (1 - \theta)^n,$$

where the equality follows directly from equations (6) and (7). Since here there is one firm that receives workers' applications with probability θ (firm j), and $m - 1$ firms that receive workers' applications with probability $\bar{\theta}$ (the rest), the term in curly brackets represents the economy-wide unemployed.

Having established the typical firm's problem, I now describe the symmetric equilibrium wage in the model with directed search and UI funds raised through System A.

Lemma 1. *a) In the unique symmetric equilibrium of the directed search model, for some given z and under Taxation System A, all firms post the wage*

$$w^A = \frac{n \left(1 - \frac{1}{m}\right)^n y + m \left[1 - \left(1 + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n\right] z}{m - \left(m + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n}. \quad (10)$$

b) The equilibrium wage is higher than the equilibrium wage in the model with no taxes, for all $n, m \geq 2$, that is, $w^A > w^$, where w^* is described in (2).*

c) The equilibrium profit, π^A , is lower than the equilibrium profit in the model with no taxes, π^ , for all $n, m \geq 2$. More precisely,*

$$\pi^A = \left[1 - \left(1 - \frac{1}{m}\right)^n\right] (y - w^A) - \left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n\right] z < \pi^*, \quad (11)$$

where π^ is described in (3).*

Proof. See the appendix. □

Lemma 1 describes the equilibrium wage under Taxation System A. Notice that equilibrium profit is decreasing in z . I restrict attention to $z > 0$, such that $\pi^A \geq 0$.¹³ The fact that $w^A > w^*$ might be striking at first. Since in the model with taxes every unemployed worker will receive an UI payment, and since workers care about expected utility, which includes both w and z , weighted by the relevant probabilities, one might think that the wages that firms post should be lower compared to the model without UI. In other words, since firms know that in equilibrium they will have to pay some taxes towards UI, one

¹³ If z exceeds this critical level, equilibrium collapses since firms do not find it worth operating.

might expect that they should at least promise lower wages.

This reasoning turns out to be wrong, and the justification is as follows. In the directed search model, a higher wage has two opposing effects: ex post it leads to a lower profit, but ex ante it leads to a higher probability of filling up the firm's vacancy. When taxes are raised in a lump-sum manner, like under System A, the positive effect of a higher wage described above is magnified. Firms post high wages in order to attract many workers, while they only have to pay for a very small part of their aggressiveness, since other firms will pay equally for the unemployed that a firm "created" by offering a high wage and attracting workers whom it could not employ. Of course, in a symmetric equilibrium all firms post the same wage and receive the same expected number of applications. However, the externality described above leads to $w^A > w^*$. Since in the model with UI, firms post a higher wage, and pay taxes, part (c) of Lemma 1 follows immediately.

I now focus on the case in which the authorities adopt the Taxation System B in order to raise UI funds. Notice that this system is personalized, in the sense that firms pay taxes that depend on how many applications it received. Once again, focus on the behavior of firm j and assume that all other firms post $\tilde{w}$. Let θ denote the probability with which the typical worker applies to firm j , and $\tilde{\theta} = (1 - \theta)/(m - 1)$ the probability with which she applies to any of the other $m - 1$ firms. Firm j 's problem is to

$$\begin{aligned} \max_{w^j} \left\{ \pi^j(w^j, \theta) \right\} &= [1 - (1 - \theta)^n] (y - w^j) - \tau_D^B, \\ \text{s.t. } \frac{1 - (1 - \theta)^n}{n\theta} w^j + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta} \right] z &= \frac{1 - (1 - \tilde{\theta})^n}{n\tilde{\theta}} \tilde{w} + \left[1 - \frac{1 - (1 - \tilde{\theta})^n}{n\tilde{\theta}} \right] z, \\ \tau_D^B &= [n\theta - 1 + (1 - \theta)^n] z. \end{aligned}$$

The only difference between the firm's problem under Systems A and B, is the definition of the tax terms τ_D^A, τ_D^B . Under System B, the firm cannot be aggressive and expect to free-ride on other firms, since now it has to pay taxes that are proportional to the number of applications it receives. As we shall see, this will be reflected in the determination of the equilibrium wage and profit. The following lemma states the relevant result.

Lemma 2. *a) In the unique symmetric equilibrium of the directed search model, for some given z and under Taxation System B, all firms post the wage*

$$w^B = \frac{n \left(1 - \frac{1}{m}\right)^n y - \left[\left(m + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n + \frac{n(m-1)}{m} - m \right] z}{m - \left(m + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n}. \quad (12)$$

b) For any $n, m \geq 2$, the equilibrium wages in the various models satisfy $w^A > w^ \geq w^B$.
c) For any $n, m \geq 2$, the equilibrium profits in the various models satisfy $\pi^* > \pi^B > \pi^A$, where*

$$\pi^B = \left[1 - \left(1 - \frac{1}{m}\right)^n \right] (y - w^B) - \left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n \right] z. \quad (13)$$

Proof. See the appendix. □

Again I restrict attention to $z > 0$, such that $\pi^B \geq 0$. Under Taxation System B, firms realize that if they advertise a very high wage, trying to maximize the probability of filling up their vacancy, they will have to pay the full cost of this aggressive behavior. This leads to an equilibrium wage that is lower compared to the model without taxes and, of course, the model where UI funds are raised through System A. Hence, under System B, which is Pigouvian in nature, the firms internalize the externality that they exert on one another under lump-sum taxation, but not fully: in the finite market, under System B, the firms cannot fully undo the effects of taxation, since $\pi^B < \pi^*$. This result will be altered in the large market case. Lastly, the term π^B is strictly decreasing in z . Hence, in the finite market with fixed n, m , a higher UI changes the sharing rule of the surplus in favor of the workers.

Before I proceed to the analysis of the limiting market, which will allow me to study free-entry and welfare under the different systems, I study a third alternative that the authorities can adopt in order to guarantee an UI payment to the unemployed. This third specification does not involve taxes. Instead, the government requires firms to post a general schedule (w, z) , where w denotes the wage paid to the employed worker, and z is an UI payment that is made to all workers who apply to the firm and are not offered a job. I refer to this specification as System C.

The typical firm j chooses a pair (w^j, z^j) , taking as given the strategies of its competitors, $(\tilde{w}, \tilde{z})$. Let θ denote the probability with which any given worker applies to firm j , and $\tilde{\theta} = (1 - \theta)/(m - 1)$ the probability with which she applies to any of the other $m - 1$ firms. Then, firm j 's problem is to

$$\begin{aligned} \max_{w^j, z^j} \left\{ \pi^j(w^j, z^j, \theta) \right\} &= [1 - (1 - \theta)^n] (y - w^j) - [n\theta - 1 + (1 - \theta)^n] z^j, \\ \text{s.t. } \frac{1 - (1 - \theta)^n}{n\theta} w^j + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta} \right] z^j &= \frac{1 - (1 - \tilde{\theta})^n}{n\tilde{\theta}} \tilde{w} + \left[1 - \frac{1 - (1 - \tilde{\theta})^n}{n\tilde{\theta}} \right] \tilde{z}, \end{aligned}$$

The firm's problem admits a similar interpretation as the ones described previously, under Systems A and B. The main difference is that now z is not a parameter chosen by the government, but an endogenous object that will be determined in equilibrium. The following lemma characterizes equilibrium under System C.

Lemma 3. *Consider a directed search model, and assume that firms have to pay themselves UI to the workers who apply to them and are not offered a job. Then, every pair (w^C, z^C) that satisfies*

$$w^C = \frac{n \left(1 - \frac{1}{m}\right)^n y - \left[\left(m + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n + \frac{n(m-1)}{m} - m \right] z^C}{m - \left(m + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n}, \quad (14)$$

$$\pi^C = \left[1 - \left(1 - \frac{1}{m}\right)^n \right] (y - w^C) - \left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n \right] z^C \in [0, S(m, n)], \quad (15)$$

constitutes a symmetric equilibrium. The expression $S(n, m)$ is defined in (1).

