Department of Economics
Working Paper Series

Does Exchange Rate Variability Matter for Welfare? A Quantitative Investigation of Stabilization Policies

Paul Bergin
University of California, Davis

Hyung-Cheol Shin
UC Davis

Ivan Tchakarov
International Monetary Fund

June 30, 2005

Paper # 05-12

This paper evaluates quantitatively the potential welfare gains from monetary policy and fixed exchange rate rules in a two-country sticky-price model. The first finding is that the gains from stabilization tend to be small in the types of economic environments emphasized in recent theoretical literature. The analysis goes on to identify two types of economies in which the welfare implications of risk are larger: where agents exhibit habits, and where international asset markets exhibit asymmetry in the form of "original sin." In the habits case, monetary policy aimed solely at inflation stabilization is optimal. But in the original sin case there are potentially large welfare gains from also eliminating exchange rate volatility.

UCDAVIS

Department of Economics
One Shields Avenue
Davis, CA 95616
(530)752-0741

http://www.econ.ucdavis.edu/working_search.cfm

Does Exchange Rate Variability Matter for Welfare?
A Quantitative Investigation of Stabilization Policies

Paul R. Bergin
Department of Economics, University of California at Davis, and NBER

Hyung-Cheol Shin
Department of Economics, University of California at Davis

Ivan Tchakarov
International Monetary Fund

This version: June 30, 2005

Abstract
This paper evaluates quantitatively the potential welfare gains from monetary policy and fixed exchange rate rules in a two-country sticky-price model. The first finding is that the gains from stabilization tend to be small in the types of economic environments emphasized in recent theoretical literature. The analysis goes on to identify two types of economies in which the welfare implications of risk are larger: where agents exhibit habits, and where international asset markets exhibit asymmetry in the form of "original sin." In the habits case, monetary policy aimed solely at inflation stabilization is optimal. But in the original sin case there are potentially large welfare gains from also eliminating exchange rate volatility.

Keywords: exchange rate risk, second order approximation

JEL classification: F41

Correspondence to Paul R. Bergin, Dept. of Economics, University of California at Davis, One Shields Ave., Davis, CA 95616 USA prbergin@ucdavis.edu, phone: (530) 752-8398, fax (530) 752-9382.

We thank Michael Devereux and Robert Kollmann for helpful comments. We also thank seminar participants at the Institute for Advanced Studies in Vienna, University of Bern, Central European University, Simon Fraser University, and the University of British Columbia.

1 Introduction

Exchange rate variability is one of the most prominent features of open economy macroeconomics, and the tendency for nominal exchange rates to move so volatilely and unpredictably has been blamed for limiting gains from international trade and for lowering welfare. A desire to moderate this uncertainty has been a motivation behind the managed exchange rate regimes of many countries as well as European monetary union. This paper conducts a quantitative examination of the welfare effects of risk in the context of a two-country general equilibrium model with sticky prices. It addresses two questions. Firstly, for what types of open economies does stabilization matter quantitatively for welfare? And secondly, under what circumstances is it optimal to focus policy on stabilizing the exchange rate in particular?

Several recent papers have employed analytically solvable models to investigate the welfare effects of exchange rate risk and the potential welfare benefits of exchange rate stabilization. Obstfeld and Rogoff (2002) demonstrated how micro-founded sticky price models could be extended to include risk premia, and thereby to trace out implications for overall economic activity and welfare. Bacchetta and Van Wincoop (2000) demonstrated that exchange rate variability may be costly or beneficial depending on the nature of household preferences. Devereux and Engel (2003) showed that results depend on the currency of price stickiness.[1]

This paper aims at a more quantitative analysis, as permitted by calibration and numerical solution. For example, asset markets and preferences are not limited here to cases of complete risk sharing, which allows us to explore the potentially important implications of exchange rate fluctuations working through the current account. This also allows us to explore reallocations of wealth between countries, which are potentially important for international welfare. The paper evaluates a number of novel but empirically relevant asset structures and consumer preferences. This numerical analysis is made possible by the second-order solution method developed in Kim et. al. (2004), which is also applied to an open economy setting in Kollmann (2002) and (2004).[2] The analysis here differs from

[1] For a sample of other work looking at welfare analysis in micro-founded models, see Benigno (2004), Benigno and Benigno (2003), Carre and Collard (2003), Clarida, Gali and Gertler (2001), Corsetti and Pesenti (2001a,b), Obstfeld and Rogoff (2000), Sutherland (2005).

[2] Related solution algorithms have been proposed in Schmitt-Grohe and Uribe (2004a), and Collard and Juillard (2001).

Kollmann mainly in that it seeks out and studies economic environments where the welfare effects can be large.[3]

The first finding of this quantitative analysis is to note that the welfare effects of uncertainty and the gains from stabilization policy are all quite small in the context of the economic environments debated in recent theoretical literature. The second contribution is to identify two cases, not previously analyzed in this literature, where welfare effects appear to be larger. The first of these cases is where household preferences exhibit habits, and the second is where international asset markets are asymmetric so as to exhibit "original sin." Under habits, households are by definition more sensitive to the risk of consumption variability. However, despite the larger welfare effects of risk in this case, the ability of policy rules to improve welfare remains small. Further, we find that it is not optimal for a policy rule in this environment to do anything to diminish exchange rate volatility.

The second case where the welfare effects of risk are larger is where one country is unable to issue debt denominated in its own currency. Termed "original sin" in recent literature, this asset asymmetry makes it difficult for the country to hedge against risk by saving, since net foreign assets expose the country to greater exchange rate risk. As a result there is a lower level of saving. In this context it can be welfare improving for this country to use monetary policy to stabilize the exchange rate.

The next section of the paper presents the two-country model, calibration, and solution method. Section 3 presents results for a range of cases for this model. Section 4 concludes and makes suggestions for future research.

2 The Model

Consider a model of two countries, denoted home and foreign. Agents consume two final goods, where each country specializes in the production of one of these. Monopolistically competitive firms produce intermediates using capital and labor, and set prices sluggishly due to adjustment costs.

2.1 Goods market structure

[3] The analysis here also differs from Kollmann in that it presents conditional welfare analysis.

Final goods (F) are a CES index over sub-indexes of the home (F_H) and foreign (F_F) intermediates. The aggregation technology for final goods is:

$$F_t = \left[a^{1/m} F_{H,t}^{(m-1)/m} + (1-a)^{1/m} F_{F,t}^{(m-1)/m}\right]^{m/(m-1)}, \text{ where} \tag{1}$$

$$F_{H,t} = \left(\int_0^1 f_{H,t}(i)^{(l-1)/l}\, di\right)^{l/(l-1)} \tag{2}$$

$$F_{F,t} = \left(\int_0^1 f_{F,t}(j)^{(l-1)/l}\, dj\right)^{l/(l-1)} \tag{3}$$

where a lower case represents output of the individual firms.

