

# CONDITIONAL DOMINANCE IN GAMES WITH UNAWARENESS\*

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## Abstract

Heifetz, Meier, and Schipper (2013) introduced dynamic game with unawareness consisting of a partially ordered set of games in extensive form. Here, we study the normal form of dynamic games with unawareness. The generalized normal form associated with a dynamic game with unawareness consists of a partially ordered set of games in normal form. We use the generalized normal form to characterize extensive-form rationalizability (resp., prudent rationalizability) in dynamic games with unawareness by iterated conditional strict (resp., weak) dominance in the associated generalized normal form. We also show that the analogue to iterated admissibility for dynamic games with unawareness depends on extensive-form structure. This is because under unawareness, a player's information set not only determines which nodes she considers possible but also of which game tree(s) she is aware of.

**Keywords:** Awareness, unknown unknowns, extensive form, normal form, extensive-form rationalizability, prudent rationalizability, strong rationalizability, iterated conditional dominance, iterated admissibility.

**JEL-Classifications:** C72, D83.

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# 1 Introduction

Recent years witnessed the extension of game theory to unawareness. The challenge was to invent formal tools for modeling asymmetric lack of conception, beliefs about other players' lack of conception etc. It goes beyond modeling standard asymmetric lack of information. Several frameworks have been proposed both for static games (Meier and Schipper, 2014, Feinberg, 2021, Sadzik, 2021, Perea, 2022) and dynamic games (Halpern and Rego, 2014, Heifetz, Meier, and Schipper, 2013, 2021, Feinberg, 2021, Grant and Quiggin, 2013, Li, 2008, Schipper, 2021, 2017, Guarino, 2020); see Schipper (2014) for a non-technical review. These tools have already lead to interesting applications such as in contract theory (Filiz-Ozbay, 2012, Auster, 2013, Auster and Pavoni, 2023, von Thadden and Zhao, 2012, Francetich and Schipper, 2022), political economy (Schipper and Woo, 2019), and finance (Schipper and Zhou, 2021).<sup>1</sup> Developing sensible solution concepts for games with unawareness has been another challenge. The problem with familiar equilibrium concepts is that a player's awareness may change endogenously during the course of play making the equilibrium assumption of an ex ante ready-made behavioral convention often implausible; see Schipper (2021) for an extensive discussion. As a remedy, rationalizability concepts have been proposed (Heifetz, Meier, and Schipper, 2013, 2021) and successfully used in applications (Schipper and Woo, 2019, Francetich and Schipper, 2022). It is well-known that rationalizability in the normal form is equivalent to the iterated elimination of strictly dominated strategies (Pearce, 1984). A similar result holds for extensive-form rationalizability and iterated elimination of conditional strictly dominated strategies (Shimoji and Watson, 1998, Chen and Micali, 2013).<sup>2</sup> Such a characterization might be useful for applications as it is often easier to eliminate strategies by dominance arguments than eliminating beliefs over opponents' strategy profiles especially in games with more than two players.<sup>3</sup> This paper studies the connection of rationalizability and dominance in games with unawareness.

We focus on dynamic games with unawareness (Heifetz, Meier, and Schipper, 2013) because, as already mentioned, the challenge under unawareness is the endogenous change of players' awareness throughout the play. This elevates the time structure of the strategic interaction embodied by the generalized extensive form. It makes an equivalence between rationalizability notions and iterated dominance notions more surprising as the latter are typically applied to the associated normal form while the former apply to the extensive form. Arguably, the normal form "folds" the time structure. Nevertheless, Kohlberg and Mertens (1986) argued that for standard games without unawareness, the normal form contains all strategically relevant information. Could a similar claim be made for strategic situations in which the very perceptions of the situation by players change endogenously in ways that ex ante are not even anticipated by all

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<sup>1</sup>See <http://www.econ.ucdavis.edu/faculty/schipper/unaw.htm> for a bibliography on unawareness.

<sup>2</sup>Extensive-form rationalizability has been introduced by Pearce (1984) and Battigalli (1997). More specifically, we refer to extensive-form correlated rationalizability because for  $n$ -player games with  $n > 2$  we allow any player to believe in correlations among her opponents' strategies.

<sup>3</sup>While finding both undominated strategies and best responses involve linear programming problems, the dimensionalities of these linear programming problems differ. For undominated strategies, we need to find mixtures over the player's *other own* strategies while for best responses, we need to find mixtures over opponents' strategy *profiles*. With more than two players, latter problem involves more computations.

players? To answer such a question in the context of extensive-form rationalizability notions, we first need to define the appropriate notion of normal form associated with dynamic games with unawareness. For standard dynamic games without unawareness, the associated normal form features strategies of the players as primitives. A strategy assigns an action to each information set of the player. Dynamic games with unawareness consist of a forest of partially ordered trees, each representing a partial view of the objectively feasible sequences of moves. A player’s information set at a node of a tree may consist of nodes of a “less expressive” tree. A player may not be aware of all strategies *ex ante* and may discover further actions during the course of play. Yet, for any tree, there is a well-defined set of partial strategies, namely objects that assign actions to all information sets of the player in the tree and any less expressive tree. The associated generalized normal form consists now of a partially ordered set of normal forms, indexed by trees, taking the sets of *partial* strategies as primitives.

With the definition of associated generalized normal form in place, we can characterize extensive-form rationalizability of Pearce (1984) and Battigalli (1997) extended to unawareness in Heifetz, Meier, and Schipper (2013) by iterated elimination of conditionally strictly dominated strategies similar to Shimoji and Watson (1998) for standard games without unawareness. Yet, the non-trivial twist is that partial strategies eliminated in less expressive normal forms must trigger also the elimination of any strategies in more expressive normal forms that they are part of. This is because strategies in more expressive normal forms also specify actions in less expressive trees. A player with an awareness level given by the less expressive tree may consider the corresponding partial strategy to be dominated. Consequently, opponents with higher awareness levels realize that their rational but unawareness opponent will not play such a strategy and thus eliminate this player’s strategy in their more expressive normal form. We illustrate this with a simple example in the next section.

In dynamic games with unawareness, extensive-form rationalizability may involve imprudent behavior. It may be rationalizable for a player to make an opponent aware of one of the opponent’s actions that is extremely bad for the player because the player is allowed to believe that the opponent will not take this action. As a remedy, Heifetz, Meier, and Schipper (2021) introduced a version of extensive-form rationalizability using the idea of prudence or caution that proved to be instrumental in applications such as disclosure of verifiable information (Heifetz, Meier, and Schipper, 2013, Li and Schipper, 2019, 2020), electoral campaigning (Schipper and Woo, 2019), and screening under unawareness (Francetich and Schipper, 2022). This solution concept, called prudent rationalizability, could be viewed as an extensive-form analogue of iterated admissibility in dynamic games with unawareness. While iterated admissibility is known to be a refinement of rationalizability in static games, prudent rationalizability is surprisingly *not* always a refinement of extensive-form rationalizability in terms of strategies (not even for standard dynamic games without unawareness); see Heifetz, Meier, and Schipper (2021) for examples.<sup>4</sup> In this paper, we characterize prudent rationalizability in dynamic games with unawareness by iterated elimination of conditionally weakly dominated strategies

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<sup>4</sup>More recently, results by Cantonini (2022) imply that prudent rationalizability is not even an outcome refinement of extensive-form rationalizability in standard games.

in the associated generalized normal form. Again, the twist is that (partial) strategies eliminated in less expressive normal forms must trigger also the elimination of any strategies in more expressive normal forms that they are part of.

One could argue that the characterization of extensive-form rationalizability (resp., prudent rationalizability) by iterated conditional strict (resp., weak) dominance falls short of showing that one can capture sophisticated extensive-form reasoning in the normal form. This is because we condition on normal-form information sets corresponding to information sets of a *given* dynamic game with unawareness. In our context, a relevant normal-form information set consists of the subset of partial strategy profiles inducing a path through an information set in a *given* dynamic game with unawareness. Thus, we implicitly make use of extensive-form structures in the definition of iterated conditional strict (resp., weak) dominance when conditioning on normal-form information sets. In contrast, for standard games without unawareness, Shimoji and Watson (1998) show that generically they can capture sophisticated reasoning embodied in extensive-form rationalizability by iterated elimination of conditionally strictly dominated strategies in the normal form using more generally any normal-form information set, i.e., normal-form information sets for which there exists *some* game in extensive-form with an information set that corresponds to it (Mailath, Samuelson, and Swinkels, 1993). Such a result is elusive for games with unawareness since normal-form information sets must also encode awareness that crucially depends on the extensive form. A potential remedy would be to use a different solution concept that does not make use of extensive-form structure even though it somehow captures sophisticated reasoning in the extensive-form. For standard games such a solution concept is iterated admissibility. Indeed, Brandenburger and Friedenberg (2011) show that for standard dynamic games with perfect recall, iterated admissibility coincides with iterated conditional weak dominance at every level. Thus, by an inductive application of Lemma 4 of Pearce (1984), in standard games without unawareness, prudent rationalizability coincides with iterated admissibility in the associated normal form at every level/iteration. However, as we demonstrate, for dynamic games with unawareness the appropriate definition of iterated admissibility must make use of information sets as well. This is due to the fact that information sets in dynamic games with unawareness do not only model a player’s information in the standard sense but also her awareness. The player’s awareness of strategies is crucial for admissibility since for instance at the first level, a player cautiously considers possible any of the opponents’ strategies only to the extent that she is aware of them. We define iterated admissibility for games with unawareness and show that in dynamic games with unawareness, iterated admissibility is conceptually closer to iterated conditional weak dominance because it cannot be independent of the awareness encoded in information sets of the extensive form. We show that prudent rationalizability is characterized by iterated admissibility in games with unawareness.

In ongoing work on mechanism design with unawareness (Pram and Schipper, 2023), we use insights from the current paper to study efficient implementation in *conditional* weak dominant equilibrium in mechanisms under unawareness. In such settings, dynamic mechanisms become crucial as the awareness contained in the agents’ reports is pooled by the mechanism

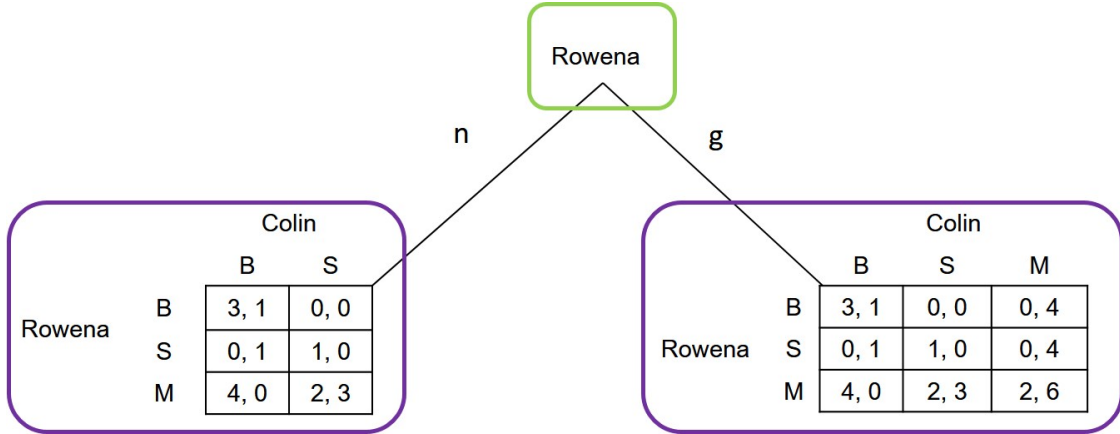
designer and communicated back to all agents who subsequently elaborate on their previously reported types at the pooled awareness level. We extend dominant strategy implementation of VCG mechanisms in the standard setting to *conditional* dominant strategy implementation of dynamic direct elaboration VCG mechanisms under unawareness. We use the insight from the current paper that admissibility in games with unawareness must still condition on the awareness embodied in information sets. It necessitates the use of *conditional* dominance under unawareness while just unconditional dominance would suffice in standard dynamic mechanism.

The paper is organized as follows: The next section provides an illustrative example. Section 3 recalls definitions of dynamic games with unawareness and presents the novel notion of associated generalized normal form. Section 4 shows the characterization of extensive-form rationalizability by iterated elimination of conditional strictly dominated strategies. The characterization of prudent rationalizability by iterated elimination of conditional weakly dominated strategies is presented in Section 5. Finally, we present the notion of iterated admissibility and show that it characterizes prudent rationalizability in Section 6. Proofs are relegated to an appendix.

## 2 Introductory Example

The purpose of the example is to illustrate the novel notion of generalized normal form associated with dynamic games with unawareness and the subtleties of the characterization of extensive-form rationalizability by iterated conditional dominance in games with unawareness. In particular, we contrast it to standard games. To this extent, we begin with a standard game given by Figure 1. There are two players, Rowena and Colin. Rowena moves first deciding

Figure 1: Example without Unawareness



between actions  $n$  and  $g$ . This is followed by a simultaneous move game. The implication of Rowena's action  $g$  is that Colin has all three actions  $B$ ,  $S$ , and  $M$ . Otherwise, if Rowena takes action  $n$ , Colin is just left with actions  $B$  and  $S$  in the simultaneous game that follows. That is,

Rowena's initial move affects Colin's availability of action  $M$ .<sup>5</sup> The information sets of players are indicated by the green and purple rectangles. The green rectangle at the root of the tree belongs to Rowena. The purple rectangles indicate information sets of both players.