Proof. See the appendix. □

Lemma 3 reveals that, under System C, equilibrium is indeterminate. A continuum of equilibria exist that are not payoff equivalent. As condition (15) indicates, any symmetric equilibrium profit in the range $[0, S(m, n)]$ can be supported (or, alternatively, any sharing rule of the surplus can be supported). The indeterminacy of equilibrium characterizing this model is in the spirit of Coles and Eeckhout (2003). In that paper, sellers who possess one unit of an indivisible good, can charge a price that depends on ex-post realized demand (i.e. how many buyers show up at their store). What drives the indeterminacy result is that sellers can post advertisements that change the utility of buyers differently under different realizations of ex-post realized demand.

Like in Coles and Eeckhout (2003), in this model under System C, firms have two control variables, w and z . To see how indeterminacy of the sharing rule arises, fix the strategy of all firms but j , and consider this firm's best response. With access to a scheme (w^j, z^j) , firm j can advertise more worker surplus in some states and less in other states, leaving the expected payoffs of other players unaffected.¹⁴ Intuitively, for any announcement of the rival firms, firm j has a best response correspondence (rather than function), and, by symmetry, this gives rise to a continuum of equilibrium schemes (w^C, z^C) . Moreover, in a small market, like the one examined here, the various best responses are associated with different levels of expected utility in the subgame (since market utility is not fixed). As a result, the equilibrium (w^C, z^C) is not uniquely pinned down, and more importantly, different equilibria are associated with different sharing rules of the surplus.

Since under System C the equilibrium wage and profit are not uniquely pinned down, it is not possible to compare them with the equilibrium values in the other specifications studied earlier. Nevertheless, in the next subsection it will be shown that, as the market grows infinitely large, π^C converge to a unique limit. Therefore, a comparison of the equilibrium profit and welfare is possible for the large market.

4.2 The Large Market Case

Like in the case of random search, in this subsection I assume that $n, m \rightarrow \infty$, and the market tightness $b \equiv n/m$ remains constant. Also, I normalize the measure of workers to the unit, letting b represent the inverse of the measure of firms. All firms who wish to enter the labor market pay an entry fee $k > 0$ in advance, and a zero-profit condition is used in order to pin down the measure of firms in equilibrium. Let $\bar{w}^A, \bar{\pi}^A$ denote the equilibrium wage and profit (net of entree fee) in the limiting market, under Taxation

¹⁴ By "states" here I refer to the different realizations of how many workers apply to a certain firm, which is stochastic. Also, when I state that the expected payoffs of other players is unaffected, I refer to the payoffs of all n workers and all the remaining $m - 1$ firms.

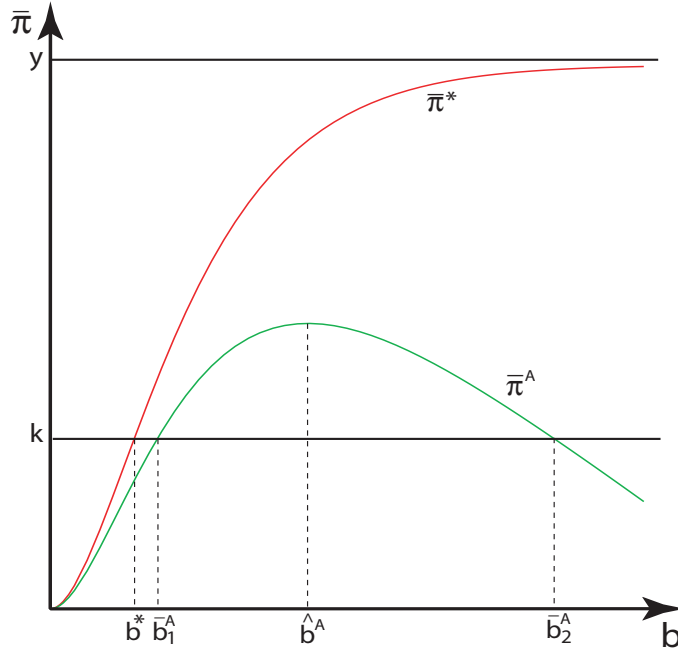


Figure 2: Equilibrium in the directed search model.

System A. Also, let $\bar{b}^A$ denote the value of b that solves $\bar{\pi}^A = k$, i.e the equilibrium market tightness. The next lemma describes these objects in detail.

Lemma 4. *Let $n, m \rightarrow \infty$, with $b = n/m$ held constant. Assume that the authorities adopt the Taxation System A in order to raise UI funds. The following are true:*

a) *The equilibrium wage and profit are given by*

$$\bar{w}^A = z + \frac{be^{-b}}{1 - e^{-b}}(y - z),$$

$$\bar{\pi}^A = (1 - e^{-b} - be^{-b})(y - z) - \left(1 - \frac{1 - e^{-b}}{b}\right)bz.$$

For any $b > 0$, $\bar{\pi}^A < \bar{\pi}^*$.

b) *If the set $\mathcal{B}^A \equiv \{b : \bar{\pi}^A = k\}$ is not empty, then it contains (generically) two elements, that is, equilibria come in pairs.*

c) *Define $\hat{b}^A \equiv \{b : be^{-b}(y - z) = (1 - e^{-b})z\}$. Equilibria with a positive measure of firms exist if and only if*

$$\bar{\pi}^A(\hat{b}^A) = \left(1 - e^{-\hat{b}^A}\right)(y - z) - \hat{b}^A z < k.$$

Proof. See the appendix. □

The main results of Lemma 4 are highlighted in Figure 2. In the large market case, under System A, the equilibrium profit is always lower compared to the model without taxes. Therefore, if an equilibrium exists, it will always involve less firms than optimal. The measure of firms that enter the labor market is not uniquely pinned down, because $\bar{\pi}^A$ is hump-shaped. Like in the random search model, the typical firm's profit eventually becomes increasing in the number of firms, because a lower b leads to fewer unemployed workers and a smaller share of the UI bill. Hence, the externality caused by the lump-sum tax is still present in the large market, and leads to suboptimal entry of firms. Finally, the condition $\bar{\pi}^A(\hat{b}^A) < k$ places an upper bound on the value of k , such that the existence of equilibria with a positive measure of firms is guaranteed.

I now turn to the analysis of equilibrium under System B. Let $\bar{w}^B, \bar{\pi}^B$ denote the equilibrium wage and profit (net of entree fee) in the limiting market, under this system. Also, let $\bar{b}^B$ denote the value of b that solves $\bar{\pi}^B = k$, i.e the equilibrium market tightness. The next lemma describes these objects in detail.

Lemma 5. *Let $n, m \rightarrow \infty$, with $b = n/m$ held constant. Assume that the authorities adopt the Taxation System B in order to raise UI funds. The equilibrium wage and profit are given by*

$$\begin{aligned}\bar{w}^B &= z + b \frac{e^{-b}y - z}{1 - e^{-b}}, \\ \bar{\pi}^B &= (1 - e^{-b} - be^{-b})y = \bar{\pi}^*.\end{aligned}$$

Hence, regardless of z , $\bar{b}^B = b^$, and the equilibrium measure of firms coincides with the Social Planner's solution.*

Proof. See the appendix. □

Lemma 5 reveals that, although the equilibrium wage under Taxation System B depends on the value of z , the equilibrium profit does not. In other words, under the personalized System B, the firms fully internalize the externality that they exert on one another under lump-sum taxation. Hence, in the large market case, under System B, the firms can fully undo the effects of taxation. Regardless of the value of z chosen by the government, the equilibrium value of profit equals π^* , which implies that the equilibrium measure of firms under System B coincides with the Social Planner's solution. Recall that this property of equilibrium is specific to the large market case, since in the finite market it was shown that $\pi^B < \pi^*$ for all $m, n \geq 2$ and $z > 0$.

One can conclude that the authorities can implement any UI payment z to the unemployed without sacrificing any efficiency, as long as they adopt Taxation System B. On the contrary, under System A, any $z > 0$ creates an externality that results into a suboptimal entry of firms, and a subsequent welfare loss. In fact, the equilibrium profit $\bar{\pi}^A$ is decreasing in z . The higher the UI payment that the government wishes to implement, the larger the wedge between $\bar{\pi}^*$ and $\bar{\pi}^A$, and the greater the distance between $\bar{b}^A$ and the optimal b^* (assuming, for simplicity, that under System A the economy ends up at the

equilibrium with the lower b).