Final goods producers behave competitively, maximizing profit each period:

$$\Pi_t = \max\left[P_t F_t - P_{H,t} F_{H,t} - S_t P_{F,t} F_{F,t}\right], \tag{4}$$

where P is the overall price index of the final good, P_H is the price index of home goods in home currency units, and P_F of foreign goods in foreign currency units. S is the nominal exchange rate, home currency units per foreign. The price indexes may be defined:

$$P_t = \left[a P_{H,t}^{1-m} + (1-a)\left(S_t P_{F,t}\right)^{(m-1)/m}\right]^{1/(1-m)}, \text{ where} \tag{5}$$

$$P_{H,t} = \left(\int_0^1 p_{H,t}(i)^{1-l}\, di\right)^{1/(1-l)}, \qquad P_{F,t} = \left(\int_0^1 p_{F,t}(j)^{1-l}\, dj\right)^{1/(1-l)}. \tag{6, 7}$$

Given the aggregation functions above, demand will be allocated between home and foreign goods according to

$$F_{H,t} = a\left(\frac{P_{H,t}}{P_t}\right)^{-m} F_t, \qquad F_{F,t} = (1-a)\left(\frac{S_t P_{F,t}}{P_t}\right)^{-m} F_t \tag{8, 9}$$

with demands for individual goods:

$$f_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-l} F_{H,t} \qquad f_{F,t}(i) = \left(\frac{p_{F,t}(j)}{P_{F,t}}\right)^{-l} F_{H,t} \tag{10, 11}$$

Analogous definitions apply to the foreign country.

2.2 Home household problem

The representative home household derives utility from consumption (C), real money balances (M/P), and labor (H). Households derive income by selling their labor (H) at the nominal wage rate (W), renting out capital to firms at the real rental rate (r), receiving real profits from home firms (?), and from government transfers (T). In addition to money, households can hold a noncontingent nominal bond denominated in home currency (B_H)

which pays an interest rate (i), or a bond denominated in foreign currency (B_F) which pays an interest rate (i*). The household determines capital accumulation (K), which involves a quadratic adjustment cost that depends upon the parameter $?_I$ and a constant rate of depreciation (d).

Household optimization for the home country may be written:

$$\max \; E_0 \sum_{t=0}^{\infty} b^t U(C_t, \frac{M_t}{P_t}, H_t)$$

subject to the budget constraint:

$$P_t C_t + P_t (K_{t+1} - (1-d) K_t) + P_t AC_{It} + M_t + B_{H,t} + S_t B_{F,t} + AC_{B,t}$$
$$= (1+i_{t-1}) B_{H,t-1} + S_t (1+i^*_{t-1}) B_{F,t-1} + M_{t-1} + W_t H_t + P_t r_t K_t + T_t + \Pi_t$$

where

$$U_t = \frac{C_t^{1-r}}{1-r} + \frac{c_t}{1-e}\left(\frac{M_t}{P_t}\right)^{1-e} - \frac{H_t^{1+y}}{1+y} \tag{12}$$

and

$$AC_{I,t} = \frac{y_I}{2} \frac{(K_{t+1} - K_t)^2}{K_t} \tag{13}$$

$$AC_{B,t} = \frac{y_B}{2}\left(\frac{(S_t(B_{Ft} - \overline{B_F}))^2}{P_{Ht} Y_t}\right).$$

Money demand shocks are represented by shifts in $?_t$. There is a small adjustment cost on bond holdings, AC_B, to ensure stationarity in the net foreign asset position.[4] Later sections of the paper will consider more general forms of preferences than those here, including habits. Later sections will also consider alternative asset market structures, such as that limited to only one type of bond.

Optimization implies a money demand equation:

$$\frac{M_t}{P_t} = \frac{c_t^{1/e} (C_t)^{r/e}}{(1-d_t))^{1/e}}, \tag{14}$$

a trade-off between consumption and leisure:

---

[4] Home and foreign bonds are treated separately in this adjustment cost to ensure that there exists a determinate allocation between home and foreign currency bonds even in a first-order approximation to the system, as is required by the second-order accurate solution method.

$$\frac{W_t}{P_t C_t^r} = H_t^y \ , \tag{15}$$

a consumption Euler equation:

$$d_t = b\ E_t \frac{P_t C_t^r}{P_{t+1} C_{t+1}^r}, \tag{16}$$

with the definition of d:

$$i_t = \frac{1}{d_t} - 1, \tag{17}$$

an interest parity condition:

$$E_t\left[\frac{P_t C_t^r}{P_{t+1} C_{t+1}^r}\frac{S_{i+1}}{S_t}(1+i_t^*)\left(1+\frac{y_B S_t(B_{Ft}-\overline{B_F})}{P_{Ht}Y_t}\right)^{-1}\right] = E_t\left[\frac{P_t C_t^r}{P_{t+1} C_{t+1}^r}(1+i_t)\right], \tag{18}$$

and finally, capital accumulation:

$$\left(1+\frac{y_I(K_{t+1}-K_t)}{K_t}\right) = b\ E_t\left[\frac{C_t^r}{C_{t+1}^r}\left(r_t + (1-d) + \left(\frac{1}{2}y_I \frac{K_{t+2}^2 - K_{t+1}^2}{K_{t+1}^2}\right)\right)\right]. \tag{19}$$

equating the benefits and costs of capital accumulation.

2.3 Home firm problem

The benchmark version of the model assumes producer currency pricing, so that firms set prices in their own currency both for sales domestically and sales abroad. They rent capital (*K*) at the rental rate r, and hire labor (*H*) at the nominal rate W. Prices are sticky because there is a quadratic cost to adjusting them. The home firm maximization problem for the domestic consumer is:

$$\max E_0 \sum_{t=0}^{\infty} x_{t,t+n} \Pi_{H,t}(i) \tag{20}$$

where

$$\Pi_{H,t}(i) = \left(P_{H,t}(i) - MC_t(i) - AC_{P,t}(i)\right)\left(f_{H,t}(i) + f^*_{H,t}(i)\right), \tag{21}$$

with the adjustment and marginal costs defined respectively as:

$$AC_{P,t}(i) = \frac{y_P}{2}\frac{\left(p_{H,t}(i) - p_{H,t-1}(i)\right)^2}{P_{H,t-1}(i)}, \tag{22}$$

$$MC_t = \frac{(r_t P_t)^a W_t^{1-a}}{q_t a^a (1-a)^{1-a}} \tag{23}$$

and subject to the demand function for $f_{H,t}(i)$ from above and the production function specifying output ($y(i)$) as:

$$f_{H,t}(i) + f^*_{H,t}(i) = y_t(i) = q_t K_t^a(i) H_t^{1-a}(i) \tag{24}$$

Here ? represents technology common to all production firms in the country, and is subject to shocks. Lastly, $?_{t,t+n}$ is the pricing kernel used to value random date $t+n$ payoffs. Since firms are owned by the representative household, they are assumed to value future profits according to the household's intertemporal marginal rate of substitution in consumption, so $x_{t,t+n} = b^n \left(U'_{C,t+n}/P_{t+n}\right)/\left(U'_{C,t}/P_t\right)$.

The optimization problem implies a trade-off between capital and labor inputs that depend on the relative cost of each:

$$P_t r_t K_t(i) = \frac{a}{1-a} W_t H_t(i), \tag{25}$$

and price setting behavior:

$$\begin{aligned} p_{Ht}(i) &= \frac{l}{l-1}(MC_t + AC_{P,t}) \\ &+ \frac{y_P}{l-1} p_{H,t}(i)\left(1 - \frac{p_{H,t}(i)}{p_{H,t-1}(i)}\right) \\ &+ \frac{1}{2}\frac{y_P}{l-1} p_{H,t}(i) E_t\left[\frac{x_{t,t+i+1}}{x_{t,t+i}}\left(1 - \frac{p^2_{H,t+1}(i)}{p^2_{H,t}(i)}\right)\frac{F_{H,t+1}}{F_{H,t}}\right]. \end{aligned} \tag{26}$$

Note that in the special case of no price stickiness ($?_P$=0), price-setting is set as a simple markup over marginal costs: $p_{Ht}(i) = \frac{l}{l-1} MC_t$. But in the presence of price adjustment costs, price-setting will deviate from this simple markup because of several additional terms. First, the resource cost of setting a price (AC_P) should be included along with the cost of production when computing the overall price of bringing a good to market. The next term in the expression above reflects the backward looking component of price setting: firms are reluctant to make large changes in price due to the marginal adjustment cost. The final term reflects the forward-looking component of price setting. If a firm expects the need to change prices further in the next period, it will tend to change the price more today, to minimize future adjustment costs. Further, there is an additional reason to raise prices today, because a higher current price means that any future changes will be a smaller percentage change. Here we see one reason for the monopolist to set a higher price on average, as a hedge against future price changes.