Extensive-form rationalizability is almost trivial in this example as the procedure concludes already after the first level. Any of Rowena's strategy that does not prescribe action  $M$  after action  $n$  is not rationalizable as there is no belief over Colin's strategies conditional on reaching the subgame after  $n$  for which any other action than  $M$  is rational. Similarly, any of Rowena's strategy that does not prescribe action  $M$  after action  $g$  is not rationalizable. This leaves strategies  $\{nMB, nMS, nMM, gBM, gSM, gMM\}$  for Rowena. For Colin, any strategy that does not prescribe  $M$  in the subgame after  $g$  is not rationalizable since there is no belief of Colin over Rowena's strategies conditional on  $g$  that would make action  $B$  or  $S$  rational in the subgame after  $g$ . This leaves strategies  $\{BM, SM\}$  for Colin. At the second level, both players believe in the opponent's first-level extensive-form rationalizable strategies. In this example it does not lead to further eliminations. However, the example is sufficiently rich to demonstrate the difference between strict dominance and *conditional* strict dominance.

Figure 2: Associated Normal Form

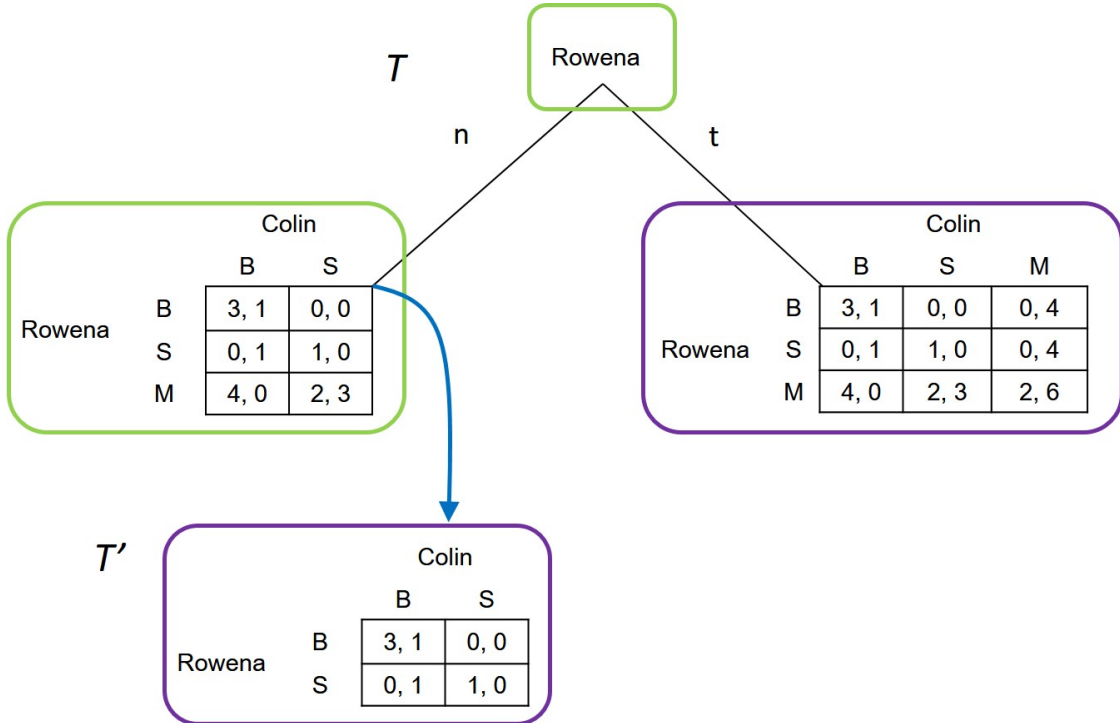
|        |     |       |      |      |      |      |      |
|--------|-----|-------|------|------|------|------|------|
|        |     | Colin |      |      |      |      |      |
|        |     | BB    | BS   | BM   | SB   | SS   | SM   |
| Rowena | nBB | 3, 1  | 3, 1 | 3, 1 | 0, 0 | 0, 0 | 0, 0 |
|        | nBS | 3, 1  | 3, 1 | 3, 1 | 0, 0 | 0, 0 | 0, 0 |
|        | nBM | 3, 1  | 3, 1 | 3, 1 | 0, 0 | 0, 0 | 0, 0 |
|        | nSB | 0, 1  | 0, 1 | 0, 1 | 1, 0 | 1, 0 | 1, 0 |
|        | nSS | 0, 1  | 0, 1 | 0, 1 | 1, 0 | 1, 0 | 1, 0 |
|        | nSM | 0, 1  | 0, 1 | 0, 1 | 1, 0 | 1, 0 | 1, 0 |
|        | nMB | 4, 0  | 4, 0 | 4, 0 | 2, 3 | 2, 3 | 2, 3 |
|        | nMS | 4, 0  | 4, 0 | 4, 0 | 2, 3 | 2, 3 | 2, 3 |
|        | nMM | 4, 0  | 4, 0 | 4, 0 | 2, 3 | 2, 3 | 2, 3 |
|        | gBB | 3, 1  | 0, 0 | 0, 4 | 3, 1 | 0, 0 | 0, 4 |
|        | gBS | 0, 1  | 1, 0 | 0, 4 | 0, 1 | 1, 0 | 0, 4 |
|        | gBM | 4, 0  | 2, 3 | 2, 6 | 4, 0 | 2, 3 | 2, 6 |
|        | gSB | 3, 1  | 0, 0 | 0, 4 | 3, 1 | 0, 0 | 0, 4 |
|        | gSS | 0, 1  | 1, 0 | 0, 4 | 0, 1 | 1, 0 | 0, 4 |
|        | gSM | 4, 0  | 2, 3 | 2, 6 | 4, 0 | 2, 3 | 2, 6 |
|        | gMB | 3, 1  | 0, 0 | 0, 4 | 3, 1 | 0, 0 | 0, 4 |
|        | gMS | 0, 1  | 1, 0 | 0, 4 | 0, 1 | 1, 0 | 0, 4 |
|        | gMM | 4, 0  | 2, 3 | 2, 6 | 4, 0 | 2, 3 | 2, 6 |

<sup>5</sup>While the names of actions and the structure bears some similarity with the games discussed in Heifetz, Meier, and Schipper (2013), the payoffs differ.

Figure 2 presents the normal form associated with the standard dynamic game of Figure 1. The rows represent Rowena's strategies. E.g.,  $nBS$  denotes the strategy that prescribes  $n$  at her initial information,  $B$  at her information set after  $n$ , and  $S$  at her information set after  $g$ . The strategies of Colin are represented by columns. E.g.,  $BS$  is the strategy that would require Colin to play  $B$  at his information set after  $n$  and  $S$  at his information set after  $g$ . We also indicate the normal-form information sets by rectangles. A normal-form information set consists of the subset of strategy profiles inducing a path through an information set in the dynamic game. E.g., the entire strategy space is consistent with Rowena's first information set. That is, the green information set in Figure 1 corresponds to the green rectangle in Figure 2.

We indicate strategies eliminated by conditional strict dominance by dashed lines. Noteworthy, strategies  $\{BB, BS, SB, SS\}$  of Colin are eliminated because they are strictly dominated *conditional* on the normal-form information set at the bottom of the figure, which corresponds to his information after  $g$ . They would not be unconditionally strictly dominated. This illustrates how conditional strict dominance goes beyond (unconditional) strict dominance. We also observe that for both players, extensive-form rationalizable strategies correspond exactly to the strategies eliminated by elimination of strictly dominated strategies. This is just an example of the general result by Shimoji and Watson (1998).

Figure 3: Example of a Dynamic Game with Unawareness



So far, in this example, both solution concepts are almost trivial as the procedure conclude after one round of elimination. The point, beside demonstrating conditional dominance, is

that in an analogous game with Colin’s unawareness of action  $M$  (rather than unavailability of action  $M$ ), the solution concepts can become more interesting. Consider the dynamic game with unawareness of Figure 3. There are two “trees”, the upper tree  $T$  and the lower “tree”  $T'$ . The upper tree looks equivalent to the tree in Figure 1. However, here action  $t$  of Rowena is interpreted as raising Colin’s awareness of action  $M$ . This is indicated by the purple information after  $t$ , an information set that belongs to both players. If Rowena takes action  $n$  instead, then Colin remains unaware of action  $M$ . This is indicated by his information set after  $n$ , which is not on  $T$  but on the lower “tree”  $T'$  (see the blue arrow). The lower “tree” is just a simultaneous move game between Rowena and Colin with actions  $B$  and  $S$ . There is no mentioning of  $M$ . Not even action  $n$  of not telling Colin about  $M$  is part of the description of the strategic situation in the lower tree  $T'$ , indicating that when a player considers the tree  $T'$ , (s)he is unaware of  $M$ . The green information after action  $n$  belongs only to Rowena. Colin’s information set after  $n$  is the purple information at  $T'$ . This purple information set is also the information set of both players in the lower “partial” game. The example illustrates two important features of dynamic games with unawareness (Heifetz, Meier, and Schipper, 2013). First, there is a ordered set of trees. In the example,  $T$  is more expressive than  $T'$ . Second, an information set of a player after a history in one tree may consist of histories in a less expressive tree. Latter feature models unawareness of that player of some action in the tree that is not represented in the less expressive tree. In the example, Colin’s information set after  $n$  in  $T$  consists of nodes of the less expressive tree  $T'$ .

In this example, extensive-form rationalizability concludes after two rounds/levels. At the first level, any of Rowena’s strategy that does not prescribe  $M$  after  $n$  is not rationalizable because there is no belief over Colin’s strategies conditional on  $n$  that would let  $B$  and  $S$  after  $n$  yield a higher expected payoff than  $M$ . An analogous argument holds for strategies with initial action  $t$ . Thus, at the first level, only strategies in the set  $\{nM **, t * M*\}$  are rationalizable for Rowena where the first component refers to her actions  $n$  and  $t$  at her initial information set, the second component refers to her action in the simultaneous move game after  $n$ , the third component specifies her action in the simultaneous game after  $t$ , and the fourth component belongs to the simultaneous move game making up lower “tree”  $T'$ . (We indicate by “\*” that the action at the corresponding information set can be arbitrary.) For Colin, the only first level extensive-form rationalizable strategy is  $MB$  where the first component refers to his action at his information set in  $T$  and the second component specifies his action at his information set of “tree”  $T'$ .

At the second level, both players can take into account first-level extensive-form rationalizable strategies of the the opponent. Hence, upon reaching the information set after  $t$ , Rowena is certain that Colin plays  $M$ , while when she selects  $n$ , Colin’s action is  $B$  because he remains unaware of  $M$  and perceives the strategic situation to be given by  $T'$ . Colin’s action  $B$  in  $T'$  induces the action  $B$  in the simultaneous move game after  $n$ . Thus, with any such belief, Rowena’s expected payoff is largest when she plays a strategy with  $n$  followed by  $M$ . Also, in the lower tree  $T'$ , the unaware incarnation of Rowena is certain that Colin plays  $B$ . Thus, her best response is  $B$  in  $T'$ . Hence, the set of second-level extensive-form rationalizable strategies

of Rowena is  $\{nM * B\}$ . For Colin, it remains  $MB$ . No further changes occur at higher levels.

To derive the strategies surviving iterated elimination of strictly dominated strategies, we first need to consider the associated normal form. For standard dynamic games, the associated game in normal form is the game in which the players' strategies of the dynamic game are the primitives. A strategy assigns to each of the player's information set an action. Since in a standard dynamic game every player is aware of all actions, in principle she can "control" her entire strategy *ex ante*. In dynamic games with unawareness, for each information set of a player her strategy specifies – from the point of view of the modeler – what the player would do if and when that information set of hers is ever reached. In this sense, a player does not necessarily 'own' her full strategy at the beginning of the game, because she might not be initially aware of all of her information sets. For instance, in Figure 3 a strategy of Colin assigns an action to the information set in the upper tree  $T$  and an action to the information set in the lower tree  $T'$ . A strategy for Rowena assigns an action to the root of the upper tree  $T$ , the left information set in upper tree, the right information set in upper tree, and the information set in the lower tree  $T'$ . When players face the game in the lower tree  $T'$ , then they can only choose partial strategies, i.e., strategies restricted to the information set in  $T'$ . For instance, the set of  $T'$ -partial strategies is just  $\{B, S\}$  for any player. With the notion of strategy and partial strategy, we can define the associated generalized normal form as usual by taking the strategies and partial strategies as primitives. When we take partial strategies in  $T'$  as primitives, we get the normal form at the bottom of Figure 4. When we take entire strategies as primitives, we obtain the upper normal form in Figure 4. That is, while a dynamic game with unawareness consists of a partially ordered set of trees, its generalized associated normal form consists of a partially ordered set of normal forms.