Finally, consider equilibrium in a large market under System C. Let $\bar{w}^C, \bar{z}^C$ denote the equilibrium wage and UI posted by firms. Also, let $\bar{\pi}^C$ denote the limiting profit (net of the entree fee), and $\bar{b}^C$ the value of b that solves $\bar{\pi}^C = k$, i.e the equilibrium market tightness. The next lemma describes these objects in detail.

Lemma 6. *Let $n, m \rightarrow \infty$ and assume $b = n/m$ is held constant. If firms have to pay themselves UI to the workers who apply to them and are not offered a job, every pair (w^C, z^C) that satisfies*

$$(1 - e^{-b})\bar{w}^C + (b - 1 + e^{-b})\bar{z}^C = be^{-b}y, \quad (16)$$

is part of an equilibrium. Moreover, the equilibrium profit is uniquely given by

$$\bar{\pi}^C = \bar{\pi}^B = \bar{\pi}^* = (1 - e^{-b} - be^{-b})y.$$

Hence, $\bar{b}^C = \bar{b}^B = b^$, and the equilibrium measure of firms coincides with the Social Planner's solution.*

Proof. See the appendix. □

Lemma 6 contains some important results. The equilibrium values w^C, z^C cannot be uniquely pinned down, but the equilibrium profit is unique.¹⁵ The intuition behind this result is as follows. Under System C, for every given strategy of the competitors, each firm has a best response correspondence, rather than a function, and this gives rise to the indeterminacy of w^C, z^C . This is true regardless of the size of the market. However, in a finite market, the various best responses are associated with different levels of expected utility in the subgame, precisely because the market is small, and each player's actions can affect the outcome in the subgame. Hence, not only there exist many equilibrium values of w^C, z^C , but the different equilibria are associated with different sharing rules of the surplus. As the market grows large, the strategic interaction among firms vanishes, and the sharing rule of the surplus converges to a unique limit.

The second important result that follows from Lemma 6 is that the unique limit of the equilibrium profit under System C, coincides with the limiting profit under System B, which, in turn, coincides with the profit in the directed search model without taxes, π^* . As a result, the equilibrium measure of firms under Systems B,C coincide with the Planner's solution. In the next section, where I consider risk averse workers, this result will only be true under a very specific restriction on the parameter z .

5 Risk Averse Workers

I now focus on the case of risk averse workers in the directed search model. In particular, I assume that the workers' preferences admit an expected utility representation through

¹⁵ Geromichalos (2009) makes a similar point.

the Bernoulli utility function $u(\cdot)$, with $u' > 0$ and $u'' < 0$. The section is organized in the following way: First I describe equilibrium under Taxation Systems A,B, and C. Then I conclude with a comparison of welfare under the various systems.

In the previous sections, I focused first on the small market case, I provided formulas for the equilibrium wage and profit, and, finally, I took the limit of these expressions as the number of market participants reached infinity. With risk averse workers it will not be feasible to follow this methodology, because one cannot obtain closed form solutions for the equilibrium wage and profit. Therefore, here I focus directly on the large market case. As a starting point I consider finite but very large n, m . This simply means that firms in this environment realize that their actions cannot affect market outcomes. Then, I assume that $n, m \rightarrow \infty$, with $b = n/m$ remaining constant. As in previous sections, firms who enter the labor market have to pay a fixed cost k in advance, and the measure of operating firms will be pinned down by a free-entry condition.

First, consider the case in which the government wishes to pay UI equal to z to every unemployed worker and adopts the Taxation System A. Let θ represent the probability with which a typical worker applies to firm j . The problem solved by firm j is

$$\begin{aligned} \max_{w^j} \left\{ \pi^j(w^j, \theta) \right\} &= [1 - (1 - \theta)^n] (y - w^j) - \tilde{\tau}_D^A - k, \\ \text{s.t. } \frac{1 - (1 - \theta)^n}{n\theta} u(w^j) + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta} \right] u(z) &= U. \end{aligned}$$

The firm's objective is to maximize expected profit, subject to promising a certain level of expected utility, U , to the workers who apply for the job. Unlike earlier sections, here the firm takes this level of expected utility as given, because a firm's own actions cannot affect market outcomes.¹⁶ Notice that the firm also takes the tax payment $\tilde{\tau}_D^A$ as given. To see why this is the case, recall that under System A the UI bill is shared equally among all firms. In a market with a very large number of firms, like this one, whether firm j behaves more or less aggressively, will have an infinitesimal effect on the amount of taxes that it will have to pay. Hence, this effect can be safely ignored. The next lemma describes the symmetric equilibrium under Taxation System A.

Lemma 7. *In the directed search model with risk averse workers, suppose that the government adopts Taxation System A, and let $n, m \rightarrow \infty$, with $b = n/m$. Define the function*

$$\begin{aligned} H^A(b) &\equiv (1 - e^{-b} - be^{-b}) \left[u \left(y + z - \frac{k + zb}{1 - e^{-b}} \right) - u(z) \right] \\ &\quad - \frac{be^{-b}[(b - 1 + e^{-b})z + k]}{1 - e^{-b}} u' \left(y + z - \frac{k + zb}{1 - e^{-b}} \right). \end{aligned}$$

a) *The equilibrium market tightness is given by*

$$\tilde{b}^A \equiv \{b > 0 : H^A(b) = 0\}, \quad (17)$$

¹⁶ In the literature, papers that adopt this assumption are said to follow the *market utility* approach. See for example Montgomery (1991), Lang (1991), and Galenianos and Kircher (2009).

and the equilibrium wage is given by

$$\tilde{w}^A = y + z - \frac{k + z\tilde{b}^A}{1 - e^{-\tilde{b}^A}}.$$

b) The function H^A is hump-shaped, hence, if equilibria exist, they come in pairs. A sufficient condition for existence of equilibrium is $\max_b H^A(b) \geq 0$.

Proof. See the appendix. □

With risk averse workers it is not possible to obtain closed form solutions for the equilibrium wage and profit. The equilibrium market tightness is given implicitly by (17). Since $H^A(b)$ is hump-shaped, if equilibria exist, they come (generically) in pairs. This multiplicity result has been present in all the models visited in this paper, under Taxation System A. It stems from the fact that, under lump-sum taxation, a firm's profit is not monotonic in the market tightness. In the high $\tilde{b}^A$ equilibrium, few firms enter, so each firm has a high matching probability. On the downside, with few firms in the market, the number of unemployed workers is high, and each firm has to pay a big part of the UI bill. In the low $\tilde{b}^A$ equilibrium, the number of operating firms is large. This implies less unemployed workers and fewer taxes, but it comes together with a lower matching probability. Of course, in both equilibria firms make zero profit, due to free-entry.

I now turn to the case in which the authorities wish to pay an UI z to all unemployed workers, and Taxation System B is adopted. Let θ denote the probability with which the typical worker applies to firm j . Firm j 's problem is to

$$\begin{aligned} \max_{w^j} \left\{ \pi^j(w^j, \theta) \right\} &= [1 - (1 - \theta)^n] (y - w^j) - \tilde{\tau}_D^B - k, \\ \text{s.t. } \frac{1 - (1 - \theta)^n}{n\theta} u(w^j) + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta} \right] u(z) &= U, \\ \tau_D^B &= [n\theta - 1 + (1 - \theta)^n] z. \end{aligned}$$

Notice that, under System B, the firm cannot take the tax rate as given any more. Since the market is very large, a firm's wage posting strategy cannot affect the market utility, U . However, a firm that advertises a very high wage is more attractive for workers, and under the personalized System B, it will have to pay taxes that are proportional to the number of applications it received. The next lemma describes the symmetric equilibrium under Taxation System B.