Note that in the symmetric equilibrium $p_{H,t}(i) = P_{H,t}$.

2.4 Government

Model experiments will consider three alternative types of policy rules. The first two simple rules can be summarized in the following money growth rule:

$$m_t = m_{t-1} + a_s(s_t - \bar{s}) \tag{27}$$

where the response parameter a_S characterizes the degree to which money supply is adjusted to stabilize the nominal exchange rate at a target level. (Lower case indicates logs.) First, as a benchmark for later comparisons we consider a_S near zero, which we refer to as a "no policy" case, since monetary policy is not responding to the exchange rate or any other endogenous variables or exogenous shocks.[5] The general money supply rule in (28) also can encompass a fixed exchange rate regime, for a_s set to a large negative value. So a simple bilateral fixed exchange rate rule will be the second case considered in experiments.[6]

Finally, to relate to recent research on optimal policy rules, we also consider a Taylor-type interest rate setting rule

$$i_t = \bar{i} + \Gamma_p p_t + \Gamma_y \hat{Y}_t + \Gamma_s \hat{S}_t \tag{28}$$

where $\bar{i}$ is steady state interest rate, p_t is inflation rate, $\hat{Y}_t = (Y_t - Y_{t-1})/\bar{Y}$ is output gap, $\hat{S}_t$ is percentage changes in the nominal exchange rate. We assume that central banks make a commitment to set these parameters at time-invariant values.

The government's budget constraint is:

$$T_t = M_t - M_{t-1} \tag{29}$$

2.5 Market clearing and equilibrium

Market clearing for the home goods market requires:

[5] Setting a_s=-10^{-6} is sufficient to maintain stationarity of the exchange rate here, which allows us to analyze this key variable in nominal form in the model solution.

[6] For the foreign country's rule, the response coefficient to the exchange rate is the negative of that for the home country.

$$F_{H,t} + F^*_{H,t} = Y_t, \tag{30}$$

and for the home bond market:

$$B_{Ht} + B^*_{Ht} = 0. \tag{31}$$

Total home final goods demand must equal final goods supply:

$$F_t = C_t + (K_{t+1} - (1-d)K_t) + AC_{I,t} + \frac{AC_{B,t}}{P_t} + Y_t \frac{\int_0^1 AC_{P,t}(i)di}{P_t}, \tag{32}$$

The home balance of payments condition may be written:

$$(B_{H,t} - B_{H,t-1}) + S_t(B_{F,t} - B_{F,t-1}) = P_{H,t}Y_t + i_{t-1}B_{H,t-1} + S_t i^*_{t-1}B_{F,t-1} - P_t F_t \tag{33}$$

Equilibrium is a set of 37 sequences: $C_t, C^*_t, P_{H,t}, P_{F,t}, P_t, P^*_t, S_t, W_t, W^*_t, H_t, H^*_t$ $Y_t, Y^*_t, F_t, F^*_t, F_{H,t}, F^*_{H,t}, F_{F,t}, F^*_{F,t}, K_t, K^*_t, r_t, r^*_t, d_t, d^*_t, MC, MC^*, B_{H,t}, B^*_{H,t}, B_{F,t}, B^*_{F,t}, q_t, q^*_t,$ M_t, M^*_t, i_t, i^*_t. The 37 equilibrium conditions are: the definition of total demand (1), demand conditions for home and foreign goods (8 and 9), the overall price index (5), the price setting rule (26), the money supply rule (27 or 28), labor supply condition (15), capital-labor trade-off (25), money demand condition (14), the interest rate parity condition (18), production function (24), definition of marginal cost (23),definition of total demand (32), definition of *d* (17), consumption Euler equation (16), market clearing conditions for goods (30) and bonds (31), capital accumulation (19), along with foreign counterparts for each of the above and the balance of payments constraint (33).

The shocks, to technology and money demand in each country, will be log-normally distributed:

$$\begin{aligned}
(\log q_t - \log \overline{q}) &= r_1(\log q_{t-1} - \log \overline{q}) + e_{1t} \\
(\log q^*_t - \log \overline{q^*}) &= r^*_1(\log q^*_{t-1} - \log \overline{q^*}) + e^*_{1t} \\
(\log c_t - \log \overline{c}) &= r_2(\log c_{t-1} - \log \overline{c}) + e_{2t} \\
(\log c^*_t - \log \overline{c^*}) &= r^*_2(\log c^*_{t-1} - \log \overline{c^*}) + e^*_{2t}
\end{aligned} \tag{34}$$

$$[e_{1t}, e^*_{1t}, e_{2t}, e^*_{2t}]' \sim N(0, \Sigma).$$

To deal with the nonstationary nominal variables in this system, they will be transformed by dividing by their respective national price level. As noted above, this does not need to be done for the nominal exchange rate.

2.6 Solution method and welfare computation

The model is solved numerically to a second order approximation. See Kim et. al. (2003) or Schmitt-Grohe and Uribe (2004a) for a detailed explanation of the methodology. This contrasts with the more standard method relying upon log-linear approximations of model equations, which would only capture the direct effects of exchange rate variability on welfare through the fact that people dislike variance in consumption and leisure. A second order approximation to the full set of model equations additionally picks up the effects of variability on welfare through the means of consumption and leisure. For example, firms may hedge against exchange rate variability by setting higher prices and lowering mean output, and households may engage in precautionary saving that affects mean consumption.

We report welfare analysis both in conditional and unconditional terms. In either case, welfare is based on a second order Taylor expansion of the utility function around the deterministic steady state, indicated here by overbars. Using unconditional expectations:

$$EU(C_t,H_t)=\overline{U}+\overline{C}^{1-r}\,E(\hat{C}_t)-\frac{1}{2}\,r\,\overline{C}^{1-r}\,\mathrm{var}(\hat{C}_t)-\overline{H}^{1+y}\,E(\hat{H}_t)-\frac{1}{2}y\,\overline{H}^{1+y}\,\mathrm{var}(\hat{H}_t)\,.$$

The unconditional welfare loss is measured in terms of the share (u) of deterministic steady state consumption the household would be willing to give up to eliminate risk; that is, which equates the utility level of the determinist steady state and the unconditional expectation under uncertainty defined above:

$$U\big((1+u)C,H\big)=EU\big(C_t,H_t\big)\,.$$

This welfare effect can be decomposed into the part due to the variance of uncertain consumption and leisure, and the part due to the effect of uncertainty on the means of these variables:

$$U\big((1+u^{\mathrm{var}})C,H\big)=\overline{U}-\frac{1}{2}\,r\,\overline{C}^{1-r}\,\mathrm{var}\big(\hat{C}_t\big)-\frac{1}{2}y\,\overline{H}^{1+y}\,\mathrm{var}\big(\hat{H}_t\big)$$

$$U\big((1+u^{mean})C,H\big)=\overline{U}+\overline{C}^{1-r}\,E\big(\hat{C}_t\big)-\overline{H}^{1+y}\,E\big(\hat{H}_t\big).$$

We also report conditional welfare measures, which have the advantage of taking into consideration the transition dynamics following the implementation of a new policy rule. To compare the welfare implications of adopting the set of alternative policy rules, we trace out the utility level of the economy as it starts out from the unconditional expectation implied by the 'no policy' rule defined above, and evolves over time in response to the alternative rule under consideration. Welfare is summarized analogously to the unconditional formulas above, except that it is the discounted sum of expected utilities over time rather than the unconditional expectation that is reported. For example, the overall welfare effect is computed:

$$U\left(\left(1+u^{conditiond}\right)C, H\right) = (1-b)\sum_{t=0}^{\infty} b^{t} E_t U\left(C_t, H_t\right).$$

To solve for the reaction parameters in the policy rule (28) that are optimal in terms of welfare, a grid search is conducted to maximize the world (sum of home and foreign) welfare level. We do this in turn for both conditional and unconditional welfare measures, as defined above, though for our cases the optimal policy parameters are unaffected. Since the benchmark case is symmetric, this is a fairly simple matter of choosing three policy parameters, $G_?$, G_y and G_e, which we accomplish by grid search.[7] (We also consider an alternative non-cooperative policy for the nonsymmetric case in the paper's final experiment, discussed below.)