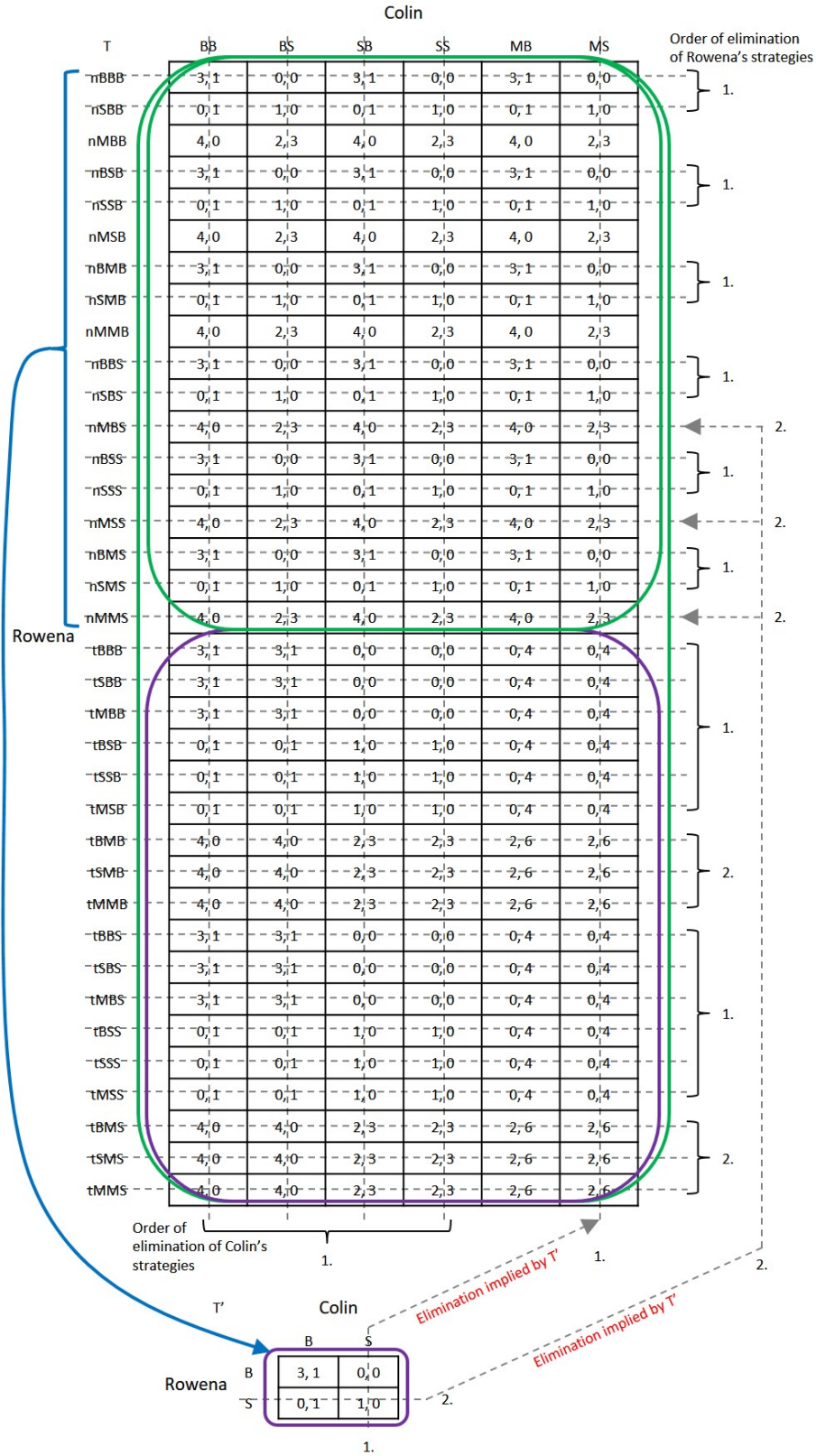
We also indicated the normal-form information sets in Figure 4 using the same color coding as for the corresponding information sets in Figure 3. Green belongs to Rowena only, purple to both players, and the blue arrow indicates that when Rowena chooses any strategy with action  $n$ , then Colin remains unaware of  $M$  and his information set is in the lower tree  $T'$ . Here, the normal-form information set after Rowena chooses action  $n$  in  $T$  is the normal-form associated with the lower tree  $T'$ .

With the dashed lines and the numbers beside them, we indicate the order of iterated elimination of conditional strictly dominated strategies. However, in games with unawareness the algorithm becomes more subtle since conditional dominance of a  $T'$ -partial strategy implies that all strategies with the same components (i.e., actions) are deleted as well. In the first round, this is the case for Colin where deletion of  $S$  in the game  $T'$  implies that all strategies with  $S$  as the second component in the game  $T$  are eliminated as well. In particular, this applies to strategy  $MS$  that is not otherwise conditionally strictly dominated in the upper normal form associated with  $T$ .<sup>6</sup> We indicate this with dashed arrows. A similar case arises

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<sup>6</sup>Again, we emphasize that while our example discussed here bears similarities with examples discussed in Heifetz, Meier, and Schipper (2013), the payoffs differ. The reason is two-fold: First, we want illustrate that a partial strategy which is conditionally dominated in a lower normal form triggers deletion of all strategies in the upper normal form that feature the partial strategy as a component. Second, while in the example of Heifetz, Meier, and Schipper (2013), extensive-form rationalizability in a game with unavailability of an action yields a

Figure 4: Associated Generalized Normal-form Game of the Example



for Colin in the second round. After two rounds, the process stops. The remaining strategies coincide exactly with the extensive-form rationalizable strategies. More precisely, each round of elimination of conditionally strictly dominated strategies corresponds to the corresponding level of extensive-form rationalization. In Section 4, we show that this is generally the case.

### 3 Dynamic Games with Unawareness

In this section we outline dynamic games with unawareness as introduced in Heifetz, Meier, and Schipper (2013).<sup>7</sup> To define a dynamic game  $\Gamma$ , consider first, as a building block, a finite perfect information game with a set of players  $I$ , a set of decision nodes  $N_0$ , active players  $I_n$  at node  $n$  with finite action sets  $A_n^i$  of player  $i \in I_n$  (for  $n \in N_0$ ) and terminal nodes  $Z_0$  with a payoff vector  $(p_i^z)_{i \in I} \in \mathbb{R}^I$  for the players for every  $z \in Z_0$ . The nodes  $\bar{N}_0 = N_0 \cup Z_0$  constitute a tree.<sup>8</sup>

Consider now a family  $\mathbf{T}$  of subtrees of  $\bar{N}_0$ , partially ordered  $(\preceq)$  by inclusion. One of the trees  $T_1 \in \mathbf{T}$  is meant to represent the modeler's view of the paths of play that are *objectively* feasible; each other tree represents the feasible paths of play as *subjectively* viewed by some player at some node at one of the trees.

In each tree  $T \in \mathbf{T}$ , denote by  $n_T$  the copy in  $T$  of the node  $n \in \bar{N}_0$  whenever the copy of  $n$  is part of the tree  $T$ . However, in what follows we will typically avoid the subscript  $T$  when no confusion may arise.

Denote by  $N_i^T$  the set of nodes in which player  $i \in I$  is active in the tree  $T \in \mathbf{T}$ . We require that all the terminal nodes in each tree  $T \in \mathbf{T}$  are copies of nodes in  $Z_0$ . Moreover, if for two decision nodes  $n, n' \in N_i^T$  (i.e.,  $i \in I_n \cap I_{n'}$ ) it is the case that  $A_n^i \cap A_{n'}^i \neq \emptyset$ , then  $A_n^i = A_{n'}^i$ .

Denote by  $N$  the union of all decision nodes in all trees  $T \in \mathbf{T}$ , by  $Z$  the union of terminal nodes, and by  $\bar{N} = N \cup Z$ . (Copies  $n_T$  of a given node  $n$  in different subtrees  $T$  are distinct from one another, so that  $\bar{N}$  is a disjoint union of sets of nodes.) For a node  $n \in \bar{N}$  we denote by  $T_n$  the tree containing  $n$ .

For each decision node  $n \in N$  and each active player  $i \in I_n$ , the information set is denoted by  $\pi_i(n)$ . It is the set of nodes that the player  $i$  considers as possible at  $n$ .  $\pi_i(n)$  will be in a different tree than in the tree  $T_n$  if at  $n$  the player is unaware of some of the paths in  $T_n$ , and rather envisages the dynamic interaction as taking place in the tree containing  $\pi_i(n)$ . We impose properties analogous to standard dynamic games with perfect recall; see Appendix A and discussions in Heifetz, Meier, and Schipper (2013) and Schipper (2021, 2019).

We denote by  $H_i$  the set of  $i$ 's information sets in all trees. For any information set  $h_i \in H_i$ , sharper prediction than in an analogous game with unawareness of that action, the opposite is the case in the example discussed here. We conclude that extensive-form rationalizability/iterated conditional strict dominance with unawareness of actions differs from unavailability of actions but one does not necessarily yield a sharper prediction than the other.

<sup>7</sup>See also Heifetz, Meier, and Schipper (2021), Schipper (2021), and Schipper (2019).

<sup>8</sup>Heifetz, Meier, and Schipper (2013) considered also moves of nature which we do not explicitly consider here. Note that we can treat nature just like another player who is indifferent among all terminal nodes.

we denote by  $T_{h_i}$  the tree containing  $h_i$ . For two information sets  $h_i, h'_i$  in a given tree  $T$ , we say that  $h_i$  precedes  $h'_i$  (or that  $h'_i$  succeeds  $h_i$ ) if for every  $n' \in h'_i$  there is a path  $n, \dots, n'$  such that  $n \in h_i$ . We denote the precedence relation by  $h_i \rightsquigarrow h'_i$ .

Standard properties imply that if  $n', n'' \in h_i$  where  $h_i = \pi_i(n)$  is an information set, then  $A_{n'}^i = A_{n''}^i$  (see Heifetz, Meier, and Schipper, 2013, Remark 1, for details). Thus, if  $n \in h_i$  we write also  $A_{h_i}$  for  $A_n^i$ .

Perfect recall property guarantees that with the precedence relation  $\rightsquigarrow$  player  $i$ 's information sets  $H_i$  form an *arborescence*: For every information set  $h'_i \in H_i$ , the information sets preceding it  $\{h_i \in H_i : h_i \rightsquigarrow h'_i\}$  are totally ordered by  $\rightsquigarrow$ .

For trees  $T, T' \in \mathbf{T}$  we denote  $T \succ T'$  whenever for some node  $n \in T$  and some player  $i \in I_n$  it is the case that  $\pi_i(n) \subseteq T'$ . Denote by  $\hookrightarrow$  the transitive closure of  $\succ$ . That is,  $T \hookrightarrow T''$  if and only if there is a sequence of trees  $T, T', \dots, T'' \in \mathbf{T}$  satisfying  $T \succ T' \succ \dots \succ T''$ .

A dynamic game with unawareness  $\Gamma$  consists of a partially ordered set  $\mathbf{T}$  of subtrees of a tree  $\bar{N}_0$  along with information sets  $\pi_i(n)$  for every  $n \in T$ ,  $T \in \mathbf{T}$  and  $i \in I_n$ , satisfying all properties imposed in Heifetz, Meier, and Schipper (2013); see Appendix A.

For every tree  $T \in \mathbf{T}$ , the  $T$ -*partial game* is the partially ordered set of trees including  $T$  and all trees  $T'$  in  $\Gamma$  satisfying  $T \hookrightarrow T'$ , with information sets as defined in  $\Gamma$ . A  $T$ -partial game is a dynamic game with unawareness, i.e., it satisfies the same properties.

We denote by  $H_i^T$  the set of  $i$ 's information sets in the  $T$ -partial game.

A (pure) strategy

$$s_i \in S_i \equiv \prod_{h_i \in H_i} A_{h_i}$$

for player  $i$  specifies an action of player  $i$  at each of her information sets  $h_i \in H_i$ . Denote by

$$S = \prod_{j \in I} S_j$$

the set of strategy profiles in the dynamic game with unawareness.

If  $s_i = (a_{h_i})_{h_i \in H_i} \in S_i$ , we denote by

$$s_i(h_i) = a_{h_i}$$

the player's action at the information set  $h_i$ . If player  $i$  is active at node  $n$ , we say that at node  $n$  the strategy prescribes to her the action  $s_i(\pi_i(n))$ .

In dynamic games with unawareness, a strategy cannot be conceived as an ex ante plan of action. If  $h_i \subseteq T$  but  $T \not\hookrightarrow T'$ , then at  $h_i$  player  $i$  may be interpreted as being unaware of her information sets in  $H_i^{T'} \setminus H_i^T$ . Thus, a strategy of player  $i$  should rather be viewed as a list of answers to the hypothetical questions “what would the player do if  $h_i$  were the set of nodes she considered as possible?”, for  $h_i \in H_i$ . However, there is no guarantee that such a question about the information set  $h'_i \in H_i^{T'}$  would even be meaningful to the player if it were asked at a different information set  $h_i \in H_i^T$  when  $T \not\hookrightarrow T'$ . The answer should therefore be interpreted as given by the modeler, as part of the description of the situation.

For a strategy  $s_i \in S_i$  and a tree  $T \in \mathbf{T}$ , we denote by  $s_i^T$  the strategy in the  $T$ -partial game induced by  $s_i$ . If  $R_i \subseteq S_i$  is a set of strategies of player  $i$ , denote by  $R_i^T$  the set of strategies induced by  $R_i$  in the  $T$ -partial game. The set of  $i$ 's strategies in the  $T$ -partial game is thus denoted by  $S_i^T$ . Denote by  $S^T = \prod_{j \in I} S_j^T$  the set of strategy profiles in the  $T$ -partial game.