Lemma 8. *In the directed search model with risk averse workers, suppose that the government adopts Taxation System B, and let $n, m \rightarrow \infty$, with $b = n/m$. Define the function*

$$\begin{aligned} H^B(b) \equiv & (1 - e^{-b} - be^{-b}) \left[u \left(y + z - \frac{k + zb}{1 - e^{-b}} \right) - u(z) \right] \\ & - b \frac{e^{-b}k - (1 - e^{-b} - be^{-b})z}{1 - e^{-b}} u' \left(y + z - \frac{k + zb}{1 - e^{-b}} \right). \end{aligned}$$

a) The equilibrium market tightness is given by

$$\tilde{b}^B \equiv \{b > 0 : H^B(b) = 0\}, \quad (18)$$

and the equilibrium wage is given by

$$\tilde{w}^B = y + z - \frac{k + z\tilde{b}^B}{1 - e^{-\tilde{b}^B}}.$$

a) Under System A a unique equilibrium exists.

Proof. See the appendix. □

Despite its complexity, the function H^B is shown to be strictly increasing in b (see the appendix for some details on parameter restrictions), leading to the existence of a unique equilibrium, $\tilde{b}^B$. As in Section 4, System B leads to a unique equilibrium because it restores the monotonic (and positive) relationship between b and the profit of the average firm. Another important result that follows from Lemmas 7 and 8, is that, under any parameter values,

$$H^B(b) - H^A(b) = b(1 - e^{-b})u' \left(y + z - \frac{k + zb}{1 - e^{-b}} \right) z > 0.$$

Therefore, for all $b > 0$, we have $H^B(b) > H^A(b)$. This result is depicted in Figure 3, and it implies that $\tilde{b}^B < \tilde{b}^A$, regardless of which of the two equilibria arises under System A. In other words, under personalized taxes, the measure of active firms is always greater.

The last environment to be studied, is the one where firms post general contracts (w, z) , i.e. System C. The typical firm j chooses a pair (w^j, z^j) , taking as given the level of utility U that it must promise to its applicants. If θ denotes the probability with which any given worker applies to firm j , the typical firm's problem is to

$$\begin{aligned} \max_{w^j, z^j} \left\{ \pi^j(w^j, z^j, \theta) \right\} &= [1 - (1 - \theta)^n] (y - w^j) - [n\theta - 1 + (1 - \theta)^n] z^j, \\ \text{s.t. } \frac{1 - (1 - \theta)^n}{n\theta} u(w^j) + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta} \right] u(z^j) &= U. \end{aligned}$$

Clearly, the main difference between this problem and the firm's problem under Systems A,B, is that here z is not a parameter chosen by the government, but the firm's control variable. The following lemma characterizes the symmetric equilibrium under System C.

Lemma 9. *Assume that firms have to pay themselves UI to the workers who apply to them and are not offered a job. Then, the equilibrium market tightness is given by $\tilde{b}^C = b^* \equiv \{b : (1 - e^{-b} - be^{-b})y = k\}$, and the equilibrium wage and UI satisfy*

$$\tilde{w}^C = \tilde{z}^C = e^{-b^*} y.$$

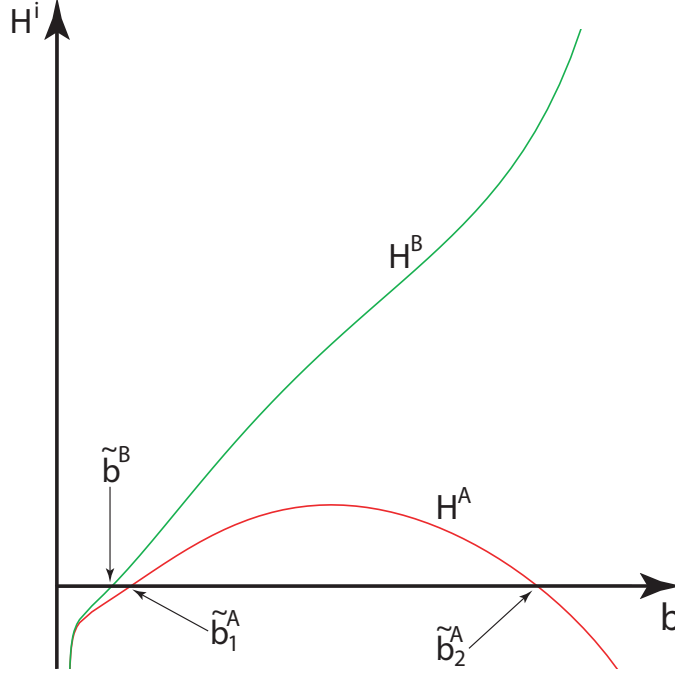


Figure 3: Equilibrium market tightness with risk averse workers.

Proof. See the appendix. □

Lemma 9 reveals that the equilibrium objects $\tilde{w}^C, \tilde{z}^C$ are uniquely determined. In Section 4.1, I showed that in a small market with risk neutral workers, w^C, z^C are not uniquely determined, and the different equilibrium values are associated with different sharing rules of the surplus (Lemma 3). This result reflects the fact that firms have a continuum of best responses, and each one of those responses is associated with a different level of expected utility in the subgame. With risk neutral workers and a large market, firms still have many ways of responding optimally to the rivals' strategies (and keeping all other players, including workers, indifferent). However, in this case the different responses do not affect the market utility, precisely because each firm is a measure zero agent. As a result, in the large market with risk neutral workers, the sharing rule of the surplus is uniquely pinned down, but this is not the case for $\tilde{w}^C, \tilde{z}^C$, which are still indeterminate (Lemma 6). When workers are risk averse, the typical firm cannot keep them indifferent by posting different levels of utility under the different realizations of ex-post realized demand or, put simply, the firm's best response is a function rather than a correspondence. As a consequence, the objects $\tilde{w}^C, \tilde{z}^C$ are uniquely determined.

Another important and very intuitive result that follows from Lemma 9, is that the risk neutral firms will fully insure the risk averse workers, by setting $\tilde{w}^C = \tilde{z}^C$. Interestingly, the equilibrium market tightness under System C, coincides with b^* , which was defined as the Social Planner's choice in the model with risk neutral workers.

The last task of this section is to study the welfare properties of equilibrium under the

different taxation systems, and facilitate comparison. The relevant results are summarized in Proposition 1. Before stating the proposition, a benchmark of efficiency for the model with risk averse workers needs to be established. As intuition may suggest, the Planner will insure the risk averse workers, and choose a b that maximizes total output net of entry fees. The next lemma states the relevant result in a formal way.¹⁷

Lemma 10. *The Social Planner's problem is to choose b, z in order maximize the workers' expected utility, subject to the zero-profit condition for the firms. More precisely, the Planner solves*

$$\max_{b,z} \{\Omega(b,z)\} \equiv \frac{1-e^{-b}}{b} u\left(y+z-\frac{k+zb}{1-e^{-b}}\right) + \left(1-\frac{1-e^{-b}}{b}\right) u(z),$$

and the solution to this problem satisfies $b^S = b^*$, and $z^S = e^{-b^*} y$.

Proof. See the appendix. □

Proposition 1. *a) Under System C, the equilibrium allocation always coincides with the Social Planner's solution.*

b) Under System B, the Social Planner's allocation can be achieved if z is chosen optimally by the authorities. The optimal level of UI is given by $z = e^{-b^} y$.*

c) Under System A, the Social Planner's allocation can never be achieved.

Proof. a) Follows immediately from comparison of Lemmas 9 and 10.

b) From Lemma 8, it can be easily verified that if $z = e^{-b^*} y$, then $\tilde{b}^B = b^*$ and $\tilde{w}^B = e^{-b^*} y$, is the unique equilibrium.

c) Clearly, $H^A(b) = 0$ cannot be satisfied at $b = b^*$ and $z = e^{-b^*} y$. □

In the model with risk aversion, efficiency requires not only a “correct” measure of firms, so that output net of entry costs is maximized, but also insuring the risk averse workers. It turns out that the system that is guaranteed to deliver the efficient outcome is the one where the authorities do not even have to pick z themselves, i.e. System C. As long as firms have to post general wage-UI contracts, as in Jacquet and Tan (2012), the incentives of firms to direct workers to their job will lead to an equilibrium that coincides with the Planner's solution. System B can potentially deliver efficiency, but this requires that the authorities choose the level of z in a way that is “just right”, $z = e^{-b^*} y$. In any other case, the equilibrium will be suboptimal. Whether the authorities are able to choose the optimal z with such precision is questionable. Finally, under System A efficiency can never be achieved, regardless of how wisely the authorities set z . With

¹⁷ In the model with risk neutral workers, the total surplus in the economy is independent of z . Hence, the Planner's problem is to choose a b that maximizes total output net of entry fees. Here, the level of z matters, so I set up the Planner's problem assuming that z is also a choice variable. Notice that the fact that firms make zero profit in equilibrium is very convenient, since it allows to write the Planner's objective as the expected utility of the workers, whose measure, as always, is normalized to the unit.

lump-sum taxation, the incentives of firms to direct workers to their jobs are always distorted, and an equilibrium with too few firms always prevails.