2.7 Calibration

We follow Bergin and Feenstra (2001) in setting *e* = *?* = 4, implying an interest elasticity of real money balances of 0.25 and an income elasticity of unity. We follow estimates of the elasticity of substitution between home and foreign goods Harrigan (1993) and Trefler and Lai (1999) in setting µ=5. A value of *?* =7 implies an average price mark-up of 16%. The share of home goods in the home final goods aggregator, *a*, is set at 0.80, reflecting the 20% share of imports in GDP on average for the G7 countries in the 1990:1-1998:4 period.

We follow Christiano at. al. (1997) in setting the labor supply at unity (*?* =1). ß=0.99, where a period in the model is one quarter. For the depreciation rate we choose *d*=0.025, and for the capital share in production *a*=0.36.

The price adjustment cost is set at $?_P$=50, which implies that 95% of the price has adjusted 4 periods after a monetary shock. Investment adjustment cost, $?_I$=4, is calibrated such that investment is about three times more volatile than output. Bond adjustment cost, $?_B$=0.000004, is necessary in order to negate the unit root associated with the incompleteness

[7] For $G_?$, we searched over a grid from 1 to 5 with a step of 0.25. For G_y, from 0 to 5 with a step of 0.25. The response to the exchange rate , G_e, was bounded above 10^{-4} to prevent nonstationarity of the exchange rate. We considered bigger values for G_e, but it did not make welfare improve for the benchmark or other symmetric cases of the model.

of the asset markets. We set lower bounds on the monetary policy reaction parameters to the exchange rate ($a_S = 10^{-6}$) in order to eliminate the unit root in the monetary policy rule. It is crucial that the first-order solution does not contain unit roots, because, otherwise, in the second-order solution the variances of the variables will grow to infinity.

The variance and persistence of the technology shock is calibrated at standard values: $\text{var}(e_1) = \text{var}(e_1^*) = .01^2$ and $r_1 = r_1^* = 0.90$, common values in the real business cycle literature and identical to Kollmann (2002). As will be seen in Table 1, these values help us to replicate the second moments of output, which we compute to be 1.80 for the 1973:1-2000:4 period in HP-filtered GDP data for the G7 countries on average.

Money demand shocks are calibrated to help replicate the observed second moments of the nominal exchange rate. While the search for an adequate theoretical explanation for exchange rate volatility is itself the subject of ongoing research, the approach taken here follows on the example of the literature discussed in the introduction; Devereux and Engel (2003), Bacchetta and van Wincoop (2000), and Obstfeld and Rogoff (2002) each use money demand or money supply shocks to generate exchange rate variability. Bergin (2003) offers some empirical support for this approach. We compute the bilateral exchange rate with the U.S. dollar of the remaining G7 countries on average to be 7.81 percent for HP-filtered data in the 1973:1 - 2000:4 period, which is between 4 and 5 times as volatile as output. The autoregressive coefficient indicated by this data is 0.79. Replicating these features requires the following values for the money demand shocks: $\text{var}(e_1) = \text{var}(e_1^*) = .03^2$ and $r_1 = r_1^* = 0.99$. For simplicity we assume that shocks are uncorrelated with each other.

3 Results

3.1 Benchmark case

Column 1 of Table 1 reports results for the benchmark specification of the model under the "no-policy" case. First note that the standard deviations of the key variables in the model match fairly well with the average among G7 economies. Consumption is about 1/2 as volatile as output, investment is more than 2 times as volatile, and the nominal exchange rate is about 4 times as volatile as output.

The next set of entries in column 1 show the effects that risk have on the means of key endogenous variables, measured as the percent difference between the unconditional mean of the second-order solution and the deterministic steady state. The level of production is lower under the presence of risk, with consumption likewise lower and leisure higher. This in part is due to the higher markup of the home goods price over marginal cost shown in the table. Note also that the trade volume is higher under the presence of risk, a possibility raised theoretically by Bacchetta and van Wincoop (2000).

The bottom set of entries in column 1 indicates that these effects of risk translate into lower welfare, about a third of which come from the shift in means noted above. But the magnitudes of these welfare effects are fairly small, amounting to a fall in utility equal to 0.14% of steady state consumption. A useful comparison is the result of Lucas (1987), which measured the effect on welfare of volatility arising from business cycle fluctuations. Finding a loss equivalent to a drop in average consumption by 0.042 percent, he concluded that this was a trivial magnitude. The result here is somewhat larger, but still of a similar order of magnitude and far below one percent of steady state consumption.

How can stabilization policies improve welfare in this context? Column 2 of Table 1 shows that the simple fixed exchange rate rule does eliminate part of the welfare loss from the "no-policy" case. Column 3 shows that welfare can be improved yet further with an optimized interest rate setting rule. The optimal rule in this case is to put weight only on inflation targeting ($G_{?}$ =5.0), with no effort to stabilize output or the exchange rate (G_Y=0.0, G_S=0.0001).[8] The conditional welfare analysis leads to the same optimal policy grid values and the same ranking of policy rules in the table. The welfare time paths for each regime for the benchmark case are depicted in Figure 1.

It is perhaps obvious that an optimal rule dominates the fixed exchange rate rule, since the choice set for the optimal rule nests a fixed exchange rate as one possibility (large G_S). More informative is the fact that the optimal rule places no more than the minimum required weight on exchange rate stabilization, instead placing all weight on the inflation term. Furthermore, the optimal rule actually raises the unconditional volatility of the exchange rate relative to the no-policy case. This may reflect the use of exchange rate movements to facilitate adjustment to real shocks, in place of prices which cannot adjust due to stickiness. These findings lead to our first significant result: exchange rate volatility per se does not seem to be harmful to welfare in the benchmark environment.

---

[8] As in Schmitt-Grohe and Uribe (2004b) we found that the inflation parameter is at the upper bound of the range considered in the grid search. Allowing yet higher values of this parameter appears to have negligible effects on the equilibrium.

The second significant conclusion is that the welfare benefits of optimal stabilization policy rules are very small. The gains relative to the no-policy case are 0.074 percent of consumption for unconditional measures and only 0.035 percent for unconditional. We might have been able to surmise this conclusion from the small welfare numbers in column 1 discussed above. If there is little loss from variability, then there is little opportunity for stabilization policy to raise welfare by stabilizing this variability.

3.2 Standard cases from the literature and sensitivity analysis

This section evaluates some of the controversies raised in the theoretical literature, regarding the impact of alternative economic environments and the implications for policy choices. Given the conclusions of the preceding paragraph and the difficulty of solving for optimal policy rules, we limit ourselves in this section to discuss unconditional deviations from deterministic steady state.