We say that a strategy profile  $s \in S$  *allows* the information set  $h_i \in H_i$  if the players' actions in  $T_{h_i}$  lead to  $h_i$ . (Notice that unlike in standard games, an information set  $\pi_i(n)$  may be contained in tree  $T' \neq T_n$ . In such a case,  $s_i(\pi_i(n))$  induces an action to player  $i$  also in  $n$  and not only in the nodes of  $\pi_i(n)$ .) We say that the strategy  $s_i \in S_i$  *allows* the information set  $h_i$  if there is a strategy profile  $s_{-i} \in S_{-i}$  of the other players such that the strategy profile  $(s_i, s_{-i})$  allows  $h_i$ . Otherwise, we say that the information set  $h_i$  is excluded by the strategy  $s_i$ . Similarly, we say that the strategy profile  $s_{-i} \in S_{-i}$  *allows* the information set  $h_i$  if there exists a strategy  $s_i \in S_i$  such that the strategy profile  $(s_i, s_{-i})$  allows  $h_i$ . A strategy profile  $(s_j)_{j \in I}$  *allows* a node  $n \in T$  if the players' actions  $s_j(\pi_j(n'))_{j \in I}$  lead to  $n$ . Since we consider only finite trees,  $(s_j)_{j \in I}$  allows an information set  $h_i \in H_i$  if and if there is a node  $n \in h_i$  such that  $(s_j)_{j \in I}$  allows  $n$ .

As it is the case also in standard games, for every given node, a given strategy profile of the players induces a terminal nodes in each tree, and hence a payoff for each player in the tree.

For an information set  $h_i$ , let  $s_i/\tilde{s}_i^{h_i}$  denote the strategy that is obtained by replacing actions prescribed by  $s_i$  at the information set  $h_i$  and its successors by actions prescribed by  $\tilde{s}_i$ . The strategy  $s_i/\tilde{s}_i^{h_i}$  is called an  $h_i$ -replacement of  $s_i$ .

### 3.1 Associated Generalized Normal Form

Consider a dynamic game with unawareness  $\Gamma$  with a partially ordered set of trees  $\mathbf{T}$ . The *associated generalized normal form*  $G$  is defined by  $\langle I, \langle (S_i^T)_{i \in I}, (u_i^T)_{i \in I} \rangle_{T \in \mathbf{T}} \rangle$ , where  $I$  is the set of players in  $\Gamma$  and  $S_i^T$  is player  $i$ 's set of  $T$ -partial strategies. Recall that if player  $i$  is active at node  $n \in T$ , then the strategy  $s_i \in S_i^T$  assigns the action  $s_i(\pi_i(n))$  to node  $n$ . Hence, each profile of strategies in  $S^T$  induces a distribution over terminal nodes in  $T$  (even if there is a player active in  $T$  with no information set in  $T$ ). We let  $u_i^T(s)$  be the payoffs associated with the terminal node in  $T$  allowed by  $s \in S^T$ . (Note that while strategy profiles in  $S^T$  allow terminal nodes also in trees  $T' \in \mathbf{T}$ ,  $T \hookrightarrow T'$ ,  $u_i^T$  concerns payoffs in the tree  $T$  only.)

Recall that  $H_i^T$  denotes player  $i$ 's set of information sets in the  $T$ -partial dynamic game. For each  $h_i \in H_i^T$ , let  $S^T(h_i) \subseteq S^T$  be the subset of the  $T$ -partial strategy space containing  $T$ -partial strategy profiles that allow the information set  $h_i$ . Define also  $S_i^T(h_i) \subseteq S_i^T$  and  $S_{-i}^T(h_i) \subseteq S_{-i}^T$  to be the set of player  $i$ 's  $T$ -partial strategies allowing  $h_i$  and the set of profiles of the other players'  $T$ -partial strategies allowing  $h_i$  respectively. For the entire game denote by  $S(h_i) \subseteq S$  the set of strategy profiles that allow  $h_i$ . Similarly,  $S_i(h_i) \subseteq S_i$  and  $S_{-i}(h_i) \subseteq S_{-i}$  are the set of player  $i$ 's strategies allowing  $h_i$  and the set of profiles of the other players' strategies allowing  $h_i$  respectively.

Given  $\Gamma$  and its associated generalized normal form  $G$ , define player  $i$ 's set of *normal-form*

information sets by

$$\mathcal{X}_i = \{S^{T_{h_i}}(h_i) : h_i \in H_i\}.$$

These are the “normal-form versions” of information sets in the dynamic game with unawareness. In the literature on standard games, normal-form information sets refer more generally to subsets of the strategy space of a pure strategy reduced normal-form for which there exists a game in extensive form with corresponding information sets (see Mailath, Samuelson, and Swinkels, 1993). For our characterization, we are just interested in the normal-form versions of information sets of a *given* dynamic game with unawareness.

For  $T \in \mathbf{T}$ , any set  $Y \subseteq S^T$  is called a *restriction for player  $i$*  (or an  *$i$ -product set*) of  $T$ -partial strategies if  $Y = Y_i \times Y_{-i}$  for some  $Y_i \subseteq S_i^T$  and  $Y_{-i} \subseteq S_{-i}^T$ . Clearly, a player’s normal-form information set is a restriction for that player. I.e., if  $S^{T_{h_i}}(h_i)$  is a normal-form information set of player  $i$ , then it is a restriction for player  $i$  of  $T_{h_i}$ -partial strategy profiles. We say that  $Y \subseteq S^T$  is a *restriction* of  $T$ -partial strategies if it is a restriction for every player.

## 4 Extensive-Form Rationalizability and Iterated Conditional Strict Dominance

Extensive-form rationalizability is an iterative procedure of refining players beliefs about opponents’ (partial) strategies. At each level  $\ell$  of the procedure, each player forms beliefs at each of her information sets about opponents’  $\ell - 1$ -level extensive-form rationalizable (partial) strategies consistent with the information set. She keeps only strategies of her own that are sequentially rational w.r.t. such a system of beliefs yielding her set of  $\ell$ -level extensive-form rationalizable strategies.

In order to define the solution concept, we first define belief systems. A *belief system* of player  $i \in I$ ,

$$b_i = (b_i(h_i))_{h_i \in H_i} \in \prod_{h_i \in H_i} \Delta(S_{-i}^{T_{h_i}})$$

is a profile of beliefs, a belief  $b_i(h_i) \in \Delta(S_{-i}^{T_{h_i}})$  about the other players’ strategies in the  $T_{h_i}$ -partial game, for each information set  $h_i \in H_i$ , with the following properties

- $b_i(h_i)$  allows  $h_i$ , i.e.,  $b_i(h_i)$  assigns probability 1 to the set of strategy profiles of the other players in the  $T_{h_i}$ -partial game that allow  $h_i$ .
- If  $h_i$  precedes  $h'_i$  ( $h_i \rightsquigarrow h'_i$ ), then  $b_i(h'_i)$  is derived from  $b_i(h_i)$  by conditioning whenever possible.

Denote by  $B_i$  the set of player  $i$ ’s belief systems.

For a belief system  $b_i \in B_i$ , a strategy  $s_i \in S_i$ , and an information set  $h_i \in H_i$ , define player  $i$ ’s expected payoff at  $h_i$  as the expected payoff for player  $i$  in  $T_{h_i}$  given  $b_i(h_i)$ , the actions

prescribed by  $s_i$  at  $h_i$  and its successors, and conditional on the fact that  $h_i$  has been reached.<sup>9</sup>

We say that with the belief system  $b_i$  and the strategy  $s_i$  player  $i$  is *rational* at the information set  $h_i \in H_i$ , if either  $s_i$  does not allow  $h_i$  in the tree  $T_{h_i}$ , or if  $s_i$  does allow  $h_i$  in the tree  $T_{h_i}$  then there exists no  $h_i$ -replacement of  $s_i$  which yields player  $i$  a higher expected payoff in  $T_{h_i}$  given the belief  $b_i(h_i)$  on the other players' strategies  $S_{-i}^{T_{h_i}}$ .

The definition of extensive-form rationalizability is stated next. It is the notion of extensive-form rationalizability of Pearce (1984) and Battigalli (1997) extended to dynamic games with unawareness by Heifetz, Meier, and Schipper (2013).

**Definition 1 (Extensive-form rationalizable strategies)** *Define, recursively, the following sequence of belief systems and strategies of player  $i$ .*

$$\begin{aligned}
B_i^1 &= B_i \\
S_i^1 &= \left\{ s_i \in S_i : \begin{array}{l} \text{There exists a belief system } b_i \in B_i^1 \text{ with which for every information set} \\ h_i \in H_i, \text{ strategy } s_i \text{ is rational at } h_i. \end{array} \right\} \\
&\vdots \\
B_i^k &= \left\{ b_i \in B_i^{k-1} : \begin{array}{l} \text{For every information set } h_i, \text{ if there exists some profile of the other} \\ \text{players' strategies } s_{-i} \in S_{-i}^{k-1} \text{ such that } s_{-i} \text{ allows } h_i \text{ in the tree } T_{h_i}, \\ \text{then } b_i(h_i) \text{ assigns probability 1 to } S_{-i}^{k-1, T_{h_i}}. \end{array} \right\} \\
S_i^k &= \left\{ s_i \in S_i : \begin{array}{l} \text{There exists a belief system } b_i \in B_i^k \text{ with which for every information set} \\ h_i \in H_i, \text{ strategy } s_i \text{ is rational at } h_i. \end{array} \right\}
\end{aligned}$$

The set of player  $i$ 's **extensive-form rationalizable strategies** is

$$S_i^\infty = \bigcap_{k=1}^{\infty} S_i^k.$$

Heifetz, Meier, and Schipper (2013) proved that  $S_i^k \subseteq S_i^{k-1}$  for every  $k > 1$ . They also proved that for every finite dynamic game with unawareness, the set of extensive-form rationalizable strategies is non-empty. Battigalli and Siniscalchi (2002) characterized extensive-form rationalizability by common strong belief in rationality. This result has been extended to dynamic games with unawareness by Guarino (2020).

Next, we turn to iterated conditional strict dominance. We say that  $s_i \in S_i^T$  is *strictly dominated* in a restriction  $Y \subseteq S^T$  if  $s_i \in Y_i$ ,  $Y_{-i} \neq \emptyset$ , and there exists a mixed strategy  $\sigma_i \in \Delta(Y_i)$  such that  $u_i^T(\sigma_i, s_{-i}) > u_i^T(s_i, s_{-i})$  for all  $s_{-i} \in Y_{-i}$ .

Denote by  $\mathbf{S} = \bigcup_{T \in \mathbf{T}} S^T$  and  $\mathbf{S}_i = \bigcup_{T \in \mathbf{T}} S_i^T$ .

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<sup>9</sup>Even if this condition is counterfactual due to the fact that the strategy  $s_i$  does not allow  $h_i$ . The conditioning is thus on the event that player  $i$ 's past actions (at the information sets preceding  $h_i$ ) have led to  $h_i$  *even if these actions are distinct from those prescribed by  $s_i$ .*

For  $T \hookrightarrow T'$  and a  $T$ -partial strategy  $s_i \in S_i^T$ , we denote the  $T'$ -partial strategy  $s_i^{T'} \in S_i^{T'}$  induced by  $s_i$ . For  $\tilde{s}_i \in S_i^{T'}$ , define

$$[\tilde{s}_i] := \bigcup_{T \hookrightarrow T'} \{s_i \in S_i^T : s_i^{T'} = \tilde{s}_i\}.$$

That is,  $[\tilde{s}_i]$  is the set of strategies in  $\mathbf{S}_i$  which at information sets  $h_i \in H_i^{T'}$  prescribe the same actions as strategy  $\tilde{s}_i$ . Intuitively,  $[\tilde{s}_i]$  is the set of  $T$ -partial strategies in the that at information sets in the  $T'$ -partial game are behaviorally indistinguishable from the  $T'$ -partial strategy  $\tilde{s}_i$ .

Let  $(Y^T)_{T \in \mathbf{T}}$  be a collection of restrictions, one for each  $T \in \mathbf{T}$ . Define  $\mathbf{Y} = \bigcup_{T \in \mathbf{T}} Y^T$ . We call this set the extended restriction.

**Definition 2 (Conditional Strict Dominance)** *Given an extended restriction  $\mathbf{Y}$ , we say that  $s_i \in S_i^T$  is conditionally strictly dominated on  $(\mathcal{X}_i, \mathbf{Y})$  if there exists  $T' \in \mathbf{T}$  with  $T \hookrightarrow T'$  and  $\tilde{s}_i \in S_i^{T'}$  with  $s_i \in [\tilde{s}_i]$  such that  $\tilde{s}_i$  is strictly dominated in  $X \cap Y^{T'}$  for some normal-form information set  $X \in \mathcal{X}_i$ ,  $X \subseteq S^{T'}$ .*

This definition implies as a special case that  $s_i \in S_i^T$  is conditionally strictly dominated on  $(\mathcal{X}_i, \mathbf{Y})$  if there exists a normal-form information set  $X \in \mathcal{X}_i$ ,  $X \subseteq S^T$  such that  $s_i$  is strictly dominated in  $X \cap Y^T$ . In particular, when there is just one tree like in standard games without unawareness, the definition reduces to standard conditional dominance. Yet, the domination “across” normal forms makes the definition a non-trivial generalization of conditional strict dominance in standard games. When player  $i$ ’s  $T'$ -partial strategy is strictly dominated conditionally on her normal-form information set in the  $T'$ -partial normal form, then also her strategies that have the same components as the  $T'$ -partial strategy should be eliminated. This is due to the fact that her opponents in  $T$ -partial games with  $T \hookrightarrow T'$  should realize that player  $i$  would not play such strategies. That is, a player with a higher awareness level can realize that a strategy is dominated for an opponent with lower awareness level even though that strategy would not be dominated if the opponent were to have the same awareness level as the player. An example is strategy “ $MS$ ” for Colin in Section 2. This strategy is not dominated in the upper normal form corresponding to the full game with unawareness. However, its component “ $S$ ” is dominated in the  $T'$ -partial normal form at the bottom of Figure 4. Using our notation, “ $MS$ ”  $\in$  [“ $S$ ”].