Proposition 1 states that efficiency can potentially be achieved under System B, and it can never be achieved under System A. However, the proposition does not claim that System B is superior to System A for any given z , as was the case with risk neutral workers (see for example Figure 2).¹⁸ For any given z , it is true that $\tilde{b}^A > \tilde{b}^B$. Moreover, the probability with which a worker stays unemployed, $1 - (1 - e^{-b})/b$, is increasing in b . Hence, if z is relatively large, the second term of the welfare function Ω , defined in the Planner’s problem, will tend to be larger under System A. In theory, this could imply that for some large z , $\Omega(\tilde{b}^A, z) > \Omega(\tilde{b}^B, z)$. However, recall that what makes System A inefficient is the distortion of incentives due to lump-sum taxes, and these taxes tend to be high when z is high. This second force is typically stronger. In a series of simulations, I have not been able to find parameter values that lead to $\Omega(\tilde{b}^A, z) > \Omega(\tilde{b}^B, z)$.

6 Conclusions

I examine the welfare implications of raising funds towards unemployment insurance through different taxation systems, and within two equilibrium models of unemployment: random search and directed search. In the random search model, the choice between lump-sum or personalized taxes does not affect equilibrium outcomes. Under certain parameter values, introducing unemployment benefits and taxes can lead to an improvement of welfare. In the directed search model, the choice of taxation is essential. In general, lump-sum taxes distort the equilibrium outcomes and decrease welfare because they create an externality: Under lump-sum taxation, firms have an incentive to post high wages, in their attempt to increase the probability of filling up their vacancy, and doing so is rational because all firms share the unemployment benefit bill equally. In equilibrium, the measure of active firms is always suboptimal. One way to resolve this inefficiency is by introducing personalized taxes, i.e. a system which dictates that firms’ tax payments will be proportional to the number of applications they receive. Another way to guarantee that the economy can achieve the Planner’s allocation, in fact without any intervention by the authorities, is to require firms to post general “wage-vacancy” contracts, *a la* Jacquet and Tan (2012).

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¹⁸ In other words, the proposition says that if the authorities can choose z correctly, then System B is definitely better than System A, since it delivers efficiency. However, if the authorities choose some suboptimal z , it is not understood that System B will lead to greater efficiency than System A.

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A Appendix

Proof of Lemma 1. a) Notice that the indifference constraint faced by firm j can be rewritten as

$$[1 - (1 - \theta)^n]w^j = [1 - (1 - \theta)^n]z + \frac{\theta}{\tilde{\theta}} \left[1 - (1 - \tilde{\theta})^n\right] (\tilde{w} - z).$$

Replacing the term $[1 - (1 - \theta)^n]w^j$ from the equation above into firm j 's profit, $\pi^j(w^j, \theta)$, allows one to write the firm's problem as a function of the variable θ . More precisely, firm j wishes to solve

$$\begin{aligned} & \max_{\theta} \left\{ [1 - (1 - \theta)^n](y - z) - \frac{\theta}{\tilde{\theta}} \left[1 - (1 - \tilde{\theta})^n\right] (\tilde{w} - z) \right. \\ & \left. - \frac{1}{m} \left\{ n\theta - 1 + (1 - \theta)^n + (m - 1) \left[n\tilde{\theta} - 1 + (1 - \tilde{\theta})^n \right] \right\} z \right\}, \end{aligned}$$

subject to $\tilde{\theta} = (1 - \theta)/(m - 1)$. Firm j 's best response satisfies the first-order condition

$$\begin{aligned} & n(1 - \theta)^{n-1}(y - z) + \left[\frac{1 - (1 - \tilde{\theta})^n}{\tilde{\theta}} - \theta \frac{n\tilde{\theta} (1 - \tilde{\theta})^{n-1} - 1 + (1 - \tilde{\theta})^n}{\tilde{\theta}^2(m - 1)} \right] (z - \tilde{w}) \\ & + \frac{n}{m} \left[(1 - \theta)^{n-1} (1 - \tilde{\theta})^{n-1} \right] z = 0 \end{aligned}$$

One can now impose the following symmetry conditions. First, all firms post the same wage, $w^j = \tilde{w} = w^A$. This, in turn, implies that all workers will apply to each firm with the same probability, $\theta = \tilde{\theta} = 1/m$. Once these observations are incorporated into the first-order condition, the latter becomes an equation with only one unknown, w^A . Solving

with respect to this variable yields the formula reported in (10).

b) Comparison of (10) with (2) reveals that $w^A > w^*$ will be satisfied *iff*

$$G(n, m) \equiv \left(1 - \frac{1}{m}\right)^n \left(1 + \frac{n}{m-1}\right) < 1. \quad (\text{a.1})$$

Notice that, for any given $n \geq 2$, the function G is strictly increasing in m , since

$$\frac{\partial G(n, m)}{\partial m} = \frac{n(n-1)}{m^2(m-1)} \left(1 - \frac{1}{m}\right)^{n-1} > 0.$$

Moreover, for any given $n \geq 2$,

$$\lim_{m \rightarrow \infty} G(n, m) = 1.$$

Since for any given $n \geq 2$, the function G is strictly increasing in m and bounded from above by the unit, the requirement in (a.1) holds true, and the proof is complete.

c) The proof follows immediately from the facts that $w^A > w^*$, and the term $n/m - 1 + (1 - 1/m)^n$ is positive. $\square$

Proof of Lemma 2. a) I follow the same method as in Lemma 1. The indifference constraint faced by firm j can be re-written as

$$[1 - (1 - \theta)^n]w^j = [1 - (1 - \theta)^n]z + \frac{\theta}{\tilde{\theta}} \left[1 - (1 - \tilde{\theta})^n\right](\tilde{w} - z).$$

Replacing the term $[1 - (1 - \theta)^n]w^j$ from the equation above into firm j 's profit, allows one to re-write the firm's problem as

$$\max_{\theta} \left\{ [1 - (1 - \theta)^n]y - n\theta z - \frac{\theta}{\tilde{\theta}} \left[1 - (1 - \tilde{\theta})^n\right](\tilde{w} - z) \right\},$$

subject to $\tilde{\theta} = (1 - \theta)/(m - 1)$. Firm j 's best response satisfies the first-order condition

$$n(1 - \theta)^{n-1}y + \left[\frac{1 - (1 - \tilde{\theta})^n}{\tilde{\theta}} - \theta \frac{n\tilde{\theta}(1 - \tilde{\theta})^{n-1} - 1 + (1 - \tilde{\theta})^n}{\tilde{\theta}^2(m - 1)} \right] (z - \tilde{w}) - nz = 0.$$

After imposing the symmetry conditions, $w^j = \tilde{w} = w^B$ and $\theta = \tilde{\theta} = 1/m$, the first-order condition becomes a simple equation in one unknown, w^B . Solving with respect to this variable yields the formula reported in (12).

b) We have shown in Lemma 1 that $w^A > w^*$. The proof of the fact that $w^* \geq w^B$ follows similar steps. Notice, however, that this inequality is weak, because for $n = m = 2$, the multiplier of z in the numerator of the right-hand side in (12) is equal to zero. For any other value of n, m that term is positive, leading to $w^* > w^B$.

c) In Lemma 1, showing that $\pi^* > \pi^A$ was trivial because, in the model with UI, not

only firms had to pay taxes $[n/m - 1 + (1 - 1/m)^n]z > 0$, but also $w^A > w^*$. Showing that $\pi^* > \pi^B$ is less straightforward because, although firms pay a positive amount of taxes, they offer a lower wage. Nevertheless, after some algebra one can show that

$$\begin{aligned}\pi^B &= \left[1 - \left(1 - \frac{1}{m}\right)^n\right] (y - w^B) - \left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n\right] z = \\ &= \pi^* - \frac{1 - \left(1 + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n}{m - \left(m + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n} z.\end{aligned}\tag{a.2}$$

The multiplier of z in (a.2), will be positive if and only if

$$1 > \left(1 + \frac{n}{m-1}\right) \left(1 - \frac{1}{m}\right)^n.$$

But this is precisely the condition that guarantees $w^A > w^*$ in the proof of Lemma 1 (see condition (a.1)). Hence, one can conclude that $\pi^* > \pi^B$.