It has been demonstrated in an analytically solvable model that price stickiness in the local currency of the buyer (LCP) can alter the welfare effects of flexible exchange rates (Devereux and Engel, 2003). Column 1 of Table 2 indicates that having price stickiness of some type indeed is important to the results here. A case with no price stickiness cuts in half the welfare loss of risk, and this mainly works by fully eliminating the extra markup in price-setting behavior attributable to risk, which now takes a zero value in the table. But column 2 indicates that LCP stickiness seems not to be a quantitatively significant distinction here. The welfare effect of risk remains small, and is even a bit smaller than under the PCP benchmark specification.

It has also been demonstrated that the substitutability between consumption and leisure in preferences can switch the effects of exchange rate variability from negative to positive (Bacchetta and van Wincoop, 2000). We address this issue by replacing the utility function in (12) with one that resembles that of Bacchetta and van Wincoop:[9]

$$U_t = \left(\frac{1}{1-r}\right)\left[(b)^{1/f} c_t^{(f-1)/f} + (1-b)^{1/f}(1-h_t)^{(f-1)/f}\right]^{(1-r)f/(f-1)} + \frac{1}{1-e}\left(\frac{M_t}{P_t}\right)^{1-e}. \quad (45)$$

[9] This utility differs from Bachetta and van Wincoop (2000) in that it includes money. Note that this utility differs from that used earlier in the paper in the way it includes labor, so that it does not collapse down to equation (12) if we assume a zero elasticity of substitution. Also note that the change in utility function requires a corresponding adjustment in the computation of u^{mean} and u^{var} in evaluating welfare effects.

The parameter b is calibrated so that the steady state share of time to labor is 0.37, and f is adjusted to replicate the consumption-leisure substitutabilities considered in Bacchetta and van Wincoop, so f is set at 0.1 and 10 for complements and substitutes respectively. Our model may be viewed as a generalization of theirs to include dynamics, technology shocks, and investment. See columns 3 and 4 in Table 2 for results. Confirming their results, risk raises trade volume when consumption and leisure are complements, and it lowers trade volume when they are substitutes. Further, welfare is worse under substitutes than under complements, though the welfare effects are negative for both cases in our model. But while the effects in general resemble those of the earlier paper in a qualitative sense, once again the main conclusion is that in a quantitative sense, the present model reveals that these effects are all quite small.

We consider some parameter values that might be expected to raise the magnitude of the small welfare effects found above. One possibility is that exchange rate variability would matter more for countries that trade more with foreign countries. Column 5 shows a case where the share of imports in GDP (1-a) is raised from 0.2 to 0.5. This makes little difference. It is also possible that risk would matter if agents were more risk averse. Column 6 shows a case where the risk aversion parameter (?) is set at 30 instead of 4. Again there is little effect. It appears that a wide range of cases of the two-country model imply that the welfare effects of risk are quantitatively small.

3.3 Habits

Past research on household responses toward risk in terms of consumption and asset choices has found that habits may be an essential part of household preferences.[10] Given that this literature has found households to be quite sensitive to risk in domestic equity markets, one might also expect them to be sensitive to risk in international asset markets. Yet papers analyzing the effects of exchange rate risk to date have not been able to consider this potentially important feature because it precludes the usual analytical solution.

To include habits in the model, we consider the utility function:

$$U_t = \frac{(C_t - g\, C_{t-1})^{1-r}}{1-r} + \frac{c_t}{1-e}\left(\frac{M_t}{P_t}\right)^{1-e} - \frac{H_t^{1+y}}{1+y}, \tag{46}$$

[10] See for example Constantinides (1990) and Campbell and Cochrane (1999) regarding the equity premium puzzle, and see Deaton (1987) and Fuhrer (2000) for a discussion in the context of consumption behavior.

which implies an intertemporal Euler equation:[11]

$$d_t = b\, E_t\left[\left(\frac{P_t}{P_{t+1}}\right)\frac{(C_{t+1}-g\,C_t)^{-r} - bg\, E_{t+1}(C_{t+2}-g\,C_{t+1})^{-r}}{(C_t-g\,C_{t-1})^{-r} - bg\, E_t(C_{t+1}-g\,C_t)^{-r}}\right].$$

As ? goes to unity, households act to smooth changes in consumption rather than the level of consumption. We calibrate the habit persistence parameter at ?=0.8, which is approximately what Deaton (1987) and Constantinides (1990) require in order to explain aggregate consumption smoothness and the equity premium puzzle.

While it is common in calibrating habit persistence models to impose a large investment adjustment cost to keep the standard deviation of consumption from falling to implausibly low levels, this device does not work in an open economy where international borrowing breaks the link between domestic investment and saving. Instead we augment the bond adjustment cost in the household budget constraint to penalize large changes in asset holding as well as large levels:

$$AC_{B,t} = \frac{y_B}{2}\left(\frac{(S_t(B_{Ft}-\overline{B_F}))^2}{P_{Ht}Y_t}\right) + \frac{y_{B2}}{2}\left(\frac{(S_t(B_{Ft}-B_{F,t-1}))^2}{P_{Ht}Y_t}\right) \qquad (47)$$

where $?_{B2}$ is calibrated at 0.0009.

Column 1 of Table 3 shows that welfare in the no-policy case does now fall a somewhat larger amount due to risk. In particular, households would be willing to trade 0.66% of annual steady state consumption to eliminate this risk.[12] We conclude that in comparison to LCP and leisure substitutability and the other features considered in the analytical literature, habits appear to be a quantitatively somewhat more interesting object of study in terms of the implications for welfare.

However, despite the fact that risk has larger welfare effects here, Table 3 shows that habits do not much affect the optimal policy rule, and that stabilization policy continues to have small effects on the level of welfare. The optimal policy parameters are the same as in the benchmark case, still with minimal weight on the exchange rate. And it again is true that exchange rate variability is increased by the optimal rule. Welfare is improved by only 0.130 percent in unconditional terms, and 0.023 percent in conditional terms. While habits indicate

[11] The welfare formula is of course updated with the second order expansion of the new utility function.

[12] Note that with a different utility function, the formulas for computing u^{mean} and u^{var} must be altered accordingly. We note also that as the effects of risk become larger, there is a greater chance that the second order solution method may run into accuracy problems.

that households are more concerned about smoothing their consumption stream, it appears that monetary policy has minimal impact on this particular source of risk in the model.

3.4 Original sin in asset markets

Welfare implications can be shaped also by the structure of a country's asset market. Eichengreen, et.al. (2003) have noted that perceptions in the international capital market make it impossible for many countries to issue international debt denominated in their own national currency. Given that such perceptions of untrustworthiness may well be beyond the control of the country to change, but simply are a feature of the international capital market, the authors have termed this condition 'original sin.'

To explore this feature, consider a version of our model where there is only one nominal bond that is traded internationally, denominated in the currency of the home country. This implies the benchmark model above, except that B_{Ft} is set to zero in all periods. The home country in our model certainly is relevant for those countries whose currencies have the status of reserve currencies, such as the U.S., Japan, and EMU countries. And the foreign country in the model is relevant to some degree for any of the remaining countries, and most especially for developing countries.

Looking at the first column of Table 4, this asymmetry in asset markets implies that risk has a significant impact on welfare of the two countries.[13] While the foreign country's welfare is hurt by the presence of risk, the home country actually benefits. The magnitude of the effect on the foreign country is even larger than that in the habits case above. Table 4 indicates that the large majority of this effect comes from a change in the mean value of variables rather than the variances. In particular there is a rise in the mean of home consumption and a fall in foreign. Clearly the asymmetry of this result distinguishes it from the analytical models with perfect risk sharing common in the literature.