Define an operator on extended restrictions by for any extended restriction  $\mathbf{Y}$ ,

$$U_i(\mathbf{Y}) = \{s_i \in \mathbf{S}_i : s_i \text{ is not conditionally strictly dominated on } (\mathcal{X}_i, \mathbf{Y})\},$$

$$U(\mathbf{Y}) = \bigcup_{T \in \mathbf{T}} \prod_{i \in I} (U_i(\mathbf{Y}) \cap S_i^T),$$

and

$$U_{-i}(\mathbf{Y}) = \bigcup_{T \in \mathbf{T}} \prod_{j \in I \setminus \{i\}} (U_j(\mathbf{Y}) \cap S_j^T).$$

The following procedure formalizes iterated elimination of conditionally strictly dominated strategies.

**Definition 3 (Iterated Conditional Strict Dominance)** *Define recursively,*

$$\begin{aligned} U^0(\mathbf{S}) &= \mathbf{S}, \\ U^{k+1}(\mathbf{S}) &= U(U^k(\mathbf{S})) \text{ for all } k \geq 0, \\ U^\infty(\mathbf{S}) &= \bigcap_{k=0}^{\infty} U^k(\mathbf{S}), \end{aligned}$$

and similarly for  $U_i^k(\mathbf{S})$  and  $U_{-i}^k(\mathbf{S})$ .  $U^\infty(\mathbf{S})$  is the maximal reduction under iterated elimination of conditional strictly dominated strategies.

This procedure generalizes iterated conditional strict dominance by Shimoji and Watson (1998) to games with unawareness.

**Example (Continued)** We will illustrate the definitions with the introductory example of Section 2. In Figure 4, the entire upper normal form is the normal-form information set (marked green) of Rowena (but not Colin) associated with Rowena's information set at the beginning of the  $T$ -partial game (but not in the  $T'$ -partial game). We denote this information set by  $X_R(\emptyset^T)$ . The upper green rectangle in the upper normal form is the normal-form information set of Rowena (but not of Colin) corresponding to her information set after the history  $n$  in the  $T$ -partial game (but not in the  $T'$ -partial game). We denote it by  $X_R(n)$ . The lower purple rectangle in the upper normal form is the normal-form information set for both Rowena and Colin corresponding to the information sets after history  $t$  in the  $T$ -partial game (but not in the  $T'$ -partial game). We denote it by  $X_i(t)$ .

Finally, the lower game in normal form is a normal-form information set (marked purple) corresponding to both Colin's and Rowena's information set in the  $T'$ -partial game. It is also the normal-form information set for Colin corresponding to his information set  $\pi_C(n)$  in the  $T$ -partial game. We indicate this with the blue arrow. We denote it by  $X_i(\emptyset^{T'}) = X_C(n)$ .

The definition of  $\mathbf{S}_i$  is illustrated by the example  $\mathbf{S}_C = \{BB, BS, SB, SS, MB, MS, B, S\}$ , while the definition  $[\tilde{s}_i]$  can be illustrated by  $[\text{"S"}] = \{BS, SS, MS, S\}$ . These are all the strategies of Colin that prescribe the action "S" ("Stravinsky") at the information set  $\pi_C(n)$ .

Iterated elimination of conditionally strictly dominated strategies proceeds as follows:

$$\begin{aligned} U_i^0(\mathbf{S}) &= \mathbf{S}_i, i \in \{R, C\}, \\ U_R^1(\mathbf{S}) &= \{nMBB, nMSB, nMMB, nMBS, nMSS, nMMS, \\ &\quad tBMB, tSMB, tMMB, tBMS, tSMS, tMMS, \\ &\quad B, S\}, \\ U_C^1(\mathbf{S}) &= \{MB, B\}. \end{aligned}$$

For instance, strategy  $nSBB$  is conditionally strictly dominated by  $nMBB$  in the normal-form information set  $X_R(\emptyset^T)$  or  $X_R(n)$ . More interestingly,  $MS$  is conditionally strictly dominated on  $(\mathcal{X}_C, \mathbf{S})$  because  $MS \in [\text{"S"}]$  and  $S$  is strictly dominated by  $B$  in  $X_C(n)$ . This makes sense

because a player who is fully aware of the game should realize that Colin when only being aware of the  $T'$ -partial game considers  $S$  to be strictly dominated. Thus, Colin will not play strategy  $MS$ . So this example demonstrates that *an action in the upper normal form may be eliminated because of strict dominance in the lower normal form*. This is one reason why we chose this game to demonstrate iterated conditional strict dominance.

Applying the definitions iteratively yields

$$\begin{aligned} U_R^2(\mathbf{S}) &= \{nMBB, nMSB, nMMB, B\}, \\ &= U_R^k(\mathbf{S}) \text{ for all } k \geq 2, \\ U_C^2(\mathbf{S}) &= U_C^2(\mathbf{S}) = \{MB, B\}, \\ &= U_C^k(\mathbf{S}) \text{ for all } k \geq 1. \end{aligned}$$

Note that  $U_i^\infty(\mathbf{S}) \cap S_i = S_i^\infty$ . That is, the set of strategies remaining after iterated elimination of conditionally strictly dominated strategies coincides with the set of extensive-form rationalizable strategies. Both solution concepts predict that Rowena will play  $n$  following by  $M$ , while Colin takes action  $B$ .  $\square$

Our main result of this section is that iterated elimination of conditionally strictly dominated strategies characterizes extensive-form rationalizability in dynamic games with unawareness.

**Theorem 1** *For every finite dynamic game with unawareness,  $U_i^k(\mathbf{S}) \cap S_i = S_i^k$ , for all  $k \geq 1$ . Consequently,  $U_i^\infty(\mathbf{S}) \cap S_i = S_i^\infty$ .*

The proof is contained in the appendix. It proceeds by induction and makes use of Lemma 3 by Pearce (1984). Theorem 1 generalizes Shimoji and Watson (1998) to dynamic games with unawareness.

## 5 Prudent Rationalizability and Iterated Conditional Weak Dominance

Heifetz, Meier, and Schipper (2021) defined prudent rationalizability, a notion that is considered to be an outcome refinement of extensive-form rationalizability.

**Definition 4 (Prudent rationalizability)** *For every player  $i \in I$ , let*

$$\bar{S}_i^0 = S_i.$$

For all  $k \geq 1$  define recursively,

$$\begin{aligned}\bar{B}_i^k &= \left\{ b_i \in B_i : \begin{array}{l} \text{For every information set } h_i, \text{ if there exists some profile } s_{-i} \in \bar{S}_{-i}^{k-1} \text{ of the} \\ \text{other players' strategies such that } s_{-i} \text{ allows } h_i \text{ in the tree } T_{h_i}, \text{ then the} \\ \text{support of } b_i(h_i) \text{ is the set of strategy profiles } s_{-i} \in \bar{S}_{-i}^{k-1, T_{h_i}} \text{ that allow } h_i. \end{array} \right\} \\ \bar{S}_i^k &= \left\{ s_i \in \bar{S}_i^{k-1} : \begin{array}{l} \text{There exists a belief system } b_i \in \bar{B}_i^k \text{ with which for every information set} \\ h_i \in H_i, \text{ strategy } s_i \text{ is rational at } h_i. \end{array} \right\}\end{aligned}$$

The set of **prudent rationalizable strategies** of player  $i$  is

$$\bar{S}_i^\infty = \bigcap_{k=1}^{\infty} \bar{S}_i^k$$

Heifetz, Meier, and Schipper (2021) proved that the set of prudent rationalizable strategies is non-empty and discussed various properties and examples. It has been useful in solving disclosure games with and without unawareness (Heifetz, Meier, and Schipper, 2021, Li and Schipper, 2019, 2020), electoral campaigning under unawareness (Schipper and Woo, 2019), and screening and disclosure under unawareness (Francetich and Schipper, 2022). Here we study how to capture this solution concept by iterated elimination of weakly dominated strategies.

We say that  $s_i \in S_i^T$  is *weakly dominated* in a restriction  $Y \subseteq S^T$  if  $s_i \in Y_i$ ,  $Y_{-i} \neq \emptyset$ , and there exists a mixed strategy  $\sigma_i \in \Delta(Y_i)$  such that  $u_i^T(\sigma_i, s_{-i}) \geq u_i^T(s_i, s_{-i})$  for all  $s_{-i} \in Y_{-i}$  and  $u_i^T(\sigma_i, s_{-i}) > u_i^T(s_i, s_{-i})$  for some  $s_{-i} \in Y_{-i}$ .

**Definition 5 (Conditional Weak Dominance)** *Given an extended restriction  $\mathbf{Y}$ , we say that  $s_i \in S_i^T$  is conditionally weakly dominated on  $(\mathcal{X}_i, \mathbf{Y})$  if there exists  $T' \in \mathbf{T}$  with  $T \hookrightarrow T'$  and  $\tilde{s}_i \in S_i^{T'}$  with  $s_i \in [\tilde{s}_i]$  such that  $\tilde{s}_i$  is weakly dominated in  $X \cap Y^{T'}$  for some normal-form information set  $X \in \mathcal{X}_i$ ,  $X \subseteq S^{T'}$ .*

Note that this definition implies as a special case that  $s_i \in S_i^T$  is conditionally weakly dominated on  $(\mathcal{X}_i, \mathbf{Y})$  if there exists a normal-form information set  $X \in \mathcal{X}_i$ ,  $X \subseteq S^T$  such that  $s_i$  is weakly dominated in  $X \cap Y^T$ . Yet, the weak domination “across” normal forms makes this definition a non-trivial generalization of conditional weak dominance. Again, the idea is that a player who perceives his opponent to be aware of less strategies than herself may realize that the opponent considers some of his partial strategies as weakly dominated. Consequently, the player knows that he will not play any strategy that has the weakly dominated partial strategies as components.

We define an operator on extended restrictions by for any extended restriction  $\mathbf{Y}$ ,

$$W_i(\mathbf{Y}) = \{s_i \in \mathbf{S}_i : s_i \text{ is not conditionally weakly dominated on } (\mathcal{X}_i, \mathbf{Y})\},$$

$$W(\mathbf{Y}) = \bigcup_{T \in \mathbf{T}} \prod_{i \in I} (W_i(\mathbf{Y}) \cap S_i^T),$$

and

$$W_{-i}(\mathbf{Y}) = \bigcup_{T \in \mathbf{T}} \prod_{j \in I \setminus \{i\}} (W_j(\mathbf{Y}) \cap S_j^T).$$

**Definition 6 (Iterated Elimination of Conditionally Weakly Dominated Strategies)**

Define recursively,

$$\begin{aligned} W^0(\mathbf{S}) &= \mathbf{S}, \\ W^{k+1}(\mathbf{S}) &= W(W^k(\mathbf{S})) \text{ for all } k \geq 0, \\ W^\infty(\mathbf{S}) &= \bigcap_{k=0}^{\infty} W^k(\mathbf{S}), \end{aligned}$$

and similarly for  $W_i^k(\mathbf{S})$  and  $W_{-i}^k(\mathbf{S})$ . The set  $W^\infty(\mathbf{S})$  is the maximal reduction under iterated elimination of conditionally weakly dominated strategies.

This extends the notion of iterated elimination of conditionally weakly dominated strategies to games with unawareness. We characterize prudent rationalizability by iterated elimination of conditionally weakly dominated strategies.

**Theorem 2** *For every finite dynamic game with unawareness,  $W_i^k(\mathbf{S}) \cap S_i = \bar{S}_i^k$ , for all  $k \geq 1$ . Consequently,  $W_i^\infty(\mathbf{S}) \cap S_i = \bar{S}_i^\infty$ .*

**Proof.** The proof is analogous to Theorem 1 in the appendix. Instead of using Lemma 3 of Pearce (1984), we now use Lemma 4 of Pearce (1984).  $\square$

## 6 Iterated Admissibility

For standard games in extensive-form with perfect recall and without unawareness, Brandenburger and Friedenberg (2011, Proposition 3.1) showed that iterated elimination of conditionally weakly dominated strategies coincides with iterated admissibility at each iteration in the associated normal form. Before we can extend the result to games with unawareness, we first need a notion of iterated admissibility for games with unawareness. How to define iterated admissibility in games with unawareness in generalized normal form?