The last claim in part (c) of the lemma is that $\pi^B > \pi^A$. In the symmetric equilibrium, the firms will always pay the same taxes, $[n/m - 1 + (1 - 1/m)^n]z > 0$. Therefore, the equilibrium profit will be higher under the system that leads to a lower equilibrium wage and, from part (b), this is Taxation System B. $\square$

Proof of Lemma 3. The indifference constraint faced by firm j can be re-written as

$$[1 - (1 - \theta)^n](w^j - z^j) + n\theta z^j = \frac{\theta}{\tilde{\theta}} \left[1 - (1 - \tilde{\theta})^n\right] (\tilde{w} - \tilde{z}) + n\theta \tilde{z}.$$

Replacing the left-hand side of the last expression into firm j 's profit, allows one to re-write the firm's problem as

$$\max_{\theta} \left\{ [1 - (1 - \theta)^n]y - n\theta \tilde{z} - \frac{\theta}{\tilde{\theta}} \left[1 - (1 - \tilde{\theta})^n\right] (\tilde{w} - \tilde{z}) \right\},$$

subject to $\tilde{\theta} = (1 - \theta)/(m - 1)$. Firm j 's best response satisfies the first-order condition

$$n(1 - \theta)^{n-1}y + \left[\frac{1 - (1 - \tilde{\theta})^n}{\tilde{\theta}} - \theta \frac{n\tilde{\theta} (1 - \tilde{\theta})^{n-1} - 1 + (1 - \tilde{\theta})^n}{\tilde{\theta}^2(m - 1)} \right] (\tilde{z} - \tilde{w}) - n\tilde{z} = 0.$$

One can now impose the following symmetry conditions: $w^j = \tilde{w} = w^C$, $z^j = \tilde{z} = z^C$, and $\theta = \tilde{\theta} = 1/m$ into the first-order condition. After some algebra the result reported in (14) emerges.

Clearly, we have only one equation in order to characterize two equilibrium objects, (w^C, z^C) . Every pair (w^C, z^C) that satisfies (14) is part of an equilibrium, as long as it leads to positive expected profit for the firms and positive expected utility for the workers. The condition in (15) guarantees precisely that. $\square$

Proof of Lemma 4. a) Consider first the wage. Using the definition of b , one can re-write the equilibrium wage in the small market (equation (10)) as

$$w^A = \frac{b \left(1 - \frac{b}{n}\right)^n y + \left[1 - \left(1 + \frac{b}{1 - \frac{b}{n}}\right) \left(1 - \frac{b}{n}\right)^n\right] z}{1 - \left(1 + \frac{b}{\frac{n}{b} - 1}\right) \left(1 - \frac{b}{n}\right)^n},$$

and since $\lim_{n \rightarrow \infty} (1 - b/n)^n = e^{-b}$, we have

$$\bar{w}^A = \lim_{n \rightarrow \infty} w_A = \frac{be^{-b}y + [1 - (1 + b)e^{-b}]z}{1 - e^{-b}} = z + \frac{be^{-b}}{1 - e^{-b}}(y - z).$$

Regarding profit, use the definition of b in equation (11) to write

$$\pi^A = \left[1 - \left(1 - \frac{b}{n}\right)^n\right] (y - w^A) - \left[b - 1 + \left(1 - \frac{b}{n}\right)^n\right] z.$$

Therefore,

$$\begin{aligned} \bar{\pi}^A &= \lim_{n \rightarrow \infty} \pi_A = (1 - e^{-b})(y - \bar{w}^A) - (b - 1 + e^{-b})z \\ &= (1 - e^{-b} - be^{-b})(y - z) - \left(1 - \frac{1 - e^{-b}}{b}\right)bz. \end{aligned}$$

The inequality $\bar{\pi}^A < \bar{\pi}^*$ will be satisfied if and only if $b(1 - e^{-b})z > 0$, which is true for all $b, z > 0$.

b) For convenience, define the large market profit as the function

$$F(b) \equiv (1 - e^{-b} - be^{-b})(y - z) - bz + (1 - e^{-b})z.$$

Notice that $F(0) = 0$ and $F'(b) = be^{-b}(y - z) - (1 - e^{-b})z$. Hence, for any $y \leq 2z$

$$F'(b) \leq 2be^{-b}z - be^{-b}z - (1 - e^{-b})z = -z(1 - e^{-b} - be^{-b}),$$

which is strictly negative for all $b, z > 0$. Therefore if $z \geq y/2$, the profit is never positive, and no firms will ever enter in the market, no matter how small k might be. From now on, I assume that $z < y/2$.

Assuming that $z < y/2$, implies that there exist b small enough, such that $F'(b) > 0$. Moreover, it is straightforward to see that $\lim_{b \rightarrow \infty} F(b) = -\infty$. Therefore, if an equilibrium exists, it will never be unique (recall that $F(0) = 0$). But the lemma claims something stronger: if equilibria exist, they always comes in pairs. This will be true if the function $F(b)$ has a unique interior maximum. I will show that this is the case, by proving that the set $\mathcal{B}'_0 \equiv \{b > 0 : F'(b) = 0\}$ is a singleton (I have already shown that it is non-empty). For future reference, notice that $F'(0) = 0$ and

$$F''(b) = e^{-b}(y - 2z) - be^{-b}(y - z). \tag{a.3}$$

Assume, by way of contradiction, that $\mathcal{B}'_0$ is not a singleton, and define $b_1 \equiv \min\{\mathcal{B}'_0\}$. Notice that since for $b \approx 0$, $F'(b) > 0$, it must be that $F''(b_1) < 0$. Under the contradictory assumption, there exists $b_2 \in \mathcal{B}'_0$, such that $b_2 > b_1$ and $F''(b_2) > 0$. Moreover, since $b_1, b_2 \in \mathcal{B}'_0$, we have

$$b_1 e^{-b_1}(y - z) = (1 - e^{-b_1})z, \quad (\text{a.4})$$

$$b_2 e^{-b_2}(y - z) = (1 - e^{-b_2})z. \quad (\text{a.5})$$

Combining (a.3) with (a.4) and (a.5), one can write

$$F''(b_1) = e^{-b_1}(y - z) - z,$$

$$F''(b_2) = e^{-b_2}(y - z) - z.$$

Since $F''(b_1) < 0$ and $F''(b_2) > 0$, one can conclude that

$$z - e^{-b_1}(y - z) > 0, \quad (\text{a.6})$$

$$e^{-b_2}(y - z) - z > 0. \quad (\text{a.7})$$

Adding up (a.6) and (a.7) implies that

$$(e^{-b_2} - e^{-b_1})(y - z) > 0,$$

which is a contradiction, since $y > z$ and $b_2 > b_1$.

c) This part is fairly trivial. The term $\hat{b}^A$ is the unique interior maximizer of F . Equilibria exist as long as $\bar{\pi}^A$, evaluated at $\hat{b}^A$, exceeds the entry fee. $\square$