The logic for what happens in this case of asymmetric incomplete markets is as follows. The introduction of risk makes households want to engage in precautionary saving to hedge. But since this is true for both countries, the main effect is to make the world interest rate on the bond fall, as can be seen clearly in column 1 of Table 4. Since the international asset is in the currency of the home country, the exchange rate risk makes it a less attractive instrument for saving for the foreign country than for the home country. Given the fall in the

---

[13] For the model to continue producing the same level of exchange rate variability under this specification, the variance of the money demand shock needed to be increased somewhat from 0.03^2 to 0.05^2.

interest rate, the foreign country chooses to save less in equilibrium, while the home country saves more, also seen in the table. So the home country has greater wealth and hence greater consumption in steady state than the foreign country.[14] See the appendix for a more detailed derivation of these points.

Column 2 shows that the simple fixed exchange rate rule does lead to large changes in unconditional welfare, eliminating about half of the large welfare loss of the foreign country discussed above.[15] Column 3 shows that in a cooperative optimal policy regime there again is no effort to stabilize the exchange rate.[16] But a cooperative policy experiment may not be appropriate here, since summing the utilities over home and foreign households cancels out the large asymmetric welfare effects found above. Iterating to compute a full Nash policy equilibrium is prohibitive here, especially due to asymmetry. But as a final experiment (column 4), we compute the optimal policy rule for the foreign (sin) country, in a case where the home country policy is assumed to be committed to an inflation targeting rule. In this case the foreign country would choose a policy rule with substantial weight on the exchange rate (G_S=0.5), and this does virtually eliminate exchange rate variability.[17] We think this experiment is informative, indicating that there are potentially large welfare gains for some types of countries from adopting policies that stabilize the exchange rate. It is economies of this type that may most warrant further investigation in the theoretical literature.

4 Conclusions

This paper has investigated quantitatively the welfare effects of risk in a two-country sticky price model, as well as the potential welfare gains from fixed exchange rates and other

---

[14] This result helps us to better understand and qualify the result in the small open economy model of Kollmann (2002). Because the model is of a small open economy, the world interest rate is taken as exogenous and is therefore unaffected by the presence of risk. As a result, the small country saves more and ends up with higher welfare in steady state. Our analysis shows that when a two-country model takes into consideration the effects of risk on the world environment, the resulting fall in interest rate reverses this result.

[15] Note also that the conditional welfare effect is small despite the large unconditional effect. Adopting a new policy does not change the fact that the foreign economy is starting off with a low share of wealth. After the adoption of the new policy, the foreign country begins saving more, but this entails a lower consumption level during the lengthy transition period where it is building up its capital stock and assets.

[16] The full policy rules are $i_t = \bar{i} + 4.5 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$ for the foreign (sin) country, and $i_t = \bar{i} + 2.75 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$ for the home country.

[17] The full policy rules are $i_t = \bar{i} + 5.0 p_t + 0.0 \hat{Y}_t + 0.5 \hat{S}_t$ for the foreign (sin) country, with the home country policy fixed at $i_t = \bar{i} + 5.0 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$, which was the optimum from the benchmark case.

policy rules. The paper takes advantage of developments in solving second-order approximations to dynamic stochastic models, to investigate a plausibly calibrated and fairly richly specified model. The first finding is that in a standard economic environment, the welfare effects are likely to be quite small. Accordingly, the welfare gains from fixed exchange rates or optimal interest rate setting rules are likewise found to me small. In particular, there appears to be no positive gain in such a setting from stabilizing exchange rates per se. The finding of small quantitative implications appears to be true also for the set of economic features that have been the focus in recent theoretical research, including local currency pricing and consumption-leisure complementarity.

However, we identify two other types of economies where risk may have larger quantitative implications. The first is where agents exhibit habits, so that volatility in the consumption stream matters more to them. However, while risk affects these agents more, we find that this has little implication for policy. The optimal policy parameters are little affected by habits, and it remains true that policy rules here have little ability to improve welfare.

Finally, we find that economies characterized by original sin, unable to internationally sell debt denominated in their domestic currency, can be strongly affected by exchange rate risk. In contrast to the earlier cases, such a country potentially can improve welfare significantly by adopting a fixed exchange rate or by adopting an interest-rate setting policy rule that stabilizes the exchange rate as well as inflation.

This research leads us to believe that it would be useful for the literature to focus greater attention on the particular cases identified here as quantitatively significant for welfare. In particular, there is a need for theory to shed light on the various implications of original sin in asset markets.

5 References

Bacchetta, P. and E. van Wincoop, 2000. Does exchange rate stability increase trade and welfare? *American Economic Review* 90, 1093-1109.

Benigno, G. and P. Benigno, 2003. Price stability in open economies, *Review of Economic Studies* 70, 743-764.

Benigno, P., 2004. Optimal monetary policy in a currency area. *Journal of International Economics*.63, 293-320.

Bergin, P. and R. Feenstra, 2001. Pricing-to-market, staggered contracts, and real exchange rate persistence, *Journal of International Economics* 54, 333-359.

Bergin, P. 2003. Putting the `new open economy macroeconomics' to a test, *Journal of International Economics* 60, 3-34.

Campbell, J. Y. and J. H. Cochrane, 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205-251.

Carre, M. and F. Collard, 2003. Monetary Union: A Welfare Based Approach, *European Economic Review* 47 (3), 521-552.

Christiano, L. J., M. Eichenbaum and C. Evans, 1997. Sticky price and limited participation models of money: a comparison. *European Economic Review* 41, 1021-1049.

Clarida, R., J. Gali and M. Gertler, 2001. Optimal monetary policy in open versus closed economies. *American Economic Review* 91, 248-252.

Collard, F. and M. Juillard, 2001. Accuracy of stochastic perturbation methods: the case of asset pricing models, *Journal of Economic Dynamics and Control* 25, 979-999.

Constantinides, G. M, 1990. Habit formation: a resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519-543.

Corsetti, G. and P. Pesenti, 2001a. Welfare and macroeconomic interdependence, *Quarterly Journal of Economics* 116, 421-445.

Corsetti, G. and P. Pesenti, 2001b. International dimensions of optimal monetary policy, NBER Working Paper No. 7665.

Deaton, A., 1987. Life-cycle models of consumption: is the evidence consistent with the theory? in Advances in Econometrics Fifth World Congress, Vol. 2, ed. by T. Bewley, Cambridge University Press, 121-148.

Devereux, Michael B. and Charles Engel, 2003, "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility," *Review of Economic Studies* 70, 765-783.

Eichengreen, B., R. Hausmann and U. Panizza, 2003. "The pain of original sin," in B. Eichengreen and R. Hausmann (eds.), *Debt, Denomination and Financial Instability*

Fuhrer, J.C., 2000. Habit formation in consumption and its implications for monetary policy models. *American Economic Review* 90, 367-390.

Harrigan, J., 1993. OECD imports and trade barriers in 1983. *Journal of International Economics* 35, 91-111.

Kim, J., S. Kim, S., E. Schaumburg, and C.Sims, 2003. Calculating and using second order accurate solutions of discrete time dynamic equilibrium models, mimeo.Princeton University.

Kollmann, R., 2002. Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics* 49, 989-1015.

Kollmann, R., 2004. Welfare effects of a monetary union: The role of trade openness. *Journal of the European Economic Association* 2, 289-301.

Lucas, R. E., 1987. Models of Business Cycles, Basil Blackwell, New York.