**Example (Continued)** To motivate our notion of iterated admissibility for games with unawareness, we consider again the example in Section 2. A first straightforward but naive approach could be to apply iterated admissibility to each of the two normal forms separately. At the first level, the set of admissible strategies coincides with all the set of strategies that are not conditionally strict dominated except for  $MS$  of Colin. (Recall that under conditional strict dominance, we were able to delete strategy  $MS$  because  $S$  was strictly dominated in the lower game  $T'$ .) Notice that at the second level, strategy  $MB$  becomes now weakly dominated

by  $MS$ . But  $MB$  is the only extensive-form rationalizable strategy and prudent rationalizable strategy of Colin in the  $T$ -partial game. What is wrong with this naive approach is that it does not eliminate a strategy in the  $T$ -partial game when it is weakly dominated by another strategy in a lower game. Strategies  $MB$  and  $MS$  differ in the second component only, the action of Colin in the lower game  $T'$ . It is in this lower game  $T'$  that  $S$  is dominated by  $B$ , and hence any strategy prescribing  $S$  at the lower game  $T'$  should be eliminated. This motivates us to define iterated admissibility as a procedure that conditions at least on normal forms instead of any normal-form information set. But not any normal form will do. We also need to ensure that for each information set, we condition on the “correct” normal form, namely the normal form that represents the player’s awareness at this information set. This highlights a role of information sets in dynamic games with unawareness. A player’s information set does not only embody information in the standard sense but also the player’s awareness that is given by the tree in which this information set is located.  $\square$

More formally, let  $\mathcal{S}_i = \{S^{Th_i} : h_i \in H_i\}$ . This is the collection of all strategy profiles and  $T$ -partial strategy profiles for any tree  $T$  for which player  $i$  has an information set. We can understand it as the set of normal forms of player  $i$ .

**Definition 7 (NF-conditional Weak Dominance)** *Given an extended restriction  $\mathbf{Y}$ , we say that  $s_i \in S_i^T$  is NF-conditionally weakly dominated on  $(\mathcal{S}_i, \mathbf{Y})$  if there exists  $T' \in \mathbf{T}$  with  $T \hookrightarrow T'$  and  $\tilde{s}_i \in S_i^{T'}$  with  $s_i \in [\tilde{s}_i]$  such that  $\tilde{s}_i$  is weakly dominated in  $S^{T'} \cap Y^{T'}$  for some normal form  $S^{T'} \in \mathcal{S}_i$ .*

Note that this definition implies as a special case that  $s_i \in S_i^T$  is NF-conditionally weakly dominated on  $(\mathcal{S}_i, \mathbf{Y})$  if there exists a normal form  $S^T \in \mathcal{S}_i$ , such that  $s_i$  is weakly dominated in  $S^T \cap Y^T$ . Again, the weak domination “across” normal forms makes this definition a non-trivial generalization of weak dominance.

Define an operator on extended restrictions by for any extended restriction  $\mathbf{Y}$  by

$$\widetilde{W}_i(\mathbf{Y}) = \{s_i \in \mathbf{S}_i : s_i \text{ is not NF-conditionally weakly dominated on } (\mathcal{S}_i, \mathbf{Y})\},$$

$$\widetilde{W}(\mathbf{Y}) = \bigcup_{T \in \mathbf{T}} \prod_{i \in I} \left( \widetilde{W}_i(\mathbf{Y}) \cap S_i^T \right),$$

and

$$\widetilde{W}_{-i}(\mathbf{Y}) = \bigcup_{T \in \mathbf{T}} \prod_{j \in I \setminus \{i\}} \left( \widetilde{W}_j(\mathbf{Y}) \cap S_j^T \right).$$

Compared to operator  $W$  defined earlier that considered strategies that are not conditionally weakly dominated, operator  $\widetilde{W}$  considers strategies that are not *NF*-conditionally weakly dominated. This means that with  $\widetilde{W}$  we condition only on the normal form but not on finer subsets of strategy profiles.

**Definition 8 (Iterated Admissibility)** *Define recursively,*

$$\begin{aligned}\widetilde{W}^0(\mathbf{S}) &= \mathbf{S}, \\ \widetilde{W}^{k+1}(\mathbf{S}) &= \widetilde{W}(\widetilde{W}^k(\mathbf{S})) \text{ for all } k \geq 0, \\ \widetilde{W}^\infty(\mathbf{S}) &= \bigcap_{k=0}^{\infty} \widetilde{W}^k(\mathbf{S}),\end{aligned}$$

and similarly for  $\widetilde{W}_i^k(\mathbf{S})$  and  $\widetilde{W}_{-i}^k(\mathbf{S})$ . The set  $\widetilde{W}^\infty(\mathbf{S})$  is the maximal reduction under iterated admissibility. We call any strategy of player  $i$  in  $\widetilde{W}_i(\mathbf{Y})$  admissible on  $(\mathcal{S}_i, \mathbf{Y})$ , and any strategy of player  $i$  in  $\widetilde{W}_i^\infty(\mathbf{S})$  as iterated admissible.

This procedure generalizes iterated admissibility to games with unawareness. While the definition boils down to admissibility (resp., iterated admissibility) when restricted to standard games without unawareness, the terminology maybe be somewhat misleading. In games with unawareness, iterated admissibility is conceptually closer to iterated conditional weak dominance because it makes explicit use of information sets. In dynamic games with unawareness, the information set does not only contain information in the standard sense but also awareness given by the tree on which it is located. This awareness determines the relevant normal form that is conditioned on in the definition of iterated admissibility for games with unawareness. That is, while iterated admissibility in standard games does not rely on any structure in the extensive form, iterated admissibility in dynamic games must rely on the awareness captured by information sets in the extensive form. Iterated admissibility as defined here allows us to neglect information but not awareness embodied in the extensive form.

We show that prudent rationalizability is captured also by iterated admissibility in dynamic games with unawareness.

**Theorem 3** *For every finite dynamic game with unawareness,  $\widetilde{W}_i^k(\mathbf{S}) = \bar{S}_i^k$ , for all  $k \geq 1$ . Consequently,  $\widetilde{W}_i^\infty(\mathbf{S}) = \bar{S}_i^\infty$ .*

The proof is contained in the appendix. There, we first define a version of prudent rationalizability that weakens dynamic properties of belief systems. We then show inductively the equivalence between that version of prudent rationalizability and iterated admissibility making use again of Lemma 4 in Pearce (1984). Finally, we show inductively the equivalence between that weakened version of prudent rationalizability and Definition 4.

As a corollary, iterated admissibility is equivalent to the iterated conditional weak dominance at every iteration in dynamic games with unawareness. This result generalizes a result for standard games by Brandenburger and Friedenberg (2011, Proposition 3.1) to dynamic games with unawareness.

**Corollary 1** *For every finite dynamic game with unawareness,  $\widetilde{W}_i^k(\mathbf{S}) = W_i^k(\mathbf{S})$ , for all  $k \geq 1$ . Consequently,  $\widetilde{W}_i^\infty(\mathbf{S}) = W_i^\infty(\mathbf{S})$ .*

## A Properties of Information Sets

In order to make the paper self-contained, we list properties of information sets satisfied in dynamic games with unawareness; see Heifetz, Meier, and Schipper (2013) and Schipper (2019, 2021) for further discussions and illustrations. Some of these properties are explicitly used in the proofs of the results.

For each decision node  $n \in N$  and active player  $i \in I_n$  there is a nonempty information set  $\pi_i(n)$  satisfying the following properties:

- I0 Confinement:  $\pi_i(n) \subseteq T$  for some tree  $T$ .
- I1 No-delusion given the awareness level: If  $\pi_i(n) \subseteq T_n$ , then  $n \in \pi_i(n)$ .
- I2 Introspection: If  $n' \in \pi_i(n)$ , then  $\pi_i(n') = \pi_i(n)$ .
- I3 No divining of currently unimaginable paths, no expectation to forget currently conceivable paths: If  $n' \in \pi_i(n) \subseteq T'$  (where  $T' \in \mathbf{T}$  is a tree) and there is a path  $n', \dots, n'' \in T'$  such that  $i \in I_{n'} \cap I_{n''}$ , then  $\pi_i(n'') \subseteq T'$ .
- I4 No imaginary actions: If  $n' \in \pi_i(n)$ , then  $A_{n'}^i \subseteq A_n^i$ .
- I5 Distinct action names in disjoint information sets: For a subtree  $T$ , if  $n, n' \in T$  and  $A_n^i = A_{n'}^i$ , then  $\pi_i(n') = \pi_i(n)$ .
- I6 Perfect recall: Suppose that player  $i$  is active in two distinct nodes  $n_1$  and  $n_k$ , and there is a path  $n_1, n_2, \dots, n_k$  such that at  $n_1$  player  $i$  takes the action  $a_i$ . If  $n' \in \pi_i(n_k)$ , then there exists a node  $n'_1 \neq n'$  and a path  $n'_1, n'_2, \dots, n'_\ell = n'$  such that  $\pi_i(n'_1) = \pi_i(n_1)$  and at  $n'_1$  player  $i$  takes the action  $a_i$ .

## B Proofs

### Proof of Theorem 1

A *general belief system* of player  $i \in I$ ,

$$\widehat{b}_i = (\widehat{b}_i(h_i))_{h_i \in H_i} \in \prod_{h_i \in H_i} \Delta(S_{-i}^{T_{h_i}})$$

is a profile of beliefs – a belief  $\widehat{b}_i(h_i) \in \Delta(S_{-i}^{T_{h_i}})$  about the other players' strategies in the  $T_{h_i}$ -partial game, for each information set  $h_i \in H_i$ , such that  $\widehat{b}_i(h_i)$  allows  $h_i$ , i.e.,  $\widehat{b}_i(h_i)$  assigns probability 1 to the set of strategy profiles of the other players that allow  $h_i$ . The difference between a belief system and a general belief system is that in the latter *we do not impose conditioning*.

For  $k \geq 1$  let  $\widehat{B}_i^k$  and  $\widehat{S}_i^k$  be defined inductively like  $B_i^k$  and  $S_i^k$  in Definition 1, respectively, the only change being that belief systems are replaced by generalized belief systems.

**Lemma 1**  $U_i^k(\mathbf{S}) \cap S_i = \widehat{S}_i^k$  for all  $k \geq 1$ . Consequently,  $U_i^\infty(\mathbf{S}) \cap S_i = \widehat{S}_i^\infty$ .

**Proof of the Lemma 1.** We proceed by induction. The case  $k = 0$  is straightforward since  $U_i^0(\mathbf{S}) \cap S_i = S_i = \widehat{S}_i^0$  for all  $i \in I$ .

Suppose now that we have shown  $U_i^k(\mathbf{S}) \cap S_i = \widehat{S}_i^k$  for all  $i \in I$ . We want to show that  $U_i^{k+1}(\mathbf{S}) \cap S_i = \widehat{S}_i^{k+1}$  for all  $i \in I$ .

“ $\subseteq$ ”: First we show, if  $s_i \in U_i^{k+1}(\mathbf{S}) \cap S_i$  then  $s_i \in \widehat{S}_i^{k+1}$ .

$s_i \in U_i^{k+1}(\mathbf{S}) \cap S_i$  if  $s_i \in S_i$  is not conditionally strictly dominated on  $(\mathcal{X}_i, U^k(\mathbf{S}))$ .

$s_i \in S_i$  is not conditionally strictly dominated on  $(\mathcal{X}_i, U^k(\mathbf{S}))$  if for all  $T' \in \mathbf{T}$  with  $T_1 \hookrightarrow T'$  and all  $\tilde{s}_i \in S_i^{T'}$  such that  $s_i \in [\tilde{s}_i]$ , we have that there does not exist a normal-form information set  $X \in \mathcal{X}_i$  with  $X \subseteq S^{T'}$  such that  $\tilde{s}_i$  is strictly dominated in  $X \cap U^k(\mathbf{S})$ .

For any information set  $h_i \in H_i$ , if  $\tilde{s}_i \in S_i^{T_{h_i}}$  is not strictly dominated in  $S^{T_{h_i}}(h_i) \cap U^k(\mathbf{S})$ , then

- (i) either  $\tilde{s}_i$  does not allow  $h_i$ , in which case  $\tilde{s}_i$  is trivially rational at  $h_i$ ; or
- (ii) by Lemma 3 in Pearce (1984) there exists a belief  $\widehat{b}_i(h_i) \in \Delta(S_{-i}^{T_{h_i}}(h_i) \cap U_{-i}^k(\mathbf{S}))$  for which  $\tilde{s}_i$  is rational at  $h_i$ . Since by the induction hypothesis  $U^k(\mathbf{S}) \cap S = \widehat{S}^k$ , we have in this case that there exists a belief at  $h_i$  with  $\widehat{b}_i(h_i)(\widehat{S}_{-i}^{k, T_{h_i}}) = 1$  for which  $\tilde{s}_i$  is rational at  $h_i$ .

By the definitions of  $[\tilde{s}_i]$  and “allow”, if  $\tilde{s}_i$  allows  $h_i$  in the tree  $T_{h_i}$  and  $s_i \in [\tilde{s}_i]$ , then  $s_i$  allows  $h_i$  in the tree  $T_{h_i}$ . Hence, if  $\tilde{s}_i \in S_i^{T_{h_i}}$  is rational at  $h_i$  given  $\widehat{b}_i(h_i)$ , then  $s_i \in [\tilde{s}_i]$  is rational at  $h_i$  given  $\widehat{b}_i(h_i)$ .