Proof of Lemma 5. a) Consider the wage. Using the definition of b , one can re-write the equilibrium wage in the small market (equation (12)) as

$$w^B = \frac{b \left(1 - \frac{b}{n}\right)^n y + \left[\left(1 + \frac{b}{\frac{n}{b}-1}\right) \left(1 - \frac{b}{n}\right)^n + b \left(1 - \frac{b}{n}\right) - 1\right] z}{1 - \left(1 + \frac{b}{\frac{n}{b}-1}\right) \left(1 - \frac{b}{n}\right)^n},$$

and since $\lim_{n \rightarrow \infty} (1 - b/n)^n = e^{-b}$, we have

$$\bar{w}^B = \lim_{n \rightarrow \infty} w^B = \frac{b e^{-b} y - (e^{-b} + b - 1) z}{1 - e^{-b}} = z + b \frac{e^{-b} y - z}{1 - e^{-b}}.$$

Regarding profit, use the definition of b in equation (13) to write

$$\pi^B = \left[1 - \left(1 - \frac{b}{n}\right)^n\right] (y - w^B) - \left[b - 1 + \left(1 - \frac{b}{n}\right)^n\right] z.$$

Therefore,

$$\bar{\pi}^B = \lim_{n \rightarrow \infty} \pi^B = (1 - e^{-b})(y - \bar{w}^B) - (b - 1 + e^{-b}) z = (1 - e^{-b} - b e^{-b}) y.$$

$\square$

Proof of Lemma 6. Equation (16) emerges after substituting $m = n/b$ into (14) and taking the limit as $n \rightarrow \infty$.

Now consider the symmetric equilibrium profit. Substituting $m = n/b$ in (15) allows one to write the finite market profit as

$$\pi^C = \left[1 - \left(1 - \frac{b}{n}\right)^n\right] (y - w^C) - \left[b - 1 + \left(1 - \frac{b}{n}\right)^n\right] z^C,$$

and as the market grows infinitely large,

$$\bar{\pi}^C = \lim_{n \rightarrow \infty} \pi^C = (1 - e^{-b}) (y - \bar{w}^C) - (b - 1 + e^{-b}) \bar{z}^C. \quad (\text{a.8})$$

Finally, substituting the left-hand side of (16) into (a.8) yields the desired result. $\square$

Proof of Lemma 7. a) Consider the problem of the typical firm j in the market with very large (yet finite) n, m . The necessary and sufficient first-order condition for profit maximization satisfies

$$- [1 - (1 - \theta)^n] + n(1 - \theta)^{n-1} (y - w^j) \frac{\partial \theta}{\partial w^j} = 0,$$

and an expression for $\partial \theta / \partial w^j$ can be obtained by applying the implicit function theorem on the expected utility constraint in the firm's problem. In particular, one can show that

$$\frac{\partial \theta}{\partial w^j} = -\frac{1}{n} \frac{[1 - (1 - \theta)^n] u'(w^j)}{(1 - \theta)^{n-1} [u(w^j) - u(z)] + u(z) - U}, \quad (\text{a.9})$$

Substituting (a.9) into the first-order condition, and obtaining the term U from the expected utility constraint, implies that the typical firm chooses a w^j that satisfies

$$\left[\frac{1 - (1 - \theta)^n}{n\theta} - (1 - \theta)^{n-1} \right] [u(w^j) - u(z)] - (1 - \theta)^{n-1} (y - w^j) u'(w^j) = 0. \quad (\text{a.10})$$

At this point, some important observations are in order. First, since the focus is on symmetric equilibria, it must be $w^j = w$. Moreover, under symmetry, the tax that each firm will have to pay is given by $\tilde{\tau}^A = [n/m - 1 + (1 - 1/m)^n]z$. Lastly, in equilibrium all firms make zero profit, which implies that

$$y - w = \frac{\tilde{\tau}^A + k}{1 - (1 - \frac{1}{m})^n} = \frac{[\frac{n}{m} - 1 + (1 - \frac{1}{m})^n] z + k}{1 - (1 - \frac{1}{m})^n}.$$

Incorporating these observations in (a.10), using the definition of market tightness in order to substitute for $m = n/b$, and taking the limit as $n \rightarrow \infty$, yields the equilibrium condition given in (17).

b) The proof is similar to the one of part (b) of Lemma 4. $\square$

Proof of Lemma 8. a) Substitute the constraint $\tau_D^B = [n\theta - 1 + (1 - \theta)^n]z$ into the objective function of the firm, and take the first-order condition with respect to w^j to obtain

$$- [1 - (1 - \theta)^n] + n(1 - \theta)^{n-1}(y - w^j) \frac{\partial \theta}{\partial w^j} - [n - n(1 - \theta)^{n-1}]z \frac{\partial \theta}{\partial w^j} = 0.$$

Since the expected utility constraint in the firm's problem is identical to the one under System A, $\partial \theta / \partial w^j$ is still given by (a.9). Substituting this expression back into the first-order condition, and obtaining the term U from the expected utility constraint, implies that the typical firm chooses a w^j that satisfies

$$\left[\frac{1 - (1 - \theta)^n}{n\theta} - (1 - \theta)^{n-1} \right] [u(w^j) - u(z)] - [(1 - \theta)^{n-1}(y + z - w^j) - z] u'(w^j) = 0. \quad (\text{a.11})$$

The rest of the proof proceeds in the same steps as the one of Lemma 7. First, since the focus is on symmetric equilibria, $w^j = w$. Moreover, under symmetry, each firm's tax bill is given by $\tilde{\tau}^B = [n/m - 1 + (1 - 1/m)^n]z$. Finally, in equilibrium all firms make zero profit, which implies that

$$y - w = \frac{\left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n \right] z + k}{1 - \left(1 - \frac{1}{m}\right)^n}.$$

Incorporating these observations in (a.11), using the definition of market tightness in order to substitute for $m = n/b$, and taking the limit as $n \rightarrow \infty$, yields the equilibrium condition given in (18).

b) Define the terms

$$\begin{aligned} f(b) &= 1 - e^{-b} - be^{-b}, \\ g(b) &= \frac{k + zb}{1 - e^{-b}}. \end{aligned}$$

Using these definitions, H^B can be re-written as

$$H^B(b) = f(b) \{u[y + z - g(b)] - u(z)\} - b[e^{-b}g(b) - z]u' \left(y + z - \frac{k + zb}{1 - e^{-b}} \right).$$

Notice that $f(b) \in (0, 1)$, for all $b > 0$, with $f'(b) = be^{-b} > 0$, for all $b > 0$.

The function $g(b)$ is key for this proof, so I discuss it in detail. It satisfies $g(b) > 0$, for all $b > 0$, and also $\lim_{b \rightarrow 0} g(b) = \infty = \lim_{b \rightarrow \infty} g(b)$. Moreover,

$$g'(b) = \frac{f(b)z - ke^{-b}}{(1 - e^{-b})^2} = - \frac{g(b)e^{-b} - z}{1 - e^{-b}}. \quad (\text{a.12})$$

Hence, $g(b)$ is u -shaped, and it attains a unique global minimum, at $b = b^m \equiv \{b : f(b)z = ke^{-b}\}$. From now on, I will assume that the following condition holds:

$$g(b^m) = \frac{z}{e^{b^m}} < y. \quad (\text{a.13})$$

In what follows, I show that this is a sufficient condition for existence and uniqueness of equilibrium.¹⁹ Notice that if (a.13) holds true, there exists a range of b 's for which $g(b) < y$. Hence, (a.13) is a fairly weak requirement on how big k, z can be: it guarantees that, given the announcement z (and the cost k), there will exist equilibria where firms post a wage which is no less than z . Assuming that (a.13) holds, there exist exactly two values of $b > 0$, for which $g(b) = y$. Let me define them as b_1^y, b_2^y , with $b_1^y < b_2^y$.