Obstfeld, M. and K. Rogoff, 2000. Do we really need a new international monetary compact? NBER Working Paper No. 7864. Forthcoming in *Quarterly Journal of Economics*.

Obstfeld, M. and K. Rogoff, 2002. Risk and Exchange Rates. In Elhanan Helpman and Effraim Sadka (eds.), Contemporary Economic Policy: Essays in Honor of Assaf Razin. Cambridge: Cambridge University Press 2002. Also, NBER Working Paper 6694.

Schmitt-Grohe, S. and M. Uribe, 2004a. Solving dynamic general equilibrium models using a second-order approximation of the policy function. *Journal of Economic Dynamics and Control* 28, 645-858

Schmitt-Grohe, S. and M. Uribe, 2004b. Optimal simple and implementable monetary and fiscal rules. mimeo, Duke University.

Sutherland, A., 2005 Incomplete pass-through and the welfare effects of exchange rate variability. *Journal of International Economics* 65 (2), 375-399.

Trefler, D. and H. Lai, 1999. The gains from trade: standard errors with the CES monopolistic competition model, University of Toronto manuscript.

6 Appendix: Asymmetric Asset Market Case

One can gain some insight into the mechanism described in the text for the asymmetric assets markets case by comparing the intertemporal Euler equations across countries. The consumption Euler equation for the foreign country is:

$$\frac{1}{1+i_t} = b\, E_t\left[\frac{S_t P_t^* C_t^{*r}}{S_{t+1} P_{t+1}^* C_{t+1}^{*r}}\left(1+\frac{y_B(B_{Ht}^* - \overline{B_H^*})}{S_t P_{Ft}^* Y_t^*}\right)^{-1}\right] \tag{48}$$

A bar over a variable denotes its deterministic steady state. The consumption Euler for the home country is:

$$\frac{1}{1+i_t} = b\, E_t \frac{P_t C_t^r}{P_{t+1} C_{t+1}^r} \tag{49}$$

Equalizing (48) and (49)

$$E_t\left[\frac{S_t P_t^* C_t^{*r}}{S_{t+1} P_{t+1}^* C_{t+1}^{*r}}\left(1+\frac{y_B(B_{Ht}^* - \overline{B_H^*})}{S_t P_{Ft}^* Y_t^*}\right)^{-1}\right] = E_t \frac{P_t C_t^r}{P_{t+1} C_{t+1}^r} \tag{50}$$

Denoting $q_t^f = \frac{S_t P_t^* C_t^{*r}}{S_{t+1} P_{t+1}^* C_{t+1}^{*r}}$ and $q_t^h = \frac{P_t C_t^r}{P_{t+1} C_{t+1}^r}$ and taking unconditional expectations of (50) yields:

$$E\left[q_t^f\left(1+\frac{y_B B_{Ht}^*}{S_t P_{Ft}^* Y_t^*}\right)^{-1}\right] = E(q_t^h) \tag{51}$$

A second-order Taylor expansion of (51) gives:

$$\frac{y_B}{\overline{\overline{S\, P_F^* Y^*}}} E(d\, B_{Ht}^*) = E(\hat{q}_t^f) - E(\hat{q}_t^h) - \frac{y_B}{\overline{\overline{S\, P_F^* Y^*}}} \operatorname{cov}(\hat{q}_t^f, d\, B_{Ht}^*) \tag{52}$$

A hat over a variable denotes a log deviation from its deterministic steady state. Since bonds are assumed to be zero in steady state, $dB\, h_t^*$ stands for the absolute deviation of bond holdings from zero. The last expression is crucial in our analysis because it demonstrates the relationship between the expected holdings of home bonds by the foreign country and the variability of the exchange rate.[18] This can be seen more clearly by further expanding q_t^f

[18] Note that among the three terms on the right hand side of the equality, the last term will be quantitatively insignificant in comparison with the others, as it is scaled by the adjustment cost parameter, ?_{B}, which is calibrated to be very small.

(using a second-order Taylor-series approximation). We also denote $q_t^* = \frac{P_t^* C_t^{*r}}{P_{t+1}^* C_{t+1}^{*r}}$. Then $E(\hat{q}_t^f) - E(\hat{q}_t^h)$ will be equal to:

$$E(\hat{q}_t^f) - E(\hat{q}_t^h) = \text{var}(\hat{S}_t) - \text{cov}(\hat{S}_t, \hat{S}_{t+1}) + \text{cov}(\hat{S}_t, \hat{q}_t^*) - \text{cov}(\hat{S}_{t+1}, \hat{q}_t^*) \quad (53)$$

where we have made use of the fact that the unconditional expectations of $\hat{S}_t$ and $\hat{S}_{t+1}$, and $\hat{q}_t^*$ and $\hat{q}_t^h$ are the same. Looking at (53) it is clear that an increase in the variance of the exchange rate in isolation would tend to make the foreign country save more by investing in the home-currency bond. This is so because the variability of the exchange rate makes the intertemporal marginal rate of substitution regarding foreign currency $\hat{q}_t^f$ higher. Moreover, it is always true that $\left(\text{var}(\hat{S}_t) - \text{cov}(\hat{S}_t, \hat{S}_{t+1})\right) > 0$ since $\text{var}(\hat{S}_t) = \text{var}(\hat{S}_{t+1})$ var(S_{t})=var(S_{t+1}). If this were the whole effect then we could have safely concluded that in the true (stochastic) steady state, the foreign holdings of home-currency denominated bonds are positive. However, the covariance terms at the end of the expression alter this result. In particular, $\text{cov}(\hat{S}_t, \hat{q}_t^*)$ and $\text{cov}(\hat{S}_{t+1}, \hat{q}_{t+1}^*)$ are both negative, where the first of these covariances dominates due to the stationarity of the model. Further, this covariance is sufficiently negative that it makes the overall expression $E(\hat{q}_t^f) - E(\hat{q}_t^h) < 0$ E(q_{t}. Notice that $q_t^* = \frac{P_t^* C_t^{*r}}{P_{t+1}^* C_{t+1}^{*r}} = \frac{C_{t+1}^{*-r}}{P_{t+1}^*} \Big/ \frac{C_t^{*-r}}{P_t^*}$ which is the ratio of the marginal utility of consumption between periods (*t+1*) and *t*. Therefore, one could interpret the covariances between that ratio and the exchange rate as a risk premium associated with the investing in a foreign currency. As a result of this risk premium, the foreign agent's desire to save is less than that of the home agent, and the stochastic steady state implies a net foreign debt for the foreign country.

Table 1: Benchmark Cases

	No Monetary Policy	Fixed Exchange Rate	Optimized Flexible Exchange. Rate[3]
Standard deviations:			
consumption	1.24	1.21	1.20
output	2.41	2.36	1.86
investment	5.09	5.09	4.93
inflation	0.70	0.48	0.02
exchange rate	8.03	0.00	20.07
Stochastic steady state deviations[1]:			
consumption	-0.036	-0.018	0.016
leisure	0.012	0.017	0.026
output	-0.015	0.007	0.046
capital stock	-0.034	0.023	0.108
interest rate	-1.481	-1.082	-0.324
markup ratio	0.341	0.215	0.001
net foreign assets	0.000	0.000	0.000
trade volume	0.299	0.256	0.277
Unconditional Welfare effects (as percentage of steady state consumption)[2]:			
u-overall	-0.144	-0.113	-0.070
u-variance	-0.099	-0.082	-0.067
u-mean	-0.046	-0.031	-0.003
Conditional Welfare effects (as percentage of steady state consumption)[2]:			
u-overall	-0.144	-0.129	-0.109
u-variance	-0.099	-0.087	-0.070
u-mean	-0.046	-0.042	-0.039

[1] Percent difference between stochastic steady state and deterministic steady state.
[2] Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.
Foreign variables are identical to home in the cases shown here.
[3] Flexible exchange rate regime with an optimal policy rule ($i_t = \overline{i} + 5.0 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$).