We need to show that beliefs in (ii) define a generalized belief system in  $\widehat{B}_i^{k+1}$ . Consider any  $\widehat{b}'_i = (\widehat{b}'_i(h_i))_{h_i \in H_i} \in \widehat{B}_i^{k+1}$ . For all  $h_i \in H_i$  for which there exists a profile of player  $i$ 's opponents' strategies  $s_{-i} \in \widehat{S}_{-i}^k$  that allow  $h_i$ , replace  $\widehat{b}'_i(h_i)$  by  $\widehat{b}_i(h_i)$  as defined in (ii). Call the new belief system  $\widehat{b}_i$ . Then this is a generalized belief system. Moreover,  $\widehat{b}_i \in \widehat{B}_i^{k+1}$ .

Hence, if  $s_i$  is not conditionally strictly dominated on  $(\mathcal{X}_i, U^k(\mathbf{S}))$  then there exists a generalized belief system  $\widehat{b}_i \in \widehat{B}_i^{k+1}$  for which  $s_i$  is rational at every  $h_i \in H_i$ . Thus  $s_i \in \widehat{S}_i^{k+1}$ .

“ $\supseteq$ ”: We show next, if  $s_i \in \widehat{S}_i^{k+1}$  then  $s_i \in U_i^{k+1}(\mathbf{S}) \cap S_i$ .

If  $s_i \in \widehat{S}_i^{k+1}$  then there exists a generalized belief system  $\widehat{b}_i \in \widehat{B}_i^{k+1}$  such that for all  $h_i \in H_i$  the strategy  $s_i$  is rational given  $\widehat{b}_i(h_i)$ . That is, either

- (I)  $s_i$  does not allow  $h_i$ , or
- (II)  $s_i$  allows  $h_i$  and there does not exist an  $h_i$ -replacement of  $s_i$  which yields a higher expected payoff in  $T_{h_i}$  given  $\widehat{b}_i(h_i)$  that assigns probability 1 to  $T_{h_i}$ -partial strategies of player  $i$ 's opponents in  $\widehat{S}_{-i}^{k, T_{h_i}}$  that allow  $h_i$  in  $T_{h_i}$ . By the induction hypothesis,  $\widehat{S}_{-i}^k = U_{-i}^k(\mathbf{S}) \cap S_{-i}^{T_{h_i}}$ . Hence  $\widehat{b}_i(h_i) \in \Delta(U_{-i}^k(\mathbf{S}) \cap S_{-i}^{T_{h_i}}(h_i))$ .

If  $s_i \in [\tilde{s}_i]$  with  $\tilde{s}_i \in S_i^{T_{h_i}}$  and  $s_i$  allows  $h_i$  in the tree  $T_{h_i}$ , then  $\tilde{s}_i$  allows  $h_i$  in the tree  $T_{h_i}$ . Hence, if  $s_i \in [\tilde{s}_i]$  with  $\tilde{s}_i \in S_i^{T_{h_i}}$  is rational at  $h_i$  given  $\widehat{b}_i(h_i)$ , then  $\tilde{s}_i$  is rational at  $h_i$  given  $\widehat{b}_i(h_i)$ .

Thus, if  $s_i$  is rational at  $h_i$  given  $\widehat{b}_i(h_i)$ , then  $\tilde{s}_i \in S_i^{T_{h_i}}$  with  $s_i \in [\tilde{s}_i]$  is not strictly dominated in  $U_{-i}^k(\mathbf{S}) \cap S_{-i}^{T_{h_i}}(h_i)$  either because  $s_i$  does not allow  $h_i$  (case (I)), or because of Lemma 3 in Pearce (1984) (in case (II)).

It then follows that if the strategy  $s_i$  is rational at all  $h_i \in H_i$  given  $\widehat{b}_i$ , then  $s_i$  is not conditionally strictly dominated on  $(\mathcal{X}_i, U^k(\mathbf{S}))$ . Hence  $s_i \in U_i^{k+1}(\mathbf{S}) \cap S_i$ .  $\square$

**Lemma 2**  $\widehat{S}_i^k = S_i^k$  for all  $k \geq 1$ . Consequently,  $\widehat{S}_i^\infty = S_i^\infty$ .

**Proof of the Lemma 2.** We have  $S_i^k \subseteq \widehat{S}_i^k$  for all  $k \geq 1$  since, if  $s_i$  is rational at each information set  $h_i \in H_i$  given the belief system  $b_i \in B_i$ , then there exists a generalized belief system  $\widehat{b}_i \in \widehat{B}_i$ , namely  $\widehat{b}_i = b_i$ , such that  $s_i$  is rational at each information set  $h_i \in H_i$  given  $\widehat{b}_i$ .

We need to show the reverse inclusion,  $\widehat{S}_i^k \subseteq S_i^k$  for all  $k \geq 1$ . The first step is to show how to construct a (consistent) belief system from a generalized belief system. Let  $s_i$  be rational given  $\widehat{b}_i \in \widehat{B}_i^1$ , i.e.,  $s_i \in \widehat{S}_i^1$ . Consider an information set  $h_i^0 \in H_i$  such that in  $T_{h_i}$  there does not exist an information set  $h_i$  that precedes  $h_i^0$ . Define  $b_i(h_i^0) := \widehat{b}_i(h_i^0)$ .

Assume, inductively, that we have already defined  $b_i$  for a subset of information sets  $H_i' \subseteq H_i$  such that for each  $h_i' \in H_i'$  all the predecessors of  $h_i'$  are also in  $H_i'$ . For each successor information set  $h_i''$  of each information set  $h_i' \in H_i'$  such that  $h_i'' \notin H_i'$  define  $b_i(h_i'')$  as follows:

- If  $b_i(h_i')$  allows  $h_i''$  define  $b_i(h_i'')$  by using conditioning, i.e., if  $s_{-i}^{T_{h_i'}} \in S_{-i}(h_i'')$

$$b_i(h_i'') \left( s_{-i}^{T_{h_i'}} \right) = \frac{b_i(h_i') \left( s_{-i}^{T_{h_i'}} \right)}{\sum_{\tilde{s}_{-i}^{T_{h_i'}} \in S_{-i}(h_i'')} b_i(h_i') \left( \tilde{s}_{-i}^{T_{h_i'}} \right)}$$

$$\text{and } b_i(h_i'') \left( s_{-i}^{T_{h_i'}} \right) = 0 \text{ else.}$$

- If  $b_i(h_i')$  does not allow  $h_i''$  let  $b_i(h_i'') \equiv \widehat{b}_i(h_i'')$ .

Since there are finitely many information sets in  $H_i$ , this recursive definition concludes in a finite number of steps.

Next, assuming that  $s_i$  is rational at each information set  $h_i \in H_i$  with the generalized belief system  $\widehat{b}_i$ , we will show that  $s_i$  is also rational at each information set  $h_i \in H_i$  according to the belief system  $b_i$ .

Consider again  $h_i^0 \in H_i$  with no predecessors in  $T_{h_i^0}$ . Since  $b_i(h_i^0) = \widehat{b}_i(h_i^0)$  and  $s_i$  is rational at  $h_i^0$  given  $\widehat{b}_i(h_i^0)$ ,  $s_i$  is also rational at  $h_i^0$  given  $b_i(h_i^0)$ .

Assume, inductively, that we have already shown the claim for a subset of information sets  $H'_i \subseteq H_i$  such that for each  $h'_i \in H'_i$  all the predecessors of  $h'_i$  are also in  $H'_i$ . Consider a successor information set  $h''_i$  of an information set  $h'_i \in H'_i$  such that  $h''_i \notin H'_i$ . Notice that each  $h''_i$ -replacement is also an  $h'_i$ -replacement. Therefore,

- If  $b_i(h'_i)$  allows  $h''_i$ ,  $b_i(h''_i)$  is derived from  $b_i(h'_i)$  by conditioning, and hence any  $h''_i$ -replacement improving player  $i$ 's expected payoff according to  $b_i(h''_i)$  would improve player  $i$ 's payoff also according to  $b_i(h'_i)$ , contradicting the induction hypothesis. Hence  $s_i$  is rational at  $h''_i$  given  $b_i(h''_i)$ .
- If  $b_i(h'_i)$  does not allow  $h''_i$ , then  $b_i(h''_i) = \widehat{b}_i(h''_i)$ . Hence,  $s_i$  is rational at  $h''_i$  also given  $b_i(h''_i)$ .

Applying the same argument inductively yields  $\widehat{S}_i^k = S_i^k$  for all  $k \geq 1$ . This concludes the proof of the lemma.  $\square$

Lemmata 1 and 2 together yield  $U_i^k(\mathbf{S}) \cap S_i = S_i^k$  for all  $k \geq 1$ . Since it applies for all  $k \geq 1$  and  $i \in I$ , this completes the proof of the proposition.  $\square$

### Proof of Theorem 3

The proof proceeds in several steps. First, we show that iterated admissibility is characterized by a variant of prudent rationalizability.

A *relaxed belief system* of player  $i \in I$ ,

$$\ddot{b}_i = \left( \ddot{b}_i(h_i) \right)_{h_i \in H_i} \in \prod_{h_i \in H_i} \Delta \left( S_{-i}^{T_{h_i}} \right)$$

is a profile of beliefs, a belief  $\ddot{b}_i(h_i) \in \Delta \left( S_{-i}^{T_{h_i}} \right)$  about the other players' strategies in the  $T_{h_i}$ -partial game for each information set  $h_i \in H_i$ . Compared to belief systems, for relaxed belief systems we neither require that  $\ddot{b}_i(h_i)$  allows  $h_i$  nor conditioning.

Denote by  $\ddot{B}_i$  the set of player  $i$ 's relaxed belief systems.

We say that with the relaxed belief system  $\ddot{b}_i$  and the strategy  $s_i$  player  $i$  is *relaxed rational* at the information set  $h_i \in H_i$  if there does not exist an alternative strategy  $s'_i$  which yields player  $i$  a higher expected in  $T_{h_i}$  given the belief  $\ddot{b}_i(h_i)$  on the other players' strategies  $S_{-i}^{T_{h_i}}$ .

**Definition 9 (Prudent relaxed rationalizability)** *Let*

$$\ddot{S}_i^0 = S_i$$

For all  $k \geq 1$  define inductively

$$\begin{aligned}\ddot{B}_i^k &= \left\{ \ddot{b}_i \in \ddot{B}_i : \begin{array}{l} \text{For every information set } h_i, \text{ if there exists some profile } s_{-i} \in \ddot{S}_{-i}^{k-1} \text{ of the} \\ \text{other players' strategies such that } s_{-i} \text{ allows } h_i \text{ in the tree } T_{h_i}, \text{ then the} \\ \text{support of } \ddot{b}_i(h_i) \text{ is the set of strategy profiles } s_{-i} \in \ddot{S}_{-i}^{k-1, T_{h_i}}. \end{array} \right\} \\ \ddot{S}_i^k &= \left\{ s_i \in \ddot{S}_i^{k-1} : \begin{array}{l} \text{There exists a belief system } \ddot{b}_i \in \ddot{B}_i^k \text{ with which for every information set} \\ h_i \in H_i, \text{ player } i \text{ is } \mathbf{relaxed\ rational} \text{ at } h_i. \end{array} \right\}\end{aligned}$$

The set of prudent relaxed rationalizable strategies of player  $i$  is

$$\ddot{S}_i^\infty = \bigcap_{k=1}^{\infty} \ddot{S}_i^k$$

**Proposition 1** For every finite dynamic game with unawareness,  $\widetilde{W}_i^k(\mathbf{S}) \cap S_i = \ddot{S}_i^k$ , for all  $k \geq 1$ . Consequently,  $\widetilde{W}_i^\infty(\mathbf{S}) \cap S_i = \ddot{S}_i^\infty$ .

**Proof of Proposition 1:** We proceed by induction. The case  $k = 0$  is straightforward since  $\widetilde{W}_i^0(\mathbf{S}) \cap S_i = S_i = \ddot{S}_i^0$  for all  $i \in I$ .

Suppose now that we have shown  $\widetilde{W}_i^k(\mathbf{S}) \cap S_i = \ddot{S}_i^k$  for all  $i \in I$ . We want to show that  $\widetilde{W}_i^{k+1}(\mathbf{S}) \cap S_i = \ddot{S}_i^{k+1}$  for all  $i \in I$ .

“ $\subseteq$ ”: First we show, if  $s_i \in \widetilde{W}_i^{k+1}(\mathbf{S}) \cap S_i$ , then  $s_i \in \ddot{S}_i^{k+1}$ .

$s_i \in \widetilde{W}_i^{k+1}(\mathbf{S}) \cap S_i$  if  $s_i \in S_i$  is not NF-conditionally weakly dominated on  $(S_i, \widetilde{W}^k(\mathbf{S}))$ .

$s_i \in S_i$  is not NF-conditionally weakly dominated on  $(S_i, \widetilde{W}^k(\mathbf{S}))$  if for all  $T' \in \mathbf{T}$  with  $T_1 \hookrightarrow T'$  and all  $\tilde{s}_i \in S_i^{T'}$  such that  $s_i \in [\tilde{s}_i]$ , we have that  $\tilde{s}_i$  is not weakly dominated in  $S_i^{T'} \cap \widetilde{W}^k(\mathbf{S})$ .