I now prove that, for all $b \in [b_1^y, b_2^y]$, H^B is strictly increasing.

$$\begin{aligned} \frac{dH^B(b)}{db} &= f'(b) \{u[y + z - g(b)] - u(z)\} - f(b) g'(b) u'[y + z - g(b)] \\ &\quad - [e^{-b}g(b) - z]u'[y + z - g(b)] - be^{-b}[g(b) - g'(b)]u'[y + z - g(b)] \\ &\quad + b[e^{-b}g(b) - z]g'(b)u''[y + z - g(b)]. \end{aligned}$$

Notice that this derivative is the sum of the following five terms:

$$\begin{aligned} h_1(b) &= f'(b) \{u[y + z - g(b)] - u(z)\}, \\ h_2(b) &= -f(b) g'(b) u'[y + z - g(b)], \\ h_3(b) &= -[e^{-b}g(b) - z]u'[y + z - g(b)], \\ h_4(b) &= -be^{-b}[g(b) - g'(b)]u'[y + z - g(b)], \\ h_5(b) &= b[e^{-b}g(b) - z]g'(b)u''[y + z - g(b)]. \end{aligned}$$

Out of these terms, only h_1 is undoubtedly positive. I claim that h_5 is also positive. To see why this is true, substitute the term $g'(b)$ from (a.12) into h_5 to obtain

$$h_5(b) = -\frac{b}{1 - e^{-b}}[e^{-b}g(b) - z]^2 u''[y + z - g(b)],$$

which is strictly positive for all $b > 0$, since $u''(\cdot) < 0$.

The proof of monotonicity of H^B will be complete, if it can be shown that $h_2(b) + h_3(b) + h_4(b) \geq 0$. Before examining this term, notice that

$$g(b) - g'(b) = \frac{g(b) - z}{1 - e^{-b}}.$$

Using this fact, one can re-write the fourth term as

$$h_4(b) = be^{-b} \frac{g(b) - z}{1 - e^{-b}} u'[y + z - g(b)].$$

Hence, the requirement that $h_2(b) + h_3(b) + h_4(b) \geq 0$ will be true if and only if

$$u'[y + z - g(b)] \left\{ -f(b)g'(b) - [e^{-b}g(b) - z] + be^{-b} \frac{g(b) - z}{1 - e^{-b}} \right\} \geq 0.$$

¹⁹ This requirement is not necessary for existence of equilibrium. It can be shown that equilibrium exists even if $g(b^m) \geq y$.

Substituting for $g'(b)$ from (a.12) and rearranging terms, implies that the latter requirement will be true if and only if

$$be^{-b}g(b) \geq 0.$$

Of course, this condition holds (with strict inequality) for all $b > 0$. Hence, $dH^B/db > 0$.

It follows that if an equilibrium $b \in [b_1^y, b_2^y]$ exists, it will be unique. To obtain existence, notice that

$$\begin{aligned} H^B(b_1^y) &= -b_1^y \left[e^{-b_1^y} g(b_1^y) - z \right] u'(z), \\ H^B(b_2^y) &= -b_2^y \left[e^{-b_2^y} g(b_2^y) - z \right] u'(z). \end{aligned}$$

Luckily, the term $e^{-b}g(b) - z$ also appears in $g'(b)$ in (a.12), and the sign of the expressions $g'(b_1^y)$ and $g'(b_2^y)$ is known (by the very definition of b_1^y, b_2^y). We have

$$\begin{aligned} \text{sign}\{H^B(b_1^y)\} &= \text{sign}\left\{-\left[e^{-b_1^y} g(b_1^y) - z\right]\right\} = \text{sign}\{g'(b_1^y)\} = -, \\ \text{sign}\{H^B(b_2^y)\} &= \text{sign}\left\{-\left[e^{-b_2^y} g(b_2^y) - z\right]\right\} = \text{sign}\{g'(b_2^y)\} = +. \end{aligned}$$

Summing up, the function $H^B(b)$ is strictly increasing and continuous in $b \in [b_1^y, b_2^y]$, with $H^B(b_1^y) < 0$, and $H^B(b_2^y) > 0$. Hence, a unique equilibrium always exists. $\square$

Proof of Lemma 9. The first-order conditions with respect to w^j and z^j , respectively, are

$$\begin{aligned} -[1 - (1 - \theta)^n] + n(1 - \theta)^{n-1}(y - w^j) \frac{\partial \theta}{\partial w^j} - [n - n(1 - \theta)^{n-1}]z \frac{\partial \theta}{\partial w^j} &= 0, \\ -[n\theta - 1 + (1 - \theta)^n] + n(1 - \theta)^{n-1}(y - w^j) \frac{\partial \theta}{\partial z^j} - [n - n(1 - \theta)^{n-1}]z \frac{\partial \theta}{\partial z^j} &= 0. \end{aligned}$$

Applying the implicit function theorem on the expected utility constraint in the firm's problem, one can show that

$$\begin{aligned} \frac{\partial \theta}{\partial w^j} &= -\frac{1}{n} \frac{[1 - (1 - \theta)^n] u'(w^j)}{(1 - \theta)^{n-1}[u(w^j) - u(z^j)] + u(z^j) - U}, \\ \frac{\partial \theta}{\partial z^j} &= -\frac{1}{n} \frac{[n\theta - 1 + (1 - \theta)^n] u'(z^j)}{(1 - \theta)^{n-1}[u(w^j) - u(z^j)] + u(z^j) - U}. \end{aligned}$$

Substituting these expressions into the first-order conditions yields

$$(1 - \theta)^{n-1}[u(w^j) - u(z^j)] + u(z^j) - U = -u'(w^j)[(1 - \theta)^{n-1}(y + z^j - w^j) - z^j], \quad (\text{a.14})$$

$$(1 - \theta)^{n-1}[u(w^j) - u(z^j)] + u(z^j) - U = -u'(z^j)[(1 - \theta)^{n-1}(y + z^j - w^j) - z^j]. \quad (\text{a.15})$$

Equations (a.14) and (a.15) reveal that the firm will fully insure the risk averse worker, by setting $w^j = z^j$. To characterize the equilibrium values, impose $w^j = z^j$ in either (a.14)

or (a.15), together with symmetry. This implies that the equilibrium value of w in the finite market solves

$$u'(w) \left[\left(1 - \frac{1}{m}\right)^{n-1} y - w \right] = 0 \Rightarrow w = \left(1 - \frac{1}{m}\right)^{n-1} y.$$

To obtain an expression for the infinite market, use the definition of market tightness in order to substitute for $m = n/b$, and take the limit as $n \rightarrow \infty$. We have

$$\tilde{w}^C = \tilde{z}^C = e^{-b} y.$$

The last step is to characterize the equilibrium level of b . The equation that pins down this object is the zero-profit condition. In the large market, under symmetry, the profit for each firm is given by

$$(1 - e^{-b})(y - \tilde{w}^C) - (b - 1 + e^{-b})\tilde{z}^C - k.$$

Equating this expression with zero, and using the fact that $\tilde{w}^C = \tilde{z}^C$, implies that the equilibrium market tightness solves $(1 - e^{-b} - be^{-b})y = k$, i.e. $\tilde{b}^C = b^*$. $\square$

Proof of Lemma 10. The first-order conditions with respect to b and z are necessary and sufficient for maximization of the Planner's problem. Notice that the first-order condition with respect to z reduces to

$$u' \left(y + z - \frac{k + zb}{1 - e^{-b}} \right) = u'(z),$$

which implies that the Planner's solution satisfies

$$k = (1 - e^{-b^S}) y - bz^S. \quad (\text{a.16})$$

The first-order condition with respect to b can be written as

$$(1 - e^{-b} - be^{-b}) \left[u \left(y + z - \frac{k + zb}{1 - e^{-b}} \right) - u(z) \right] - \\ -b \frac{e^{-b}k - (1 - e^{-b} - be^{-b})z}{1 - e^{-b}} u' \left(y + z - \frac{k + zb}{1 - e^{-b}} \right) = 0. \quad (\text{a.17})$$

However, (a.16) implies that the term in square brackets in the first line of (a.17) is zero. Hence, (a.17) can only be satisfied if

$$e^{-b^S} k = (1 - e^{-b^S} - b^S e^{-b^S}) z^S. \quad (\text{a.18})$$

Combining (a.16) and (a.18), one can solve for the two unknowns b^S, z^S . It can be easily verified that the unique solution to this system of equations is given by

$$\begin{aligned} b^S &= b^*, \\ z^S &= e^{-b^*} y. \end{aligned}$$

$\square$