Table 2: Standard Cases

	(1) Flexible Prices	(2) LCP	(3) Cons.-Leis. Complements	(4) Cons.-Leis. Substitutes	(5) Higher Import share	(6) Higher Risk Aversion
Standard deviations:						
consumption	1.22	1.04	1.06	2.41	1.25	0.20
output	1.37	3.91	3.45	3.12	4.17	4.78
investment	5.00	6.16	5.78	6.87	4.66	5.82
exchange rate	7.45	9.16	8.03	8.01	8.05	8.02
Stochastic steady state deviations[1]:						
consumption	0.016	-0.053	-0.032	0.017	-0.036	-0.006
leisure	0.035	-0.025	-0.022	0.082	0.006	0.054
output	0.055	-0.041	-0.040	0.062	-0.029	0.029
capital stock	0.118	-0.014	-0.070	0.276	-0.062	0.051
interest rate	-0.031	-0.103	-0.018	-0.119	-0.051	-0.051
markup ratio	0.000	0.214	0.652	0.391	0.316	0.370
net foreign assets	0.000	0.000	0.000	0.000	0.000	0.000
trade volume	0.293	0.488	1.183	-0.037	0.068	0.279
Welfare effects (as percentage of steady state consumption)[2]:						
u-overall	-0.078	-0.090	-0.181	-0.242	-0.133	-0.147
u-variance	-0.071	-0.056	-0.174	-0.202	-0.092	-0.101
u-mean	-0.008	-0.035	-0.007	-0.039	-0.041	-0.048

[1] Percent difference between stochastic steady state and deterministic steady state.
[2] Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.
Foreign variables are identical to home in the cases shown here.

Table 3: Habit Persistence Cases

	No Monetary Policy	Fixed Exchange Rate	Optimized Flexible Exchange. Rate[3]
Standard deviations:			
consumption	1.25	1.24	1.25
output	1.41	1.41	1.39
investment	7.35	7.32	7.35
inflation rate	0.73	0.50	0.14
exchange rate	8.25	0.00	26.11
Stochastic steady state deviations[1]:			
consumption	0.018	0.023	0.030
leisure	-0.003	-0.013	-0.020
output	0.113	0.123	0.140
capital stock	0.513	0.571	0.630
interest rate	-8.235	-7.833	-8.560
markup ratio	0.360	0.186	0.020
net foreign assets	0.000	0.000	0.000
trade volume	0.576	0.561	0.570
Unconditional Welfare effects (as percentage of steady state consumption)[2]:			
u-overall	-0.659	-0.580	-0.530
u-variance	-0.692	-0.674	-0.680
u-mean	0.033	0.095	0.150
Conditional Welfare effects (as percentage of steady state consumption)[2]:			
u-overall	-0.659	-0.655	-0.636
u-variance	-0.692	-0.688	-0.683
u-mean	0.033	0.032	0.047

[1] Percent difference between stochastic steady state and deterministic steady state.
[2] Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.
Foreign variables are identical to home in the cases shown here.
[3] Flexible exchange rate regime with an optimal policy rule ($i_t = \overline{i} + 5.0 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$).

Table 4: Asymmetric Asset Market Cases

	No Monetary Policy	Fixed Exchange Rate	Coordinated Optimized Flexible Exchange Rate[3]	Foreign Country Optimized Flexible Exchange Rate[4]
Standard deviations:				
Consumption	1.29	1.29	1.38	1.38
Output	1.77	1.78	1.80	1.80
Investment	5.89	5.92	5.89	5.89
inflation rate	0.64	0.38	0.02	0.02
exchange rate	8.58	0.00	11.73	0.00
Stochastic steady state deviations[1]:				
consumption (home)	0.202	0.092	0.105	0.108
consumption (foreign)	-0.187	-0.043	-0.024	-0.027
Leisure (home)	-0.539	-0.134	-0.126	-0.135
Leisure (foreign)	0.692	0.294	0.285	0.294
Output (home)	-0.542	-0.148	-0.114	-0.122
Output (foreign)	0.592	0.247	0.264	0.273
Capital stock (home)	-0.507	-0.148	-0.073	-0.079
Capital stock (foreign)	0.400	0.168	0.230	0.236
interest rate	-0.488	-0.216	-0.179	-0.166
markup ratio	0.151	0.068	0.002	0.001
net foreign assets	78.53	27.41	26.28	27.42
trade volume	0.624	0.326	0.320	0.326
Unconditional Welfare effects (as percentage of steady state consumption)[2]:				
u-overall (home)	0.416	0.013	0.022	0.032
u-overall (foreign)	-0.879	-0.438	-0.409	-0.419
u-variance	-0.183	-0.178	-0.175	-0.175
u-mean (home)	0.599	0.191	0.198	0.207
u-mean (foreign)	-0.696	-0.260	-0.234	-0.243
Conditional Welfare effects (as percentage of steady state consumption)[2]:				
u-overall (home)	0.416	0.441	0.477	0.421
u-overall (foreign)	-0.879	-0.863	-0.816	-0.850
u-variance	-0.183	-0.174	-0.173	-0.172
u-mean (home)	0.599	0.619	0.650	0.593
u-mean (foreign)	-0.696	-0.689	-0.644	-0.674

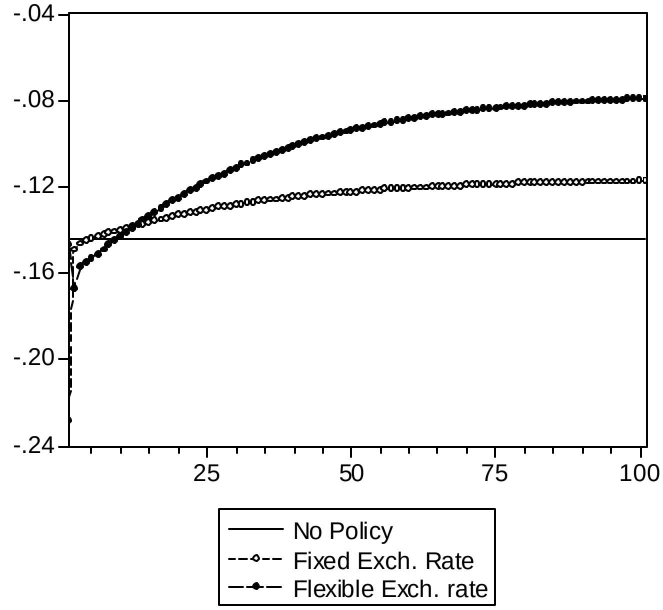
[1] Percent difference between stochastic steady state and deterministic steady state.

[2] Difference between stochastic steady state and deterministic steady state, shown as a share of deterministic steady state consumption.

[3] The policy rules are $i_t = \bar{i} + 4.5 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$ for the foreign (sin) country, and $i_t = \bar{i} + 2.75 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$ for the home country

[4] Flexible exchange rate regime with an optimal policy rule for foreign country: $i_t = \bar{i} + 5.0 p_t + 0.0 \hat{Y}_t + 0.5 \hat{S}_t$, home country policy fixed at $i_t = \bar{i} + 5.0 p_t + 0.0 \hat{Y}_t + 0.0001 \hat{S}_t$.

Figure 1: Welfare Paths (Benchmark Cases)



The paths are for overall-welfare computed in terms of the change in steady state consumption that would have the equivalent effect.