For any information set  $h_i \in H_i$ , if  $\tilde{s}_i \in S_i^{T_{h_i}}$  is not weakly dominated in  $S_i^{T_{h_i}} \cap \widetilde{W}^k(\mathbf{S})$ , then by Lemma 4 in Pearce (1984) there exists a full support belief  $\ddot{b}_i(h_i) \in \Delta(S_i^{T_{h_i}} \cap \widetilde{W}_{-i}^k(\mathbf{S}))$  for which  $\tilde{s}_i$  is relaxed rational at  $h_i$ . Since by the induction hypothesis  $\widetilde{W}^k(\mathbf{S}) \cap S = \ddot{S}^k$ , we have in this case that there exists a full support belief at  $h_i$  with  $\ddot{b}_i(h_i) \in \Delta(\ddot{S}_{-i}^{k, T_{h_i}})$  for which  $\tilde{s}_i$  is relaxed rational at  $h_i$ .

By definition of  $[\tilde{s}_i]$ , if  $\tilde{s}_i \in S_i^{T_{h_i}}$  is relaxed rational at  $h_i$  given  $\ddot{b}_i(h_i)$ , then  $s_i \in [\tilde{s}_i]$  is relaxed rational at  $h_i$  given  $\ddot{b}_i(h_i)$ .

Note that the profile of beliefs  $(\ddot{b}_i(h_i))_{h_i \in H_i}$  are a relaxed belief system in  $\ddot{B}_i^{k+1}$ .

Hence, if  $s_i$  is not NF-conditionally weakly dominated on  $(S_i, \widetilde{W}^k(\mathbf{S}))$ , then there exists a relaxed belief system  $\ddot{b}_i \in \ddot{B}_i^{k+1}$  for which  $s_i$  is relaxed rational at every  $h_i \in H_i$ . Thus  $s_i \in \ddot{S}_i^{k+1}$ .

“ $\supseteq$ ”: We show next, if  $s_i \in \ddot{S}_i^{k+1}$ , then  $s_i \in \widetilde{W}_i^{k+1}(\mathbf{S}) \cap S_i$ .

If  $s_i \in \ddot{S}_i^{k+1}$ , then there exists a relaxed belief system  $\ddot{b}_i \in \ddot{B}_i^{k+1}$  such that for all  $h_i \in H_i$  the strategy  $s_i$  is relaxed rational given  $\ddot{b}_i(h_i)$ . That is, there does not exist an alternative strategy

$s'_i$  which yields a strictly higher expected payoff in  $T_{h_i}$  given  $\ddot{b}_i(h_i)$  that has full support on  $\ddot{S}_{-i}^{k, T_{h_i}}$ .

By the induction hypothesis,  $\ddot{S}_{-i}^k = \widetilde{W}_{-i}^k(\mathbf{S}) \cap S_{-i}^{T_{h_i}}$ . Hence,  $\ddot{b}_i(h_i)$  has full support on  $\widetilde{W}_{-i}^k(\mathbf{S}) \cap S_{-i}^{T_{h_i}}$ .

If  $s_i$  is relaxed rational at  $h_i$  with  $\ddot{b}_i(h_i)$ , then for any  $\tilde{s}_i \in S_i^{T_{h_i}}$  with  $s_i \in [\tilde{s}_i]$ ,  $\tilde{s}_i$  is relaxed rational at  $h_i$  with  $\ddot{b}_i(h_i)$ . By Lemma 4 in Pearce (1984),  $\tilde{s}_i$  is not weakly dominated in  $\widetilde{W}_{-i}^k(\mathbf{S}) \cap S_{-i}^{T_{h_i}}$ .

It then follows that if the strategy  $s_i$  is rational at all  $h_i \in H_i$  given  $\tilde{b}_i$ , then  $s_i$  is not NF-conditionally weakly dominated on  $(\mathcal{S}_i, \widetilde{W}^k(\mathbf{S}))$ . Hence,  $s_i \in \widetilde{W}_i^{k+1}(\mathbf{S}) \cap S_i$ . This completes the proof of Proposition 1.  $\square$

Next, we show that prudent rationalizability is equivalent to prudent relaxed rationalizability.

**Proposition 2** *For every finite dynamic game with unawareness and every  $i \in I$ ,  $\bar{S}_i^k = \ddot{S}_i^k$ , for all  $k \geq 0$ . Consequently,  $\bar{S}_i^\infty = \ddot{S}_i^\infty$ .*

**Proof of Proposition 2** The proof proceeds by induction. By definition  $\bar{S}_i^0 = \ddot{S}_i^0$ .

Assume  $\bar{S}_i^k = \ddot{S}_i^k$ . We show that  $\bar{S}_i^{k+1} = \ddot{S}_i^{k+1}$ .

“ $\supseteq$ ”: Fix  $s_i \in \ddot{S}_i^{k+1}$  and  $h_i \in H_i$  such that  $s_i$  allows  $h_i$  (otherwise  $s_i$  is trivially rational at  $h_i$ ). There exists a relaxed belief system  $\ddot{b}_i \in \ddot{B}_i^{k+1}$  such that  $s_i$  is relaxed rational at  $h_i$ . By definition of  $\ddot{B}_i^{k+1}$ ,  $\ddot{b}_i(h_i)$  has full support on  $\ddot{S}_{-i}^{k, T_{h_i}}$ . Assume  $\ddot{S}_{-i}^{k, T_{h_i}} \cap S_{-i}^{T_{h_i}}(h_i) \neq \emptyset$ . Then  $\ddot{b}_i(h_i) \left( \ddot{S}_{-i}^{k, T_{h_i}} \cap S_{-i}^{T_{h_i}}(h_i) \right) > 0$ . Thus, we can consider conditional probabilities  $\ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right)$ . (Otherwise, if  $\ddot{S}_{-i}^{k, T_{h_i}} \cap S_{-i}^{T_{h_i}}(h_i) = \emptyset$ , then  $s_i$  is trivially rational at  $h_i$ .)

Suppose by contradiction that  $s_i$  is not rational at  $h_i$  with  $\ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right)$ . There exists an  $h_i$ -replacement of  $s_i$ , call it  $s_i/\tilde{s}_i^{h_i}$ , such that  $s_i/\tilde{s}_i^{h_i}$  yields a strictly higher expected payoff in  $T_{h_i}$  given the belief  $\ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right)$ . That is,  $u_i^{T_{h_i}} \left( s_i/\tilde{s}_i^{h_i}, \ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right) \right) > u_i^{T_{h_i}} \left( s_i, \ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right) \right)$ . Since  $u_i^{T_{h_i}} \left( s_i, \ddot{b}_i(h_i) \right) \geq u_i^{T_{h_i}} \left( s_i/\tilde{s}_i^{h_i}, \ddot{b}_i(h_i) \right)$ , we must have

$\ddot{b}_i(h_i) \left( S_{-i}^{T_{h_i}}(h_i) \right) < 1$ . Now

$$\begin{aligned}
u_i^{T_{h_i}} \left( s_i, \ddot{b}_i(h_i) \right) &\geq u_i^{T_{h_i}} \left( s_i / \tilde{s}_i^{h_i}, \ddot{b}_i(h_i) \right) \\
&= \ddot{b}_i(h_i) \left( S_{-i}^{T_{h_i}}(h_i) \right) u_i^{T_{h_i}} \left( s_i / \tilde{s}_i^{h_i}, \ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right) \right) \\
&\quad + \left( 1 - \ddot{b}_i(h_i) \left( S_{-i}^{T_{h_i}}(h_i) \right) \right) u_i^{T_{h_i}} \left( s_i / \tilde{s}_i^{h_i}, \ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right) \right) \\
&> \ddot{b}_i(h_i) \left( S_{-i}^{T_{h_i}}(h_i) \right) u_i^{T_{h_i}} \left( s_i, \ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right) \right) \\
&\quad + \left( 1 - \ddot{b}_i(h_i) \left( S_{-i}^{T_{h_i}}(h_i) \right) \right) u_i^{T_{h_i}} \left( s_i, \ddot{b}_i(h_i) \left( \cdot \mid S_{-i}^{T_{h_i}}(h_i) \right) \right) \\
&= u_i^{T_{h_i}} \left( s_i, \ddot{b}_i(h_i) \right),
\end{aligned}$$

a contradiction.

“ $\subseteq$ ”: Let  $\bar{S}_{-i}^{k, T_{h_i}}(h_i)$  denote the set of opponents’ level  $k$ -prudent rationalizable  $T_{h_i}$ -partial strategy profiles that allow  $h_i$ . For each  $T \in \mathbf{T}$  for which  $H_i^T$  is nonempty, let  $\bar{S}_{-i}^{k, T}(H_i^T) = \bigcup_{h_i \in H_i^T} \bar{S}_{-i}^{k, T}(h_i)$ . Note that if  $\bar{S}_{-i}^{k, T}(H_i^T) \subsetneq \bar{S}_{-i}^{k, T}$ , then there is a terminal history in  $T$  that is excluded by any information set  $h_i \in H_i^T$ . (In general,  $\bar{S}_{-i}^{k, T}(H_i^T)$  may not be a cross-product set).

We claim that there exists a nonempty subset of information sets  $G_i^T \subseteq H_i^T$  such that  $\left\{ \bar{S}_{-i}^{k, T}(h_i) \right\}_{h_i \in G_i^T}$  forms a partition of  $\bar{S}_{-i}^{k, T}(H_i^T)$ . To define  $G_i^T$ , we first define the rank of an information set  $h_i \in H_i^T$  as the maximal number of information sets in  $H_i^T$  required to pass in order to allow a terminal node in  $T$ . Note that if  $h_i \in H_i^T$  with  $n \in h_i$  there is path from  $n$  to  $n'$  in  $T$  with  $i \in I_{n'}$ , then Property I3 (see Appendix A) ensures that there exists an information set  $h'_i \in H_i^T$  with  $n' \in h'_i$ . Note further that there may be terminal nodes in  $T$  that are excluded by any information set in  $H_i^T$ .

Using the definition of rank of an information set, we construct a subset of information sets  $G_i^T$  as follows: For any information set  $h_i \in H_i^T$  of lowest rank, let  $h_i \in G_i^T$  if there is no information set  $h'_i \in H_i^T$  that precedes  $h_i$ . Otherwise, consider any information set  $h'_i \in H_i^T$  of second lowest rank and let  $h'_i \in G_i^T$  if there is no information set  $h''_i \in H_i^T$  that precedes  $h'_i$ , etc. Since  $T$  is finite,  $H_i^T$  is finite, and the procedure terminates after finite steps. (In particular, if player  $i$  moves at the root of  $T$ ,  $G_i^T$  is a singleton whose only information set contains the root.)

By construction,  $\bigcup_{h_i \in G_i^T} \bar{S}_{-i}^{k, T}(h_i)$  covers  $\bar{S}_{-i}^{k, T}(H_i^T)$ . Moreover, from Property I6 (see Appendix A) follows that for any two  $h_i, h'_i \in G_i^T$  with  $h_i \neq h'_i$  there is no profile of opponents’  $T$ -partial strategies  $s_{-i}^T \in \bar{S}_{-i}^{k, T}(h_i) \cap \bar{S}_{-i}^{k, T}(h'_i)$ .

Fix  $s_i \in \bar{S}_i^{k+1}$  and  $h_i \in H_i$  such that  $s_i$  allows  $h_i$  (otherwise  $s_i$  is trivially rational at  $h_i$ ). There exists a belief system  $\bar{b}_i \in \bar{B}_i^{k+1}$  such that  $s_i$  is rational at  $h_i$ . By definition of  $\bar{B}_i^{k+1}$ , the support of  $\bar{b}_i(h_i)$  is the set  $\bar{S}_{-i}^{k, T_{h_i}}(h_i)$ .

Construct a belief system  $\ddot{b}_i$  by setting for arbitrary  $\varepsilon \in (0, 1)$ ,

$$\ddot{b}_i(h_i)(s_{-i}) := \begin{cases} \frac{1-\varepsilon}{|G_i^{T_{h_i}}|} \bar{b}_i(h_i)(s_{-i}) & \text{if } s_{-i}^T \in \bar{S}_{-i}^{k, T_{h_i}}(h_i), \text{ and} \\ \frac{\varepsilon}{|\bar{S}_{-i}^{k, T_{h_i}} \setminus \bar{S}_{-i}^{k, T_{h_i}}(H_i^{T_{h_i}})|} & \text{if } s_{-i}^T \in \bar{S}_{-i}^{k, T_{h_i}} \setminus \bar{S}_{-i}^{k, T_{h_i}}(H_i^{T_{h_i}}). \end{cases}$$

This defines a probability measure  $\ddot{b}_i(h_i)$  with full support on  $\bar{S}_{-i}^{k, T_{h_i}}$ , for each information set  $h_i \in H_i$ .

$\ddot{B}_i^{k+1}$  is the set of all belief systems defined as above from any  $\bar{b}_i \in \bar{B}_i^{k+1}$  and  $\varepsilon \in (0, 1)$ .

Finally, note that any strategy  $s_i \in \bar{S}_i^{k+1}$  that with  $\bar{b}_i \in \bar{B}_i^{k+1}$  is rational at every information set  $h_i \in H_i$  continues to be rational with a belief system  $\ddot{b}_i \in \ddot{B}_i^{k+1}$  at every information set  $h_i \in H_i$ . Thus  $s_i \in \ddot{S}_i^{k+1}$ . This completes the proof of Proposition 2  $\square$

The proof of the theorem now follows from Propositions 1 and 2.  $\square$

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